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OPTIMAL CONTROL OF A ROBOTIC SYSTEM WITH TWO DEGREE OF FREEDOM

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ABSTRACT

A simple improvisation technique for designing a linear quadratic regulator (LQR) optimal controller for a robotic pan and tilt platform (PTP) with two degrees of freedom (DOF) has been proposed in this paper. Newton-Euler linear model of this robotic system has been stabilized to obtain the desired performance criteria via LQR. The performance of the proposed LQR controller is highlighted through comparisons with the existing proportional derivative (PD) and lead Compensator controllers on account of both steady state and transient response parameters.

KEYWORDS: Linear Quadratic Regulator, Newton-Euler Equation, Robotic System With 2- DOF, Transient Response Analysis

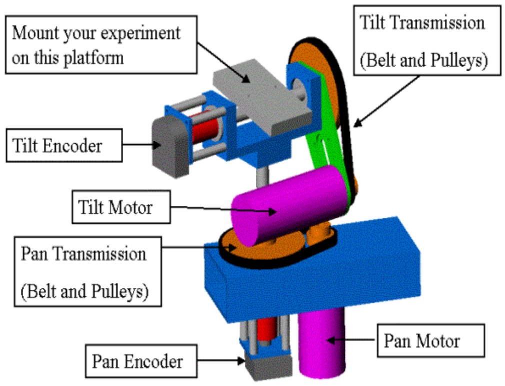
1. INTRODUCTION

Camera robotics is an enormous field of engineering for identifying threat, reducing catastrophic events, to follow a moving object and also for automation and manufacturing. Pan and tilt platform (PTP) are widely used for these purpose. Pan and Tilt mechanism is basically a robotic manipulator having two degree of freedom [1]. A camera can be mounted on tilt platform as shown in Figure 1.

Figure 1: A Digital Camera Mounted on PTP

These cameras have been consistently used for representation of entire space. They are also used in border patrolling, recording of a moving object, search and rescue operation, automation and manufacturing. Modelling of PTP has been done by Newton-Euler equation [2,3].

Three dimensional representation of PTP has been presented in Figure 2. All physical parameters of the system havebeen obtained from experiments using Computer aided design (CAD) [2,3]. A linear model has been obtained after neglecting centrifugal forces and coulomb friction.



† Courtesy:http://catsfs.rpi.edu/~wenj/ECSE446S06/project_overview.html

Figure 2: Three Dimentional View of PTP

In order to capture maximum workspace and to obtain desired position and orientation of the camera, there is a need to design a robotic system which meets the following transient response specifications [2, 3].

- Settling time (T_s) $0.1 \le T_s \le 0.5$ seconds.
- Steady state error e_{ss} within ±2%.
- Percentage overshoot (M_p) < 22%.

The paper is arranged in following manner: Section 2 narrates dynamic and state space model of a pan and tilt mechanism. Section 3 describes response analysis of the system obtained in section 2. An optimal controller has been designed for both pan and tilt mechanisms in section 4 along with discussion on the step response analysis of the compensated system. Comparative study of various controllers have been shown in Section 5. Future scope of the obtained results has been discussed in the section 6.

2. PHYSICAL MODELING OF PTP

2.1. Dynamic Linear Model for Tilt Mechanism

The nonlinear model of Pan mechanism has been developed using Newton-Euler equation with few assumptions [2,3] as:

$$\tau = j_{eff}\ddot{\theta} + F_v\dot{\theta} + F_c \operatorname{sgn}(\dot{\theta}) + MGL.\sin(\theta), \tag{1}$$

Where τ=Torque, j_{eff} =Effective inertial load, F_v = Viscous friction, F_c= Coulomb friction, θ = Angle between the force (mg) and arm length L. The nonlinearity of the system is due to coulomb-friction and coriolis forces. These nonlinearities are neglected to obtain a linear model for stability analysis of the system in [2, 3, 4] as:

$$\tau = j_{eff}\ddot{\theta} + F_v\dot{\theta}, \tag{2}$$

Dynamic linear model of PTP forms a second order differential equation which can be determined by the Jacobean matrix [2,3]. Dynamic model can be converted into state space representation for the following state variables: $x_1 = \theta, x_2 = \dot{\theta}$ and $\dot{x}_1 = \dot{\theta}, \dot{x}_2 = \ddot{\theta}$ [5] as:

$$\dot{x}=\begin{bmatrix}0 & 1\\0 & -0.1578\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}u, \quad (3)$$

$$y=\begin{bmatrix}31.2495 & 0\end{bmatrix}x, \quad (4)$$

Transfer function for tilt mechanism can be obtained by state equations (3) and (4) as G(s) =C(S.I-A)$^{-1}$.B from [2,3,4] as:

$$G_{tilt}(s)=\frac{31.24954}{s^2+0.1578s}, \quad (5)$$

Similar procedure has been followed to obtain the transfer function of the pan mechanism.

2.2 Dynamic Linear Model For Pan Mechanism

The dynamic linear model of the pan mechanism which resembles the same second order differential equation after neglecting nonlinearities as

$$\tau = j_{eff}\ddot{\theta} + F_v\dot{\theta}, \quad (6)$$

State space model [5] for the pan mechanism has been obtained for the same state variables used for the tilt mechanism given in [2, 3, 4] as:

$$\dot{x}=\begin{bmatrix}0 & 1\\0 & -0.0168\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}u, \quad (7)$$

$$y=\begin{bmatrix}2.3964 & 0\end{bmatrix}x, \quad (8)$$

3. TRANSIENT RESOPNSE OF UNCOMPENSATED PTP

3.1 Step Response of Tilt Mechanism

Closed loop system for the tilt mechanism, with open loop transfer function given in (5), is shown in Figure 3. Unity feedback step response of the tilt mechanism has been obtained as shown in Figure 4.

Settling time and peak overshoot are obtained for this response as 49.5sec and 95.6% respectively. These results are far away from the required performance specifications.

Similarly step response for pan mechanism has also been obtained.

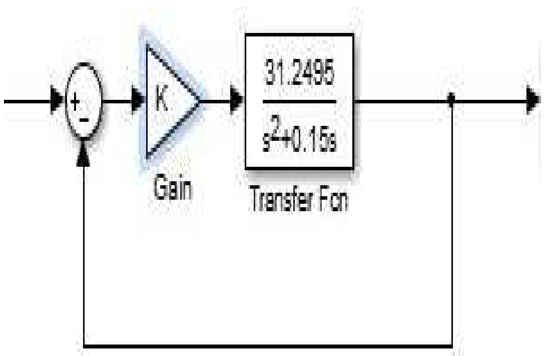


Figure 3: Block Diagram of Close Loop Uncompensated Tilt Mechanism

3.2 Step Response of Pan Mechanism

Closed loop system for the pan mechanism, with open loop transfer function given in (9), is shown in Figure 5. Unity feedback step response of the tilt mechanism has been obtained as shown in Figure 6.

Settling time and peak overshoot are obtained for the uncompensated pan mechanism as 464.8sec and 98.3% respectively. These results are also far away from the required performance specifications.

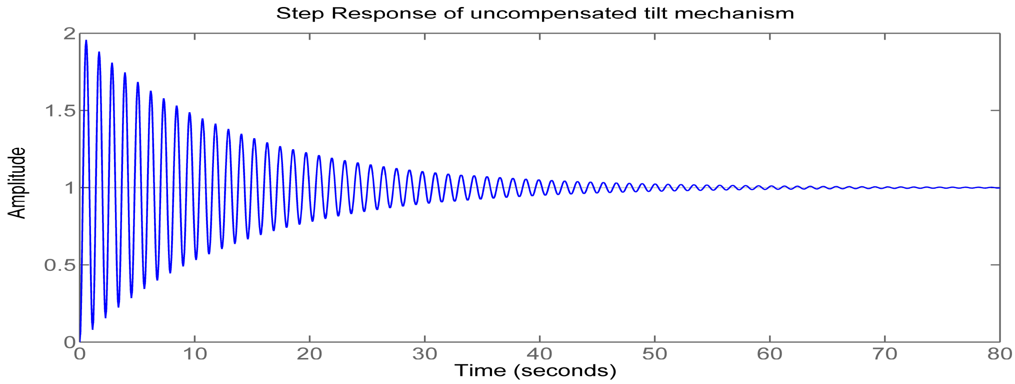


Figure 4: Step Response of Uncompensated Tilt Mechanism

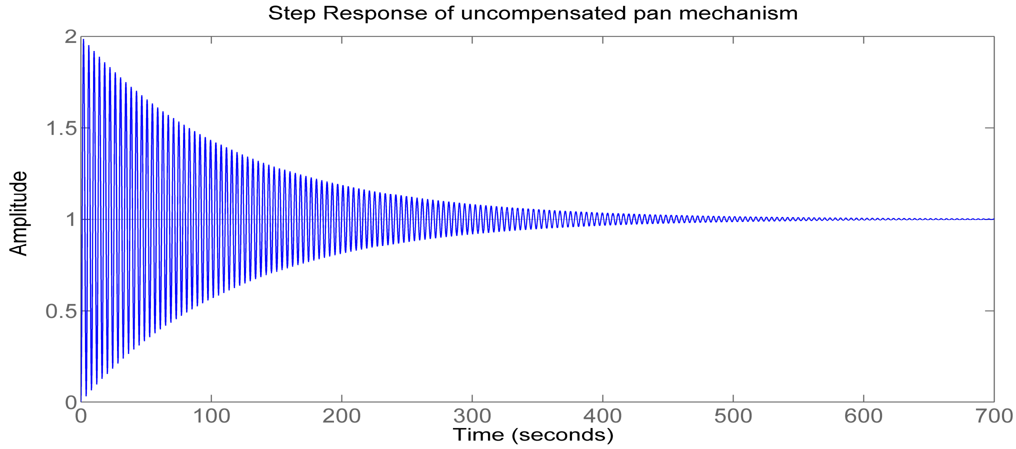


Figure 5: Step Response Uncompensated Pan Mechanism of

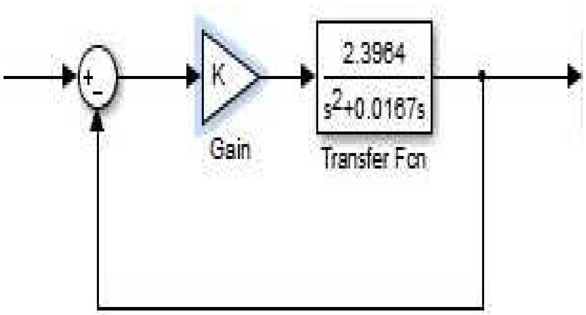


Figure 6: Block Diagram of Close Loop Uncompensated Pan Mechanism

Therefore, both the systems need a compensator for meeting the desired performance criteria. Lead compensator [2] and PD controller [3], already reported in the literature, compensate the system in frequency domain and have some limitations.

- The order of the system increases after compensated by both controllers because they add one more pole and zero in its transfer function.

- There is no clarification to choose a particular method of optimizing K_p, K_i and K_d for PID compensator which optimises the designers need.

- Lead compensator becomes less effective when the system operates at different frequency from that at which it is providing the required phase and gain shift.

Hence, modern control techniques[5,6] like pole placement method, linear quadratic regulator (LQR), full state feedback, and reduced order observer may be applied. LQR technique is reported to perform best out of these techniques [5]. Robotic system, both pan and tilt, dynamics are described by a set of linear differential equation and cost is described by a quadratic function to model it into a LQR problem. A detailed procedure for designing LQR [6] for the system is discussed in the section 4.

4. LINEAR QUARDATIC REGULATOR DESIGN FOR THE ROBOTIC SYSTEM [5,6]

State variable feedback is a direct and powerful technique for analysis of dynamic system because it guarantees desired close loop response. System must be controllable for state feedback control applications. This technique can be also applied for time varying and nonlinear systems. In this technique a control signal is chosen from an instantaneous state and feedback to the system as shown in Figure 7. The control signal u is chosen as u= - (K.x), where K is state feedback gain.

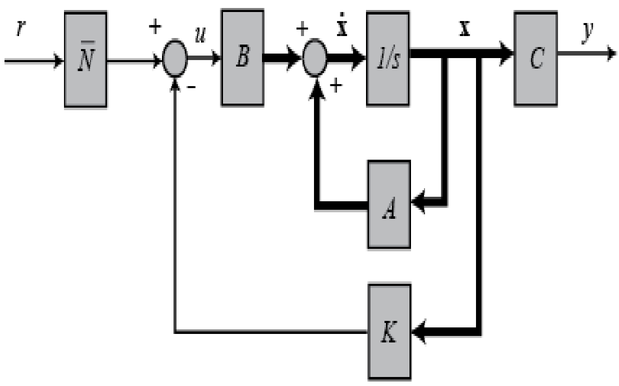


Figure 7: Block Diagram of LQR [6]

Ackermann's formula [5] determines the value of feedback gain K for desired location of closed loop poles for both system stability and its specified performance. Corresponding performance index [7,8]is given as

$$J = \int_0^{\infty} (X^T QX + U^T RU)dt, \tag{10}$$

The optimal solution of J is found by choosing the state weighing matrix Q and control matrix R to be positive or semi-positive definite [8,9].

Optimal state feedback controller $K= R^{-1}.B^T.P$ has a stabilizing control property if P is a symmetric positive definite in the formulated Algebraic Riccati Equation [10]. An unstable system must fulfil the following two conditions for design of LQR controller [8].

- LQR Theorem I (A, B) must be controllable.
- LQR TheoremII (√Q, A) must be observable.

Where A, B are the system state variable model matrices.

4.1. Linear Quardratic Regulator Design for Tilt Mechanism [5,6]

Robotic system is controllable so LQR can be designed for this system [8, 9]. The objective of the regulator is to achieve steady state time less than 0.5 second and to reduce maximum overshoot below 5%. For designing a LQR controller, designer has to choose matrices Q and R.

The simplest case is to assume R=1 and Q $=C^TC$ [7]. Corresponding performance index J places equal importance on the state variables. But choosing Q matrix as C^TC does not satisfy LQR theorem II.

Next different values of matrices Q and R are iterated for the compensation and analysis of the robotic system in the manner that both LQR theorems I and II are satisfied starting with Q =[1 1 ;1 1] and R=1. Transient response specification can further be improved by changing R from the set of values {1, 0.1, 0.01, and 0.001} in combination with the tuning of Q_{11} element of Q matrix keeping all other elements unchanged. Algebraic riccati equation has been solved for all these combinations to obtain optimal feedback gain K.

Different cases are given below:

Case -1

For $Q=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ & R=1, the obtained parameters are

K_{1tilt}= [3.5355 4.2689], T_s=4.38 sec, M_p=0.395%.

Case -2

For $Q=\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ & R=1, the obtained parameters are

K_{2tilt}=[1.414 1.80], T_s=3.33sec, M_p=1.01%.

Case-n

For $Q=\begin{bmatrix} 976 & 1 \\ 1 & 1 \end{bmatrix}$ & R=0.001, the obtained parameters are K_{ntilt} =[987.9271 54.3938], T_s =0.139 sec, M_p=0.414 %

where pre-compensation is used in this case to scale

the reference input so that output matches it from [11] as N (Pre gain multiplier)=31.5.

Since K_{ntilt} gain satisfies all the specifications, stabilized state space model of the tilt mechanism is derived using it as:

$$\dot{x}=\begin{bmatrix} 0 & 1 \\ -987.9 & -54.55 \end{bmatrix}x+\begin{bmatrix} 0 \\ 31.5 \end{bmatrix}u, \quad (11)$$

$$y=\begin{bmatrix} 31.25 & 0 \end{bmatrix}x, \quad (12)$$

Step response of the stabilized tilt mechanism system and its states are shown in Figure 8 and Figure 9 respectively.

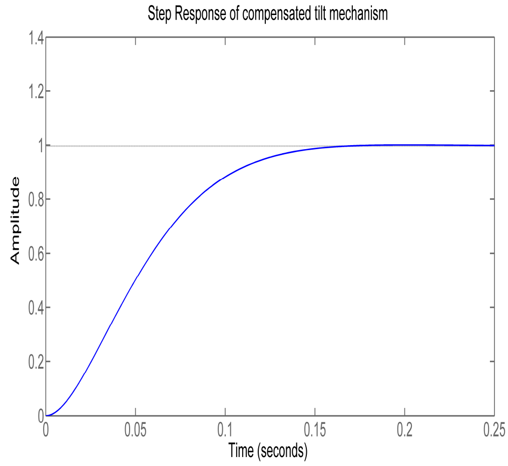


Figure 8: Step Response of Compensated Tilt Mechanism

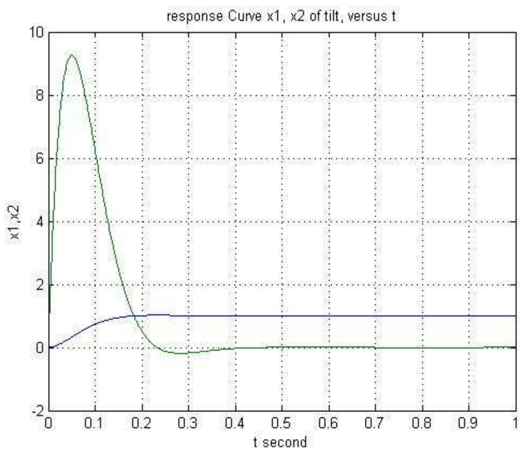


Figure 9: Step Response of Tilt Mechanism States

Similar procedure is followed for the pan mechanism also.

4.2. Linear Quardratic Regulator design for Pan Mechanism[7,8]

Different case studies are:

Case -1

For $Q=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ & R= 1, the obtained parameters are

K_{1pan}=[1 1.7], T_s=4.35 sec, M_p=0.433%

Case -2

For $Q=\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ & R=1, the obtained parameters are K_{2pan}=[1.4142 1.9399], T_s=3.32 sec, M_p=1.06%

Case-n

For $Q=\begin{bmatrix} 982 & 1 \\ 1 & 1 \end{bmatrix}$ & R=0.001, the obtained parameters are K_{npan} =[990.9591 54.5901], T_s=0.138 sec, M_p=0.419 %

Here, N (Pre gain multiplier)=412 is used.

Since K_{npan} gain satisfies all the specifications, stabilized state space model of the pan mechanism is derived using it as

$$\dot{x}=\begin{bmatrix} 0 & 1 \\ -991 & -54.61 \end{bmatrix}x+\begin{bmatrix} 0 \\ 412 \end{bmatrix}u, \tag{17}$$

$$y=\begin{bmatrix} 2.396 & 0 \end{bmatrix}x \tag{18}$$

Step response of the stabilized pan mechanism system and its states are shown in Figure 10 and Figure 11 respectively.

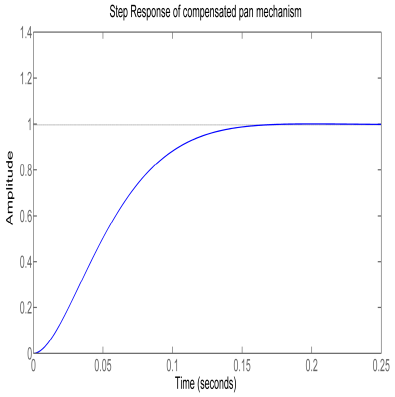


Figure 10: Step Response of Compensated Pan Mechanism

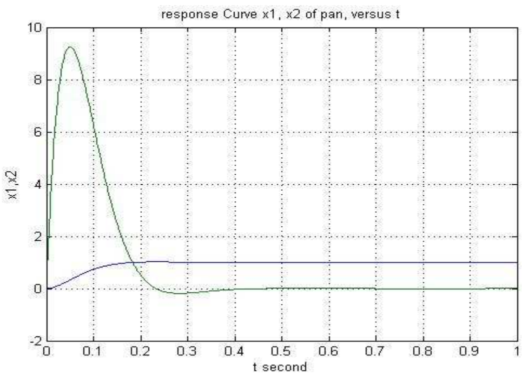


Figure 11: Step Response of Pan Mechanism States

5. RESULTS

The LQR optimized models of both pan and tilt mechanisms obtained in this study are compared with those reported in the literature i.e. with Lead compensator [2] and with PD controller [3] in Table 1.

Table 1: Compration of Different Control Techniques

	Tilt Mechanism			Pan Mechanism		
System with	T_S (Sec)	E_{ss}	% OS	T_s (Sec)	E_{ss}	% OS
LQR	0.139	0	0.41	0.138	0	0.41
Lead compensator	0.196	0.757	4.44	0.207	0.08	4.96
PD Controller	0.206	.0002	21.7	0.19	$2x10^{-6}$	21.5

It is observed from Table 1 that settling time, steady state error and maximum overshoot are best in the LQR case. Also, the order of the system is not increased in LQR case while it increases to three in other cases.

6. CONCLUSIONS

An optimal control action LQR has been modified successfully to achieve optimized modelling of both pan and tilt mechanisms for the desired specifications. The optimized robotic system is analysed and compared with other control techniques reported in the literature. It is observed that the proposed modification has best performance in comparison to other techniques as judged from the values of various specifications.

REFERENCES

1. Davis James, Chen Xing, "Calibrating pan-tilt cameras in wide area surveillance networks" *IEEE International Conference on Computer Vision*, vol.1, pp. 144 - 149,2003.

2. Imran S. Sarwar, Afzaal M. Malik, “Stability analysis and simulation of a two DOF robotic system based on linear control system” *15th International conference on mechatronics and machine vision,* pp. 263 – 268,2008.

3. Imran S. Sarwar, Afzaal M. Malik, “ Modeling analysis and simulation of a Pan Tilt Platform based on linear and nonlinear systems “, *IEEE China*,pp. 147 – 152, 2008.

4. Atul Kumar Pandey, Monika Mittal, “Analysis of Robotic System with two DOF Using Haar wavelet”, *6th India International Conference on Power Electronics*, IEEE,2014.

5. Katsuhiko Ogata, “Modern Control Engineering”, PHI, fifth edition, pp. 793-806, 2002.

6. Norman S. Nise, "Control systems engineering", Wiley student edition, Fourth edition, pp. 515-524, 2004.

7. Ashish Tewari, “Modern Control design”, John Wiley & Sons, ltd, pp 283-288, 2002.

8. F.L Lewis., D. Vrabie, V. L. Syrmos, “Optimal Control”. 3rd edition, John Wiley 2013.

9. Monika Garg, Lillie Dewan,“Non-recursive Haar Connection Coefficients Based Approah for Linear Optimal Control” *Springer Science*,vol.153, pp.320-337,2012.

10. C. E. M. Pearce, “On the Solution of a Class of Algebraic Matrix Riccati Equation”, *IEEE transcation on automatix control*, vol, AC no.3, 1983.

11. The Mathworks (www.mathworks.com), Control System Toolbox documentation Version V5.2 (R2009b).