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Combined Moho Estimators

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ABSTRACT

In this study, we developed three estimators to optimally combine seismic and gravimetric models of Moho surface. The first estimator combines them by their special harmonic coefficients; the second one uses the spherical harmonic coefficients of the seismic model and use integral formula for the gravimetric one. The kernel of the integral terms of this estimator shows that a cap size of 20° is required for the integration, but since this integral is presented to combine the low frequencies of the gravimetric model, a low resolution model is enough for the integration. The third estimator uses the gravity anomaly and converts its low frequencies to those of the gravimetric Moho model, meanwhile combining them with those of seismic one. This integral requires an integration domain of 30° for the gravity anomalies but since the maximum degree of this kernel is limited to a specific degree, the use of its spectral form is recommended. The kernel of the integral involving the gravity anomalies, developed for recovering high frequencies of Moho, is written in a closed-from formula and its singularity is investigated. This kernel is well-behaving and decreases fast, meaning that it is suitable for recovering the high frequencies of the Moho surface.

Keywords: Seismic and Gravimetric Model, Spectral Combination, Optimal Estimation, Integral Estimators.

1. INTRODUCTION

The Mohorovičić discontinuity (Moho) is the boundary between the Earth's crust and mantle. This boundary can be determined by isostatic/gravimetric and seismic methods. The masses above Moho are called the Earth's crust. In 1909, Andrija Mohorovičić, a seismologist, used the seismic waves to discover the crust-mantle boundary. Moho separates the oceanic as well as the continental crusts from the underlying mantle. By accurate definition, it is a physical/chemical boundary between the crust and mantle defined by their material properties. The value of physical quantities of seismic wave velocity, density, pressure and temperature will change from one environment to another (Mooney et al. 1998; Martinec 1994) and Moho is the boundary at which these quantities change.

Several isostatic hypotheses exist for the shape of Moho and the density of the Earth's crust. In most of them, the crust is assumed as columns with a specific density floating on the viscous mantle located in a certain depth inside of it, which is called compensation depth. Pratt (1855) and Airy (1855) models are well-known in isostatic theories. In the former, the mass of crustal column is assumed to be variable and the columns have the same compensation depth. The latter assumes that the columns have the same density, but with different compensation depths. Both hypotheses are highly-idealised due to assuming that the compensation is strictly local. Vening Meinesz (1931) modified Airy's hypothesis by defining a regional instead of the local compensation. Parker (1972) presented a practical iterative gravimetric method based on a constant density contrast and a varying Moho depth, similar to Vening Meinesz's hypothesis, in the Fourier domain and planar approximation. Due to the instability of this method, Oldenburg (1974) added a filter in the frequency domain to stabilise the solution. The combination of these two methods was generalised to three dimensions by Gomes-Ortiz and Agarwal (2005) and Shin et al. (2007). Kiamehr and Gomes-Ortiz (2009) applied this three-dimensional method and estimated the Moho depth in Iran, based on the terrestrial gravimetric data and the Earth gravity model EGM08 (Pavlis et al. 2008). Čadak and Martinec (1991) presented the first global model of Moho in terms of the spherical harmonics to degree and order 30, based on different sources of seismic data. Martinec (1993 and 1994) studied the determination of the density contrast between the mantle and crust by minimising the sum of the squares of the gravitational potentials induced by the Earth's topographic masses and Moho. Sünkel (1985) converted the Airy-Heiskanen Moho depth to the Vening Meinesz by smoothing it further, so that the global mean squared error of the difference between disturbing and topographic-isostatic potentials is minimised. Moritz (1990, Sect. 8) generalised the Vening Meinesz method to a global compensation. Sjöberg (2009) formulated this problem as a nonlinear Fredholm integral of equation of the first kind, and he presented some approximate and practical solutions for crustal thickness by gravimetric data. Sjöberg and Bagherbandi (2011) presented a method for the estimation of the density contrast using EGM08 and CRUST2.0 (Bassin et al. 2000). Similarly, Tenzer et al. (2012) studied this issue by including the ice model, ICE-5G. Eshagh (2009, 2010) studied the effect of lateral density variation of the crustal and topographic masses on the satellite gradiometry data (SGD).

The current lack of knowledge about the density structure within the Earth's crust is a major limiting factor of estimating the Moho density interface accurately. The crustal density contrast stripping corrections were applied systematically to the topographically-corrected gravity field using CRUST2.0 in Tenzer et al. (2009). Moritz (1990) improved the Vening Meinesz hypothesis by developing it to a spherical Earth model with some approximations which were not suitable for inverting the gravity data. Sjöberg (2009) reformulated this problem in a more proper way and presented a new solution for the theory of Moritz, naming it the Vening Meinesz-Moritz (VMM) inverse problem in isostasy. He presented some iterative as well as approximate spherical harmonic solutions. Bagherbandi and Sjöberg (2011) compared the gravimetric local Airy-Heiskanen and the VMM models with the seismic CRUST2.0 global model for estimating the Moho depth. They found that the VMM model is consistent with the Moho model of CRUST2.0 with a global uncertainty of 7 km. Braitenberg et al. (2000) presented an iterative inversion method to obtain the variation of Moho in the Tibet plateau. Their investigation shows that the results of the gravity inversion for the Moho recovery and the seismic Moho model have an uncertainty of about ±5 km. This tolerance is because of disregarding dynamic effects involved in Tibet such as uplift is underlying of Tibet by the Indian plate. In another study, Braitenberg et al. (2006) formulated a crustal model of South China Sea by constrained forward and inverse gravity modelling from the combined analysis of models of satellite gravity field, bathymetric, sediment and crustal thicknesses and the isostatic flexure. In Shin et al. (2007) a recovery of the Moho depth from satellite data was performed. They presented an updated model of the Moho undulations in an extended area using an improved gravity data set, which at long wavelengths relies entirely on the new results of the gravity recovery and climate experiment (GRACE) (Tapley et al. 2005) mission. They tried to invert the gravity anomalies using the Parker-Oldenburg method (Oldenburg 1974). Sampietro (2009) studied the problem of recovering the Moho depth from the SGD data in a simulation study and found that the Moho model can be recovered from SGD with an accuracy of about 2 km.

Braitenberg and Ebbing (2009) used the GRACE data and terrestrial gravity data to study the structure of the crust. They presented a three-dimensional method based on forward modelling which uses a priori known crystal structure. The gravity data of the study area are estimated using this method and are compared to the observed ones. Their residuals were used for identification of density anomalies not previously recognised. Sampietro (2011) considered the local inversion of the SGD by simulating a Moho surface and generating the SGD from them. However, some planar approximations were used in his formulation and the problems of spatial truncation error of the integral formulae and the behaviour of their kernels were not considered. Sampietro and Reguzzoni (2011) implemented a method based on collocation and fast Fourier transform to evaluate the gravity field and steady-state ocean circulation explorer (GOCE) (ESA 1999) data in the Moho estimation. They considered the spatial truncation error by applying a region larger than the study area. Reguzzoni and Sampietro (2012) presented a global crustal model based on the GOCE data. Bagherbandi and Eshagh (2011 and 2012) investigated this issue by reformulating the VMM theory according to the second-order derivative of the disturbing potential instead of the gravity anomaly and Earth gravity models. They obtained a nonlinear integral equation and solved it iteratively, using Tikhonov Regularisation (Tikhonov 1963). Barzaghi et al. (2013) presented a collocation-based method for combining the global Moho model derived from GOCE data and the local one by terrestrial data. Eshagh (2014a) presented a linear approach for estimating, the Moho discontinuity in Iran from the SGD.

The main assumption in the gravimetric-isostatic way of determining the Moho model is the compensation of the topographic potential by the potential of the masses beneath it. A shortcoming of the gravimetric method is the unrealistic assumption of a constant density contrast between the crust and mantle. Moreover, a proper selection of the mean depth of Moho is another critical issue as the Moho undulations are generated around it from the variation of the gravity. Obviously, such simple assumptions are not enough for determining such a surface. Some geophysical phenomenae, such as tectonic motion, post-glacial rebound, and mantle convection/thermal compensation and so on are influential as well. Such dynamic isostatic effects should be considered (Bagherbandi 2011) as additional corrections to Moho for its quality improvement. Nevertheless, formulating these effects is not usually straightforward. Seismic data measurements are costly with a limited coverage and most of them are located on lands. The CRUST2.0 model (Mooney et al. 1998) has a resolution of $2^{\circ}\times 2^{\circ}$, but if we look at the coverage of the seismic measurements used by Mooney et al. (1998), we will see even some land areas which are not covered well and the Moho model of these areas are mainly computed just by interpolation/extrapolation. Missing the aforementioned geophysical phenomenae for gravimetric Moho modelling and the sparseness of the seismic data motivated Eshagh et al. (2011) to combine the gravimetric and seismic models in an optimal way globally. Later, Eshagh and Bagherbandi (2012) combined these models locally and described quality measures for them. Reguzzoni et al. (2013) combined the seismic model of CRUST2.0 and the one derived from GOCE and presented a new combined Moho model.

Here, our idea is to combine these two surfaces in another way using the Butterworth filter. Since the signal spectra of the gravimetric and seismic models of Moho do not completely coincide, we can determine which model should be used to which degree by using this filter. Cutting the seismic spectra to a certain degree and replacing the higher degrees from gravimetric spectra is not reasonable as a jump will occur in the signal spectra of Moho. The Butterworth filter will somehow smooth the signal so that the spectra migrate gradually from seismic to the gravimetric signal. This idea has been applied by Haagmans (2000) and Bagherbandi (2011) for the generation of synthetic Earth models.

Amongst different methods for data assimilation and combination, there is a method called spectral combination (Sjöberg 1980, Wenzel, 1982) and widely used for geoid determination. This method has also been used for data combination and gravity data refinement by Kern et al. (2003), Eshagh (2013, 2014b) and for mixing the boundary-value problems (Eshagh 2011, 2012). In this study, we will use this idea to combine the signal spectra of the Moho models and after that we use their error spectra for their optimal weighting using the spectral combination theory. Moreover, a combining integral estimator is presented to assimilate the gravity anomaly and a global model of seismic Moho.

2. RESULTS AND DISCUSSION

2. 1. COMBINATION OF TWO EXISTING GRAVIMETRIC AND SEISMIC MOHO MODELS

In this section, we present two types of combined Moho estimator. The first one uses the spherical harmonic coefficients of the Moho models, meaning that the spherical harmonic coefficients and their corresponding errors should be primarily available. We call this method combination in the spectral domain. The second type of the estimator uses the spherical harmonic coefficients of the seismic model, but uses the gravimetric Moho model instead of its harmonic coefficients. In the following section, we will present the theoretical issues of these methods.

2. 1. 1. COMBINATION IN SPECTRAL DOMAIN

Consider the following combined Moho estimator:

$$\tilde{T}=\sum_{n=0}^{M}a_n B_n T_n^{\mathrm{S}}+\sum_{n=0}^{M}b_n B'_n T_n^{\mathrm{G}}+\sum_{n=M+1}^{\infty}T_n^{\mathrm{G}} \quad (1)$$

where T_n^{S} and T_n^{G} are the Laplace harmonics of the seismic and gravimetric Moho models, M is the maximum degree of them, a_n and b_n are the spectral coefficients to be estimated, B_n is the Butterworth filter and $B'_n=\sqrt{1-B_n^2}$ (cf. Haagmans 2000). If we consider $\varepsilon_n^{\mathrm{S}}$ and $\varepsilon_n^{\mathrm{G}}$ as the errors of T_n^{S} and T_n^{G}, respectively, the error of the estimator (1) will be:

$$\delta\tilde{T}=\sum_{n=0}^{M}a_n B'_n\varepsilon_n^{\mathrm{S}}+\sum_{n=0}^{M}b_n B_n\varepsilon_n^{\mathrm{G}}+ +\sum_{n=M+1}^{\infty}\varepsilon_n^{\mathrm{G}}+\sum_{n=0}^{M}(a_n+b_n-1)T_n \quad (2)$$

where T_n is Laplace harmonic of the true value of the Moho depth.

Note that we consider B'_n and B_n as the coefficients of $\varepsilon_n^{\mathrm{S}}$ and $\varepsilon_n^{\mathrm{G}}$, respectively, for the Butterworth filter B_n acts on the decreasing signal of Moho, while the error of Moho is an increasing function. If B_n is applied to the Moho signal, it reduces the errors after the degree of the filter and higher frequencies of the signal will get higher weights than the lower ones. This makes the high frequencies of the seismic Moho become stronger in the combination which is not a desired property for our estimator. In the case of using B'_n instead of B_n, the lower frequencies will have smaller errors and stronger contribution to the estimator. A similar argument can be made for the gravimetric Moho model. According to the properties of $\mathrm{E}\left\{\varepsilon_n^{\mathrm{S}}\right\}=\mathrm{E}\left\{\varepsilon_n^{\mathrm{G}}\right\}=0$ and $\mathrm{E}\left\{\varepsilon_n^{\mathrm{S}}\varepsilon_{n'}^{\mathrm{S}}\right\}=0$ for $n \neq n'$, E{} stands for the statistical expectation, and by squaring Eq. (2), taking the statistical expectation and the global average operator M, we have:

$$\mathrm{M}\left(\mathrm{E}\left\{\delta\tilde{T}\right\}^{2}\right)=\frac{1}{4\pi}\iint_{\sigma}\mathrm{E}\left\{\delta\tilde{T}\right\}^{2}d\sigma= \\ =\sum_{n=0}^{M}a_n^2B_n'^2dc_n^{\mathrm{S}}+\sum_{n=0}^{M}b_n^2B_n^2dc_n^{\mathrm{G}}+ \\ +\sum_{n=M+1}^{\infty}dc_n^{\mathrm{G}}+\sum_{n=0}^{M}\left(a_n+b_n-1\right)^2c_n\left(T\right) \tag{3}$$

where $dc_n^{\mathrm{S}}=\mathrm{M}\left(\mathrm{E}\left\{\varepsilon_n^{\mathrm{S}}\varepsilon_n^{\mathrm{S}}\right\}\right)$, $dc_n^{\mathrm{G}}=\mathrm{M}\left(\mathrm{E}\left\{\varepsilon_n^{\mathrm{G}}\varepsilon_n^{\mathrm{G}}\right\}\right)$ and $c_n\left(T\right)=\mathrm{M}\left(\mathrm{E}\left\{T_nT_n\right\}\right)$ are the error degree variances of the seismic and gravimetric models and $c_n\left(T\right)$, the signal degree variance of the Moho. Equation (3) is the global mean squared error of the estimator (1), in which the first three terms are related to the random errors and the last one is the bias of the estimator due to its deviation from the true value of Moho. In order to estimate a_n and b_n under the condition that the bias term has no effect on the estimator (1), we use the following constraint:

$$a_n+b_n-1=0\cdot \tag{4}$$

If we solve Eq. (4) for a_n and insert a_n back into Eq. (3), we obtain:

$$\mathrm{M}\left(\mathrm{E}\left\{\delta\tilde{T}\right\}^{2}\right)=\sum_{n=0}^{M}\left(1-b_n\right)^2B_n'^2dc_n^{\mathrm{S}}+ \\ +\sum_{n=0}^{M}b_n^2B_n^2dc_n^{\mathrm{G}}+\sum_{n=M+1}^{\infty}dc_n^{\mathrm{G}}\cdot \tag{5}$$

Taking the derivative of Eq. (5) with respect to b_n, equating the result to zero and solving it for b_n, we have:

$$b_n=\frac{B_n'^2dc_n^{\mathrm{S}}}{B_n'^2dc_n^{\mathrm{S}}+B_n^2dc_n^{\mathrm{G}}}. \tag{6}$$

According to Eqs. (6) and (4), it is straightforward to obtain:

$$a_n=\frac{B_n^2dc_n^{\mathrm{G}}}{B_n'^2dc_n^{\mathrm{S}}+B_n^2dc_n^{\mathrm{G}}}. \tag{7}$$

2. 1. 2. COMBINED MOHO INTEGRAL ESTIMATOR

The estimator (1) can be written in terms of an integral formula in the spatial domain when the gravimetric Moho model is locally available. To do so, let us write the estimator (1) in the following form:

$$\tilde{T}=\sum_{n=0}^{M}a_nB_nT_n^{\mathrm{S}}+\sum_{n=0}^{M}u_nT_n^{\mathrm{G}} \tag{8}$$

where

$$u_n=\begin{cases}b_nB_n' & n\le M\\ 1 & n<M\end{cases}$$

by considering (Heiskanen and Moritz 1967, P. 34)

$$T_n^{\mathrm{G}}=\frac{2n+1}{4\pi}\iint_{\sigma}P_n\left(\cos\psi\right)T'^{\mathrm{G}}d\sigma \tag{9}$$

we can rewrite the estimator (8) as:

$$\tilde{T}=\sum_{n=0}^{M}a_nB_nT_n^{\mathrm{S}}+ \\ +\frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}\left(2n+1\right)u_nP_n\left(\cos\psi\right)T'^{\mathrm{G}}d\sigma\cdot \tag{10}$$

The kernel of the integral term in the r.h.s of Eq. (10) does not have any closed-form formula and one has to use its spectral form and generate it to very high degrees. Instead, we use the following practical technique to avoid such a burden:

$$\tilde{T}=\sum_{n=0}^{M}a_nB_nT_n^{\mathrm{S}}+ \\ +\frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}\left(2n+1\right)\left(u_n-1\right)P_n\left(\cos\psi\right)T'^{\mathrm{G}}d\sigma+ \\ +\frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}\left(2n+1\right)P_n\left(\cos\psi\right)T'^{\mathrm{G}}d\sigma\cdot \tag{11}$$

The kernel of the third term in the r.h.s of Eq. (11) is nothing but the spherical Dirac function, therefore; we can simply write:

$$\tilde{T}=T^{\mathrm{G}}+\sum_{n=0}^{M}a_nB_nT_n^{\mathrm{S}}+ \\ +\frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}\left(2n+1\right)\left(u_n-1\right)P_n\left(\cos\psi\right)T'^{\mathrm{G}}d\sigma\cdot \tag{12}$$

On the other hand, the value of the kernel of the estimator (12) will be zero for the degrees higher than M and consequently, we do not have to compute it to high degrees.

If the seismic and gravimetric Moho models are locally available, we can use the following integral estimator to combine them:

$$\tilde{T} = T^{\mathrm{G}} + \frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}(2n+1)a_n B_n P_n(\cos\psi)T'^{\mathrm{S}}d\sigma + \frac{1}{4\pi}\iint_{\sigma}\sum_{n=0}^{\infty}(2n+1)(u_n-1)P_n(\cos\psi)T'^{\mathrm{G}}d\sigma \cdot \tag{13}$$

One issue that should be mentioned here is that the error spectra of the gravimetric Moho can simply be generated by the Sjöberg's (1986) method. However, this method is very simple and may not be realistic.

2. 2. COMBINED INTEGRAL ESTIMATORS WITH GRAVITY ANOMALY AND SEISMIC MODEL

Now the idea is to use the spherical harmonic expansion of the gravimetric Moho model presented by Sjöberg (2009). We should state that this expression delivers an approximate Moho model and two more corrections are required to increase its high frequencies. However, since the Moho surface is smooth, we can consider that the contribution of high frequencies is small. Also, Eshagh (2014a) showed that the effect of neglecting these two terms causes an error of less than 2 km which is small compared to the magnitude of the Moho depth. Sjöberg's (2009) spherical harmonic expansion of the Moho model is:

$$T^{\mathrm{G}} = \sum_{n=0}^{\infty} T_n^{\mathrm{G}} \tag{14}$$

where

$$T_n^{\mathrm{G}} = \frac{-A_{C_0}}{4\pi k}\delta_{n0} + 2\pi v_n(\mu H)_n - v_n \Delta g_n \tag{15}$$

and

$$v_n = \frac{1}{4\pi k}\frac{2n+1}{n+1}. \tag{16}$$

Since the zero- and first-degree harmonics of Moho come from the topographic information, we try to separate them from the formula. If we assume that the topographic information is errorless, we can write the combined estimator as:

$$\tilde{T} = B_0 T_0^{\mathrm{S}} + B_1 T_1^{\mathrm{S}} - \frac{A_{C_0}}{4\pi k} + 2\pi\sum_{n=0}^{M} v_n B'_n(\mu H)_n + \sum_{n=2}^{M} a_n B_n T_n^{\mathrm{S}} - \sum_{n=2}^{M} v_n b_n B'_n \Delta g_n - \sum_{n=M+1}^{\infty} v_n \Delta g_n \cdot \tag{17}$$

In order to transfer Eq. (17) from the spectral domain to the spatial domain, we use Eq. (9) for Δg_n. If we merge the last two terms in the r.h.s of Eq. (17), we obtain:

$$\tilde{T} = B_0 T_0^{\mathrm{S}} + B_1 T_1^{\mathrm{S}} - \frac{A_{C_0}}{4\pi k} + 2\pi\sum_{n=0}^{M} v_n B'_n(\mu H)_n + \sum_{n=2}^{M} a_n B_n T_n^{\mathrm{S}} - \frac{1}{4\pi}\iint_{\sigma}\sum_{n=2}^{\infty}(2n+1)v_n k_n P_n(\cos\psi)\Delta g' d\sigma \tag{18}$$

where

$$k_n = \begin{cases} b_n B'_n & n \le M \\ 1 & n < M \end{cases}. \tag{19}$$

There is no closed-form formula for the kernel of the integral term of Eq. (18) and the kernel should be generated by its spectral form.

Now, the issue is the proper weighting of the seismic Moho model and gravity anomalies for the degrees higher than two. The error of the estimator (18) is:

$$\delta\tilde{T} = B_0\varepsilon_0^{\mathrm{S}} + B_1\varepsilon_1^{\mathrm{S}} + \sum_{n=2}^{M} a_n B'_n \varepsilon_n^{\mathrm{S}} - \sum_{n=2}^{M} v_n b_n B_n \varepsilon_n^{\Delta g} - \sum_{n=M+1}^{\infty} v_n \varepsilon_n^{\Delta g} + \sum_{n=2}^{M} a_n B_n T_n + \sum_{n=2}^{M} b_n B'_n v_n\left(2\pi(\mu H)_n - \Delta g_n\right) + \sum_{n=M+1}^{\infty} v_n\left(2\pi(\mu H)_n - \Delta g_n\right) - \sum_{n=2}^{\infty} T_n \tag{20}$$

and after further simplifications:

$$\delta\tilde{T} = B_0\varepsilon_0^{\mathrm{S}} + B_1\varepsilon_1^{\mathrm{S}} + \sum_{n=2}^{M} a_n B'_n \varepsilon_n^{\mathrm{S}} - \sum_{n=2}^{M} v_n b_n B_n \varepsilon_n^{\Delta g} - \sum_{n=M+1}^{\infty} v_n \varepsilon_n^{\Delta g} + \sum_{n=2}^{M}(a_n + b_n - 1)T_n \cdot \tag{21}$$

The spectral coefficients are defined for the degrees between 2 and *M*+1. Therefore, the errors spectra of the data amongst these degrees should be considered and the error of the first- and second-degree terms of the seismic model will be added to the error model but they do not have any role in the computation of the spectral coefficients. In fact, they just contribute to the total error of the estimate and have no influence on the estimated spectral coefficients. It should be stated that the last term in Eq. (21) is nothing but the bias of the estimator (18).

The mean squared error of Eq. (21) becomes:

(22)
$$\mathrm{M}\left(\mathrm{E}\left\{ \delta\tilde{T}^2 \right\} \right) = B_0 dc_0^{\mathrm{S}} + B_1 dc_1^{\mathrm{S}} + \sum_{n=2}^{M} a_n^2 B_n'^2 dc_n^{\mathrm{S}} + \sum_{n=2}^{M} v_n^2 b_n^2 B_n^2 dc_n^{\Delta g} + \sum_{n=M+1}^{\infty} v_n^2 c_n^{\Delta g} + \sum_{n=2}^{M} \left(a_n + b_n - 1 \right)^2 c_n(T).$$

In order to estimate a_n and b_n in such a way that the estimator (18) becomes unbiased, we consider the following constraint:

(23)
$$a_n + b_n - 1 = 0\,.$$

This constraint removes the dependency of the estimated a_n and b_n to the true degree variance of Moho which is never available. If we solve Eq. (23) for a_n, insert the result back into Eq. (22), take the derivative with respect to b_n and solve the final result for it, we obtain:

(24)
$$b_n = \frac{B_n^2 v_n^2 dc_n^{\mathrm{G}}}{B_n'^2 dc_n^{\mathrm{S}} + B_n^2 v_n^2 dc_n^{\mathrm{G}}}\,.$$

and according to Eq. (23) we have:

(25)
$$a_n = \frac{B_n'^2 dc_n^{\mathrm{S}}}{B_n'^2 dc_n^{\mathrm{S}} + B_n^2 v_n^2 dc_n^{\mathrm{G}}}\,.$$

The integral in the r.h.s of the estimator (18) has a kernel in the spectral form and is time-consuming to generate it to high degrees. However, the kernel of this integral formula can be written as:

(26)
$$\sum_{n=2}^{\infty} (2n+1) v_n k_n P_n(\cos\psi) = \sum_{n=2}^{\infty} \frac{(2n+1)^2}{n+1} P_n(\cos\psi) + \sum_{n=2}^{\infty} \frac{(2n+1)^2}{n+1} \left(b_n B_n' - 1 \right) P_n(\cos\psi)\cdot$$

The first series in the r.h.s is shown by $K(\psi)$ and can be written as:

(27)
$$K(\psi) = \sum_{n=2}^{\infty} \left(4n + \frac{1}{n+1} \right) P_n(\cos\psi) = 4\sum_{n=2}^{\infty} n P_n(\cos\psi) + \sum_{n=2}^{\infty} \frac{P_n(\cos\psi)}{n+1}\,.$$

According to Martinec (2003):

(28)
$$\sum_{n=0}^{\infty} n P_n(\cos\psi) = -\frac{\sin^2\frac{\psi}{2}}{8\sin^3\frac{\psi}{2}}$$
$$\sum_{n=0}^{\infty} \frac{P_n(\cos\psi)}{n+1} = \ln\left(\frac{1+\sin\frac{\psi}{2}}{\sin\frac{\psi}{2}} \right).$$

It should be noted that these closed-form formulae contain the zero- and first-degree terms which should be removed from the kernels presented in Eq. (28). Substituting Eq. (28) into Eq. (29) considering Eq. (26) and removing the zero- and first-degree terms leads to:

(29)
$$K(\psi) = -\frac{1}{2\sin\frac{\psi}{2}} + \ln\left(\frac{1+\sin\frac{\psi}{2}}{\sin\frac{\psi}{2}} \right) - \frac{9}{2}\cos\psi - 1\cdot$$

Finally, the combined Moho estimator becomes:

(30)
$$\tilde{T} = B_0 T_0^{\mathrm{S}} + B_1 T_1^{\mathrm{S}} - \frac{A_{C_0}}{4\pi k} + 2\pi \sum_{n=0}^{M} v_n B_n' \left(\mu H \right)_n + \sum_{n=2}^{M} a_n B_n T_n^{\mathrm{S}} - \frac{1}{16\pi^2 k} \iint_{\sigma} K(\psi)\, \Delta g' d\sigma - \frac{1}{16\pi^2 k} \iint_{\sigma} \sum_{n=2}^{\infty} \frac{(2n+1)^2}{n+1} \left(b_n B_n' - 1 \right) P_n(\cos\psi)\, \Delta g' d\sigma\cdot$$

$K(\psi)$ is singular at $\psi = 0$; therefore, this singularity should be removed from the integral by the following technique:

(31)
$$\frac{1}{16\pi^2 k} \iint_{\sigma} K(\psi)\, \Delta g' d\sigma = \frac{1}{16\pi^2 k} \iint_{\sigma-\sigma_0} K(\psi)\, \Delta g' d\sigma + \frac{\Delta g}{16\pi^2 k} \iint_{\sigma_0} K(\psi)\, d\sigma$$

σ_0 is the small integration domain around the singular point and its size depends in the resolution of the data being integrated.

The advantage of using Eq. (30) instead of Eq. (18) is to have the spectral form of a kernel up to the maximum degree M and the generation and integration using such an integral is more efficient that the one presented in Eq. (18).

2. 3. NUMERICAL INVESTIGATIONS

Here, we divide our numerical studies into two parts. First, the combination of the Moho models in the spectral domain is considered and in the second part the behaviour of the kernels of the integral estimators is presented and interpreted.

2. 3. 1. NUMERICAL ASPECTS OF COMBINATION OF SPECTRAL FORMS OF MOHO MODELS

We suppose that spherical harmonic expansions of the seismic and gravimetric models of Moho are available with their errors. Our goal is to combine these two models in an optimum way, but since the signal spectra of them do not coincide, we should somehow relate them together. We know that the seismic data of CRUST2.0, which have been used for computing the spherical harmonic coefficients, are very geographically-limited, i.e. there are large gaps in the data over oceans and in some continental parts. Therefore, considering even the degree and order of 90 for the seismic Moho model seems to be unreasonable, as there are unrealistic frequencies in the signal. Consequently, our estimator should take the frequencies from the seismic model and low higher ones from the gravimetric one. Figure 1a presents the error spectra of both models and, as it is observed, the error spectra of the gravimetric model is smaller than the seismic one and there is no doubt that if the spectral combination method is used, as a combination strategy, the results will be closer to the gravimetric model due to its small errors. This is not the property that we are going to have for our combined Moho model as we want to take the low degrees from the seismic and the high ones from the gravimetric model. Therefore, neither of methods of spectral combination, and the Butterworth filter, is suited for our purpose, for in the former the gravimetric Moho will have more contribution to the results than the seismic one, even for the low degrees and the former keeps the low degrees but cannot add more frequencies to the combined signal. The combination of the Butterworth and spectral combination approaches can be useful to keep both properties. Figure 1b shows the signal spectra of seismic and gravimetric signals, combined signal using the Butterworth filter to degree and order 20 and the Butterworth spectrally-combined signal. As the plot illustrates, the combined signal, more or less, coincides with the seismic model to degree and order 20 and after that reduces its power to degree and order 90. The Butterworth spectrally-combined signal reduces in degrees lower than 20 due to the signal weighting and since the error spectra of the gravimetric signal is smaller than those of seismic one, it is normal to see that the combined signal gets closer to the gravimetric signal in the high degrees. This could be the reason for the signal power reduction before degree 20. However, we should mention that the Butterworth spectrally-combined signal is an optimal estimation for the Moho depths from a statistical point of view. Furthermore, the combined signal is closer to the gravimetric signal at high degrees and for degrees higher than 90, one can simply use the gravimetric signal without using another filtering step.

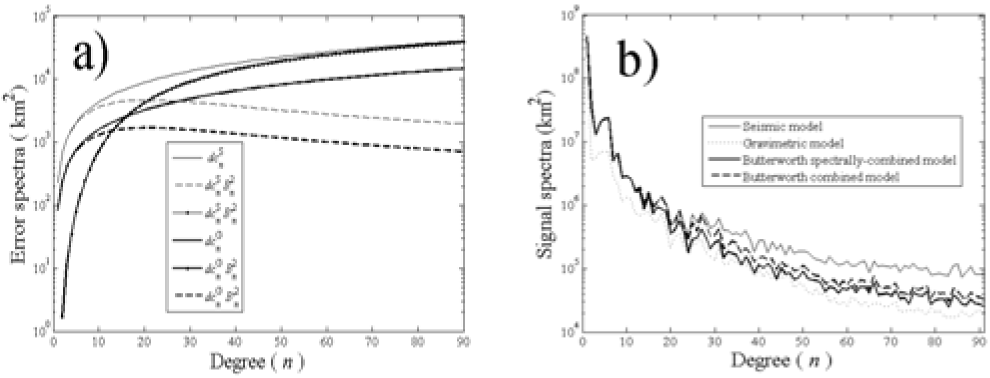

Fig 1. a) Error spectra of seismic and gravimetric models,
b) seismic, gravimetric, Butterworth and Butterworth and spectrally-combined model

Table 1. Statistics of global Moho models and their combined ones. Unit: 1 km.

	Max	Mean	Min	STD
Seismic	73.0	23.0	2.1	12.7
Gravimetric	51.5	22.2	8.1	8.3
Butterworth	74.1	23.0	5.2	12.7
Butterworth+spectral combinstion	73.9	23.0	6.3	12.4

Table 1 shows the statistics of seismic and gravimetric Moho models and their combined ones. Large differences between the statistics of the seismic and gravimetric models are seen. The mean value of these models differs by about 10 km. Also, the maximum value of the gravimetric model is smaller than the seismic one by about 20 km and its minimum is 6 km larger. The gravimetric model seems to be much smoother than the seismic one. In the case of combining them using the Butterworth filter, we observe that by taking the low frequencies of Moho from the seismic data, the mean value of the combined model becomes the same as those of seismic model and the maximum value becomes closer to the maximum value of it. However, the minimum of the combined model becomes larger. Since the error spectra of the gravimetric model is smaller than the seismic one, it is quite normal as by considering the errors of both models the combined model will be slightly closer to the gravimetric one.

2. 3. 2. BEHAVIOUR OF KERNELS OF INTEGRAL MOHO ESTIMATORS

The behaviour of the kernels of the combined Moho integral estimators is very important, as they show the significance of the contribution of the data to the estimated Moho depth. If the kernel has large values around the computation points, or a geocentric angle of zero, therefore the contribution of the far-zone data at the integration points will not be significant in the result. In such a case, a small cap size is required for performing the integration. In our study, two types of combined Moho estimators were presented. In the first one, we assume that two types of seismic and gravimetric Moho models are already available which are combined. Since the seismic Moho model has not a dense distribution of data, its spherical harmonic expansion cannot be higher than the data resolution. In our case, we use CRUST2.0 which has a resolution of $2^\circ\times2^\circ$, corresponding to the maximum degree and order of 90 in its spherical harmonic expansion. Unlike the seismic model, the gravimetric one is assumed to be dense, but without a global coverage. The estimator (12) uses the gravimetric Moho model for recovering the high frequencies of the combined Moho and the spherical harmonic coefficients of the seismic model to the degree and order 90 for low frequencies. The behaviour of kernel of the integral term of this estimator is presented in Figure 2a and it shows that the kernel becomes close to zero around a geocentric angle of 20° which is quite large. However, since the kernel is in the spectral form and limited to degree 90, using a denser resolution for the gravimetric model is not required and the integration of a grid of $2^\circ\times2^\circ$ of them for this integral should be enough. In fact, the kernel acts as a low pass filter and integration of such a data will not be difficult practically, even in large areas. The estimator (30) is different from Eq. (12) as it uses gravity anomaly directly to estimate combined Moho depth. It contains two integral terms and the first one has a closed-form formula for integrating the gravity anomalies and the second one a kernel in spectral form up to degree and order 90 which computes a gravimetric Moho model from gravity anomalies, meanwhile combining it with a seismic model of Moho to this degree. The kernel of the first integral is plotted in Figure 2b, and as it shows, the kernel decreases, which means that the contribution of far-zone anomalies is not very significant because this part of the estimator is carrying the high frequencies of the result. The opposite is true for the kernel of the second integral term. The kernel goes to zero after a geocentric angle of 30° meaning that the coverage of gravity anomaly should be 30° larger than the desired area. Having such large extent terrestrial gravity anomalies is not easy and on the other hand the kernel acts as a low pass filter of the anomalies to degree 90. Therefore, it is easier to write the estimator (30) in the following form:

$$\tilde{T} = B_0 T_0^{S} + B_1 T_1^{S} - \frac{A_{C_0}}{4\pi k} + 2\pi \sum_{n=0}^{M} v_n B'_n \left(\mu H\right)_n + \sum_{n=2}^{M} a_n B_n T_n^{S} - \frac{1}{64\pi^3 k} \iint_{\sigma} K\left(\psi\right) \Delta g' d\sigma - \frac{1}{16\pi^2 k} \sum_{n=2}^{M} \frac{2n+1}{n+1} \left(b_n B'_n - 1\right) \Delta g_n . \quad (32)$$

Therefore, a gravity model can be simply used for generating Δg_n to degree and order $M = 90$.

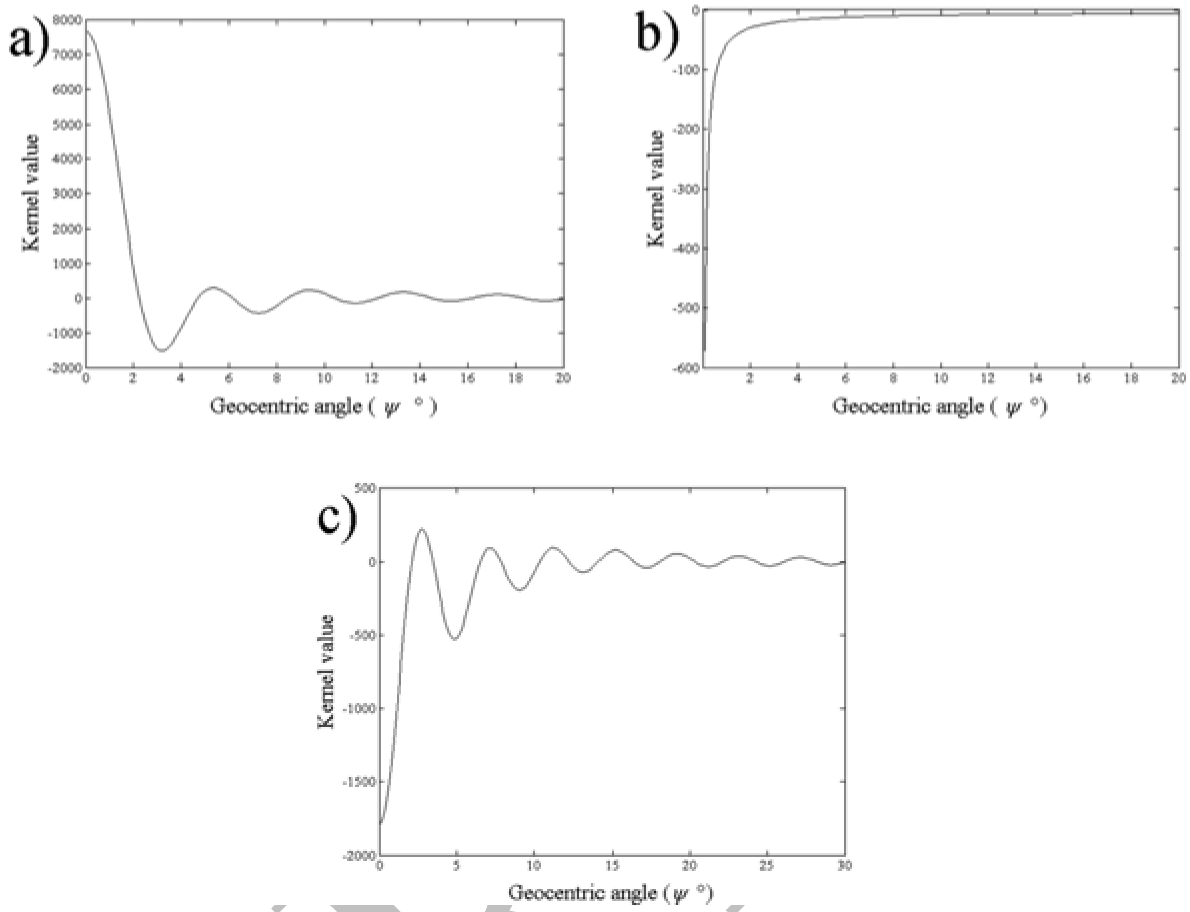

Fig 2. Kernel behaviour of a) integral of Eq. (12), b) Eq. (28) and c) last integral of Eq. (30)

3. CONCLUSIONS

Amongst the three Moho estimators, presented in this study, the first one combines two seismic and gravimetric models in the spectral domain based on the accuracies of their spherical harmonic coefficients. Since the gravimetric Moho can have high resolution, its related term was transferred into the spatial domain or the integral form. The kernel of the integral goes to zero around a geocentric angle of 20°, for this integral is responsible for combining those frequencies of the gravimetric Moho model which are lower than the maximum degree of the seismic model. The high frequencies are taken directly from the gravimetric model. The estimator (30) converts the terrestrial gravity anomalies to gravimetric Moho by an integral formula and at the same time combines its low frequencies with those of the seismic model. The kernel of the integral carrying the high frequencies of the Moho model decreases fast by the distance from the computation point, whilst the one for combining and integrating the low frequencies needs the anomalies to a geocentric angle of 30°, which is not practical. The integral part was transferred into the spectral from so that an existing global gravity model can be used for expressing the low frequencies instead of the gravity anomalies.

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