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Complementary Connected Vertex Edge Domination

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Abstract. Let D be a vertex edge dominating set of G. If $< V - D >$ is connected, then D is called a complementary connected vertex edge dominating set(ccved-set) of G. The complementary connected vertex edge domination number $\gamma_{ccve}(G)$ of G is the minimum cardinality of a ccved-set of G. Bounds for this variant of vertex edge domination in terms of various graph theoretic parameters are obtained. The graphs attaining these bounds are characterized in some cases. Also graphs having ccved-numbers as $1, p-1, p-2, p-3$ are characterized. Complementary connected vertex edge domination numbers for some of the standard graphs are given.

1. Introduction and Preliminaries.

In this paper all our graphs will be finite, undirected and without loops or multiple edges having p vertices and q edges. Any undefined term in this paper, may be found in Harary [1].

If in a graph $G = (V, E)$, each vertex in $V - D(D \subset V)$ is adjacent to a vertex in D, then D is said to be a dominating set of G. The minimum cardinality of a dominating set of G is said to be a domination number of G and is denoted by $\gamma(G)$ [2]. If each edge in $E - F$ is adjacent to an edge in F for some $F \subseteq E$, then F is said to be an edge domianting set of G. The edge domination number $\gamma'(G)$ is the cardinality of a minimum edge dominating set of G [2].

A set D of vertices in a graph G is said to vertex edge dominate G, if for each edge in G one of the end vertices is from D or one of the end vertices is adjacent to a vertex in D. The smallest cardinality of any such vertex edge dominating set is said to be vertex edge domination number of G and is denoted by $\gamma_{ve}(G)$ [3]. A vertex edge dominating set D' is said to be a minimal vertex edge dominating set of G if

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and only if there is no vertex edge dominating set D'' of G such that $D'' \subset D'$ [3]. If D is a vertex edge dominating set of G such that $< D >$ is connected, then D is said to be a connected vertex edge dominating set of G. The minimum cardinality of a connected vertex edge dominating set of G is said to be the connected vertex edge domination number of G and is denoted by $\gamma_{cve}(G)$ [5]. If D is a vertex edge dominating set of G such that $< D >$ is a tree, then D is said to be a complementary tree vertex edge dominating set of G. The minimum cardinality of a complementary tree vertex edge dominating set of G is said to be the complementary tree vertex edge domination number of G and is denoted by $\gamma_{ctve}(G)$ [7]. If D is a vertex edge dominating set such that $V - D$ is not a vertex edge dominating set, then D is said to be complementary nil vertex edge dominating set(cnved - set). The minimum cardinality of the complementary nil vertex edge dominating set of G is said to be complementary nil vertex edge domination number of G and is denoted by $\gamma_{cnve}(G)$ [8].

A graph G is said to be semi complete if and only if there is a path of length two between any pair of vertices in G [4]. A graph G is said to be unicyclic if and only if it has exactly one cycle. The friendship graph F_p is the graph obtained by joining p copies of C_3 to a common vertex. The clique number $\omega(G)$ of a graph G is the maximum size of the clique in G.

In this paper, we define a new variant of vertex edge domination namely complementary connected vertex edge domination whose definition is as follows.

Let D be a vertex edge dominating set of G. Then D is said to be complementary connected vertex edge dominating set if and only if $< V - D >$ is connected. The complementary connected vertex edge domination number $\gamma_{ccve}(G)$ of G is the cardinality of a minimum vertex edge domination number of G. By a $\gamma_{ccve}(G) - set$ we mean a minimum complementary connected vertex edge dominating set.

Throughout this paper complementary connected vertex edge domination set is abbreviated as ccved - set. Here after, we assume that G is a connected graph.

2. Main Results.

Now, we give the characterization result for a proper subset D of V to be a *ccved-set.*

THEOREM 2.1. *A subset D of V is a ccved - set for G if and only if the following conditions hold:*

(1) $\{xy \in E(G) : \textit{atleast one of } x, y \textit{ is in } D\}$ *is an edge dominating set of G.*
(2) *D is not a vertex cut in G.*

PROOF. The proof is trivial. □

THEOREM 2.2. *A ccved - set D of G is minimal if and only if for each v in D one of the following conditions holds:*

(1) *For all u in $V - D$ adjacent to v, $N(u) \bigcap D = \{v\}$.*
(2) *there is an edge v_1v_2 in $E - F$ for which $(N(v_1) \bigcup N(v_2)) \bigcap D = \{v\}$, where $F = \{xy \in E(G) : \textit{atleast one of } x, y \textit{ is in } D\}$.*
(3) *$< (V - D) \bigcup \{v\} >$ is disconnected.*

Proof. Assume that D is a minimal $ccved - set$ of G.
Suppose that there is a vertex v in D which does not satisfy any of the conditions. By (i) and (ii) $D - \{v\}(= D')$ is a vertex edge dominating set for G. By (iii), $< V - D' >$ is connected. This implies D' is a $ccved - set$ of G, contradicting our assumption.

Conversely, suppose that D is a $ccved - set$ and for each v in G, one of the three conditions holds. Suppose D is not minimal $ccved - set$. Then, $D - \{v\}$ is a $ccved - set$. This implies, for all u in $V - D$ adjacent to v, $N(u) \bigcap D \neq \{v\}$,a contradiction to (i). If $D - \{v\}$ is a $ccved - set$, then, there is no edge v_1v_2 in $E - F$ for which $(N(v_1) \bigcup N(v_2)) \bigcap D = \{v\}$. This implies a contradiction to (ii). Also, since $D - \{v\}$ is a $ccved - set$, $< V - (D - \{v\}) >$ is connected, a contradiction to (iii). □

Proposition 2.1. *For a graph G, $1 \leqslant \gamma_{ccve}(G) \leqslant p - 1$.*

Proof. The proof follows from the fact that for a complete graph K_2 both the bounds hold. □

Note:

For characterizing the graphs having $\gamma_{ccve}(G) = 1$, we define a family $\mathcal{F}$ of graphs as follows.

A graph G of order $p \geqslant 4, \delta(G) \geqslant 2$ is in $\mathcal{F}$ if and only if there is a vertex v in G satisfying the following properties:

(1) Each edge in G lies on a n - cycle through v for some $n \leqslant 4$.
(2) Any pair of vertices lie on a cycle through v.

Theorem 2.3. *For a graph G with $p \geqslant 4, \delta(G) \geqslant 2$, $\gamma_{ccve}(G) = 1$ if and only if $G \in \mathcal{F}$.*

Proof. Assume that $\gamma_{ccve}(G) = 1$. Then, there is a v in G vertex edge dominating the edges in G and $G - v$ is connected.
Let v_1v_2 be an edge in G.

Case:1: $v_1 = v$ or $v_2 = v$.
W.l.g assume that $v_1 = v$. By our assumption $deg(v_2) \geqslant 2$. So, there is v_3 in G such that v_2v_3 is an edge in G. If v_3 is not adjacent to v or to a vertex adjacent to v, then there is an edge which is not vertex edge dominated by v, a contradiction to our assumption. Then in either case vv_2 lies on an n - cycle for some $n \leqslant 4$.

Case:2: $v_1 \neq v, v_2 \neq v$.
By the construction in Case:1, we get v_1v_2 lies on an n - cycle for some $n \leqslant 4$.

So, in any case (1) holds.
Let v_1, v_2 be a pair of vertices in G.

Case:1: v_1v_2 is an edge in G.
By (1), v_1, v_2 lie on a cycle.

Case:2: v_1v_2 is not an edge in G.
For G is connected and $\{v\}$ is a ccved - set of G, through v there exists

$v_1 - v_2$ path of length atmost 4. For $deg(v_1), deg(v_2) \geqslant 2$, there exists v_3, v_4 such that v_1v_3, v_2v_4 are edges in G. If $v_3 = v_4$, then we are through. Suppose not. By (1), each of v_1v_3, v_2v_4 lie on a cycle through v. Since $G - \{v\}$ is connected, v_1, v_2 lie on a cycle through v.

Hence (2) holds.
Converse is clear. □

COROLLARY 2.1. *For any complete bipartite graph* $K_{m,n}(m, n \geqslant 2)$ *,* $\gamma_{ccve}(K_{m,n}) = 1$.

PROOF. The proof follows from the fact that in $K_{m,n}(m, n \geqslant 2)$ any edge(pair of vertices) lie on a cycle of length 4.
Also observe that $\gamma_{ccve}(K_{m,n}) = 1$ for $m + n \leqslant 3$. □

THEOREM 2.4. $\gamma_{ccve}(G) = p - 1$ *if and only if* $G = K_2$.

PROOF. Suppose that $\gamma_{ccve}(G) = p - 1$.
If $diam(G) \geqslant 2$, then $V - \{v_1, v_2\}$ is a *ccved - set* of G for an arbitrary edge v_1v_2 in G. This implies that $\gamma_{ccve}(G) \leqslant p - 2$, which is a contradiction to our assumption. Hence $diam(G) = 1$.

$$\Rightarrow G \cong K_p \; for \; some \; p \geqslant 2.$$

If $p > 2$, then $\gamma_{ccve}(G) = 1 \neq p - 1$, a contradiction to our assumption. Hence $G = K_2$.
The converse is clear. □

THEOREM 2.5. *For a graph* G*,* $\gamma_{ccve}(G) = p - 2$ *if and only if* $G = P_3$ *or* K_3 *or* P_4.

PROOF. Assume that $\gamma_{ccve}(G) = p - 2$.
Suppose that $diam(G) \geqslant 4$. Let $< v_1v_2v_3...v_kv_{k+1} >$ be a diammetral path in G. Clearly $k \geqslant 4$. Since $< \{v_2, v_3, v_4\} >$ is connected, $V - \{v_1, v_5, v_6, ..., v_kv_{k+1}\}$ is a ccved - set of cardinality $p - 3$ a contradiction. So, $diam(G) \leqslant 3$.

Suppose $diam(G) = 3$. Then there is a diammetral path of length 3, say $< v_1v_2v_3v_4 >$. Assume that there is a v_5 in $V - \{v_1, v_2, v_3, v_4\}$ adjacent to one of the vertices in $\{v_1, v_2, v_3, v_4\}$. If v_5 is adjacent with v_4 or v_1, then $V - \{v_2, v_3, v_4\}$ is a ccved - set, a contradiction. If v_5 is adjacent to v_2 or v_3 , then $V - \{v_1, v_2, v_5\}, V - \{v_3, v_4, v_5\}$ is a ccved - set respectively, a contradiction. Hence $G =< v_1v_2v_3v_4 >= P_4$.

Suppose that $diam(G) = 2$, by the above construcion we get that $G = P_3$.
Suppose that $diam(G) = 1$. Then $G = k_n$ for $n \geqslant 2$. Except for $n = 3, \gamma_{ccve}(G) \neq 3$. Hence $G = K_3$.
The converse part is clear. □

COROLLARY 2.2. *For a semi complete graph* G*,* $\gamma_{ccve}(G) = p - 2$ *if and only if* $G = K_3$.

PROOF. Since K_3 is the only semi complete graph among the class of graphs having $\gamma_{ccve}(G) = p - 2$, the proof follows. □

Note: For a tree T, $\gamma_{ctve}(T) = \gamma_{ccve}(T)$.

Now, we give the necessary and sufficient condition for a *ccved - set* to be a *ctved - set*.

THEOREM 2.6. *A ccved - set D of G is a ctved - set if and only if each cycle in G has a vertex from D.*

PROOF. The proof is trivial. $\square$

THEOREM 2.7. *For any spanning subgraph H of G, $\gamma_{ccve}(G) \leqslant \gamma_{ccve}(H)$.*

THEOREM 2.8. *For any connected (p, q) graph G,*

$$\frac{3}{2}(p + k - 1) - q \leqslant \gamma_{ccve}(G).$$

where k is the number of edge disjoint cycles in $< V - (\gamma_{ccve}(G) - set) >$.

PROOF. D be a $\gamma_{ccve}(G) - set$. Then $< V - D >$ has $p - \gamma_{ccve}(G)$ vertices and atleast $p - \gamma_{ccve}(G) - 1 + k$ edges. Let t be the number of edges having one end in D and another in $V - D$. Hence,

$$\begin{aligned} 2[q - (p - \gamma_{ccve}(G) - 1 + k)] &= \sum_{v \in D} deg(v) + t \\ &\geqslant \gamma_{ccve}(G)\delta(G) + t \\ &\geqslant \gamma_{ccve}(G) + p - \gamma_{ccve}(G) - 1 + k \\ &= p + k - 1. \end{aligned}$$

This implies,

$$2q - 2p + 2\gamma_{ccve}(G) - 2k + 2 \geqslant p + k - 1$$

Hence the result follows. $\square$

THEOREM 2.9. *Let G be a (p, q) graph,then*

$$\gamma_{ccve}(G) \leqslant 2(p + k - 1) - \frac{2q}{\Delta(G)}.$$

where k is the number of cycles in $< V - (\gamma_{ccve}(G) - set) >$.

PROOF. By the construction in the above theorem,

$$\begin{aligned} 2[q - (p - \gamma_{ccve}(G) - 1 + k)] &= \sum_{v \in D} deg(v) + t \\ &\leqslant \Delta(G)\gamma_{ccve}(G) + 2(\Delta(G) - 1)(p - \gamma_{ccve}(G) - 1 + k). \end{aligned}$$

This implies,

$$2q \leqslant -\Delta(G)\gamma_{ccve}(G) + 2\Delta(G)(p + k - 1)$$

Hence the result follows. $\square$

THEOREM 2.10. *Let G be a (p, q) graph which has $(\omega(G)+1)-$regular spanning subgraph, then*

$$\gamma_{ccve}(G) \leqslant p - \omega(G).$$

where $\omega(G)$ is the clique number of G.

PROOF. Let S be the set of vertices such that $< S >$ is complete and $|S| = \omega(G)$. By the hypothesis it follows that, $V - S$ is a $ccved - set$ of G. Hence the result follows.

Note: Since for a (p, q) graph G, $(V - S)\bigcup\{v\}_{v\in S}$ is a $ccved - set$ of G

$$\begin{aligned}\gamma_{ccve}(G) &\leqslant |V - S| \\ &\leqslant |(V - S)| + |\{v\}| \\ &\leqslant p - \omega(G) + 1.\end{aligned}$$

Here $< S >$ is complete and $|S| = \omega(G)$. □

THEOREM 2.11. *For a semi complete graph G with $p \geqslant 3$,*

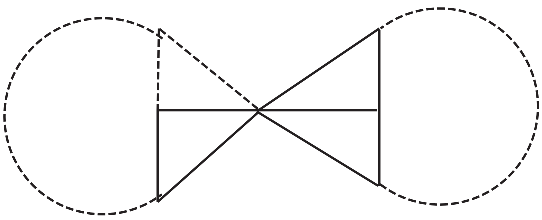
$$1 \leqslant \gamma_{ccve}(G) \leqslant p - 2.$$

PROOF. Since in a semi complete graph each edge lies on a triangle and also every complete graph with atleast three vertices is semi complete, by the above note the inequality follows. □

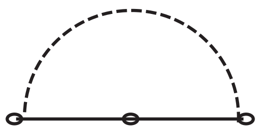
Note:

1. For a semi complete graph G having four vertices, $\gamma_{ccve}(G) = p - 3$ if and only if G is a union of two triangles having a common edge.
2. There is no semi complete graph with three, five vertices having, $\gamma_{ccve}(G) = p - 3$.

THEOREM 2.12. *G be a semi complete graph with $p > 5$, then $\gamma_{ccve}(G) = p - 3$ if and only if G is isomorphic to*



PROOF. Assume that $\gamma_{ccve}(G) = p - 3$. This implies that there is a ccved - set $D \subset V$ of cardinality $p - 3$ and $< V - D >$ is isomorphic to

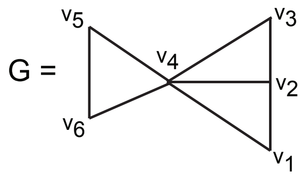


Case:1: $< V - D > = P_3(< v_1v_2v_3 >)$.

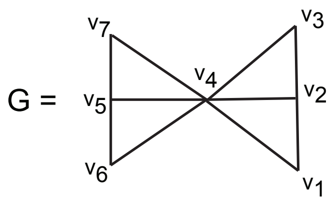
Since G is semi complete v_1v_2,v_2v_3 lie on two different triangles, say $< v_1v_2v_4 >, < v_2v_3v_5 >$.

Subcase:a: $v_4 = v_5$.

So, both the triangles have a common edge v_2v_4(say). Since $p > 5$, choose a vertex adjacent to v_4 say, v_5. For, G is semi complete v_4v_5 lies on a triangle $< v_4v_5v_6 >$. If $v_6 \in \{v_1, v_2, v_3\}$, then $D - \{v_5\}$ is a ccved - set of cardinality less than $p-3$, a contradiction to our assumption. If then the result is clear. Suppose not. We can find



a vertex say v_7 adjacent to either v_5 or v_6. W.l.g assume that v_7 is adjacent to v_5. Since G is semi complete v_4v_7 is an edge in G. If then



the result is clear. Suppose not. If there is a vertex v_8 in G, then by the semi completeness of G, v_4v_8 is an edge in G. This implies that $V - \{v_1, v_2, v_3, v_4\}$ is a ccved- set in G of cardinality $4 (= p - 4)$, a contradiction. Hence $V = \{v_1, v_2, ..., v_7\}$.

Since none of v_5, v_6, v_7 can be adjacent to one of v_1, v_2, v_3, the result holds.

Subcase:b: $v_4 \neq v_5$.

Since $p > 5$, choose a vertex v_6 from $V - \{v_1, v_2, ..., v_5\}$. If v_6 is adjacent to one of v_1, v_2, v_3, then $D - \{v_5\}, D - \{v_4, v_5\}, D - \{v_4\}$ is a ccved - set of G respectively, a contradiction to $\gamma_{ccve}(G) = p - 3$. Since G is semi complete, v_6 is adjacent with v_4 and v_5. Then, $D - \{v_4\}$ or $D - \{v_5\}$ is a ccved - set of G, a contradiction. So, $v_4 = v_5$.

Case:2: $< V - D > \neq P_3$.

Then $< V - D > = K_3$. By the construction as in the case:1, the result follows.

The converse part is clear.

□

COROLLARY 2.3. *G be a semi complete graph. Then* $\gamma_{ccve}(G) = p - 3$ *if and only if* $4 \leqslant p \leqslant 7, p \neq 5$.

COROLLARY 2.4. *For a semi complete graph G with $\gamma_{ccve}(G) = p-3$,*

$$3 \leqslant \gamma_{ccve}(G) + \Delta(G) \leqslant 10.$$

PROOF. For a semi complete graph G, $\Delta(G) \geqslant 2$ and $\gamma_{ccve}(G) \geqslant 1$. Also by Theorem 2.12 , for a semi complete graph G with $\gamma_{ccve}(G) = p-3$, $\Delta(G) \leqslant 6$. Hence the result follows. □

Note: The bounds are sharp as the lower and upper bounds are attained in the case of K_3, W_9 respectively.

THEOREM 2.13. *G be a semi complete graph with $p \geqslant 4$. Then, $\gamma_{ccve}(G) = 1$ if and only if $G \in \mathcal{F}$.*

PROOF. Since for a semi complete graph $\delta(G) \geqslant 2$, by Theorem 2.3 the proof follows. □

THEOREM 2.14. *Let G be a (p,q) graph with $\delta(G) \geqslant 3, g(G) \neq 3$,then*

$$\gamma_{ccve} G \leqslant p - \Delta(G).$$

PROOF. Suppose that $deg(v) = \Delta(G)$ for some $v \in V$. Then, $(V - N[v])$ is a $ccved - set$ of G. Hence the result. □

THEOREM 2.15. *For a (p,q) graph G with $\delta(G) \geqslant 3, g(G) > 4$,*

$$\gamma_{ccve}(G) \leqslant p - k - 3$$

where k is the diameter of G.

PROOF. Let u and v be two vertices with $d(u,v) = k = diam(G)$. Let $< u = v_1 v_2 ... v_{k-1} v_k v_{k+1} = v >$ be a diammetral path in G. Since each edge in G is vertex edge dominated by a vertex in $V - \{v_1, v_2, ..., v_{k+1}, u_1, u_{k+1}\}$(where $u_i v_i (1 \leqslant i \leqslant k+1), u_i \notin \{v_1, v_2, ..., v_{k+1}\}$) and $< \{v_1, v_2, ..., v_{k+1}, u_1, u_{k+1}\} >$ is connected, the former is a $ccved - set$ in G. Hence,

$$\begin{aligned} \gamma_{ccve}(G) &\leqslant |V - \{v_1, v_2, ..., v_{k+1}, u_1, u_{k+1}\}| \\ &\leqslant p - ((k+1)+2) \\ &= p - k - 3. \end{aligned}$$

We make use of the following result in proving the next result. □

Theorem 1.[7] If both G and $\overline{G}$ are connected with $p \geqslant 6$, then

$$4 \leqslant d + \overline{d} \leqslant p + 1$$

where $\overline{d}$ is the diameter of $\overline{G}$.

COROLLARY 2.5. *Suppose both $G, \overline{G}$ are connected with $\delta(G), \delta(\overline{G}) \geqslant 3$ and $p \geqslant 6$, then*

$$\gamma_{ccve}(G) + \gamma_{ccve}(\overline{G}) \leqslant 2p - 10.$$

PROOF. By Theorem 1.[6], the proof follows. □

THEOREM 2.16. *For a (p, q) graph G with $\delta(G) \geqslant 3, g(G) \geqslant 7$,*

$$\gamma_{ccve}(G) \leqslant p - \frac{2}{3}k - 2$$

where k is the diameter of G.

PROOF. Let u and v be two vertices with $d(u, v) = k = diam(G)$. Let $< u = v_1v_2...v_{k-1}v_kv_{k+1} = v >$ be a diammetral path in G. Since each edge in G is vertex edge dominated by a vertex in $V - \{v_1, v_2, ..., v_{k+1}, u_1, u_4, u_7, ..., u_{k+1}(k = 3m), u_k(k = 3m+1), u_{k-1}(k = 3m+2)\}$(where $u_iv_i(1 \leqslant i \leqslant k+1), u_i \notin \{v_1, v_2, ..., v_{k+1}\}$) and $\{v_1, v_2, ..., v_{k+1}, u_1, u_4, u_7, ..., u_{k+1}(k = 3m), u_k(k = 3m+1), u_{k-1}(k = 3m+2)\}$ is connected, the former is a $ccved - set$ in G. Hence,

$$\begin{aligned} \gamma_{ccve}(G) &\leqslant |V - \{v_1, v_2, ..., v_{k+1}, u_1, u_4, u_7, ..., u_{k+1}(or)u_k(or)u_{k-1}\}| \\ &\leqslant p - (k+1) - (\frac{k}{3} + 1) \\ &= p - \frac{4}{3}k - 2. \end{aligned}$$

□

THEOREM 2.17. *If D is a* ccved - set *such that no two edges in $< V - D >$ are* ve - dominated *by the same vertex in D, then*

$$\gamma_{ccve}(G) \leqslant \frac{2p - q - 2}{2}$$

.

THEOREM 2.18. *For a graph G,*

$$\gamma_{ccve}(G) + \gamma_{cve}(G) \leqslant p$$

PROOF. Let D be a $\gamma_{ccve}(G) - set$. By definition $< V - D >$ is connected and each edge in G is dominated by a vertex in $V - D$. So, $V - D$ is a connected $ve - dominating$ set of G. This implies,

$$\begin{aligned} \gamma_{cve}(G) &\leqslant |V - D| \\ &= p - \gamma_{ccve}(G). \end{aligned}$$

Hence the result follows. □

THEOREM 2.19. *For any graph G, the following conditions are equivalent*

(1) *The set of all pendant vertices form a $ccved - set$.*
(2) *The set of all pendant edges in G form an edge dominating set for G.*
(3) *Each non pendant vertex is a support vertex or adjacent to a support vertex.*

PROOF. Suppose that (1) holds.

Take $S = \{uv : u \text{ or } v \text{ is a pendant vertex}\}$. Let $e(= v_1v_2) \in E - S$. By our supposition there is atleast one pendant vertex(say,v_3) adjacent to one of the end vertices of e. W.l.g assume that $v_1v_3 \in E$. Since v_3 is a pendant vertex, $v_1v_3 \in S$. Hence (2) holds.

Suppose that (2) holds.

Let v_1 be a non pendant vertex which is neither a support vertex nor adjacent to a support vertex. Then there is a non pendant vertex(say v_2) in $V - \{support\ vertices\ of\ G\}$ adjacent to v_1. This implies v_1v_2 is not dominated by S, a contradiction to our assumption. Hence (3) holds.

Suppose that (3) holds.

Let v_1v_2 be an arbitrary non pendant edge in G. By (3), either v_1 or v_2 is a support vertex in G. Then, there is a pendant vertex v_3 such that v_1v_3 or v_2v_3 is an edge in G. This implies v_1v_2 is $ve - dominated$ by v_3. Hence (1) holds. $\square$

Corollary 2.6. *If a graph G satisfies any of the conditions mentioned in Theorem 2.19, then*

$$\gamma_{ccve}(G) \leqslant m.$$

where m is the number of pendant vertices in G.

Proof. By hypothesis, it follows that the set of all pendant vertices form a *ccved - set* of G. Hence the result follows.

Furthermore, the bound is sharp as it is attained in the case of $C_p \circ K_1$, where $C_p \circ K_1$ is the corona of C_p and K_1.

$\square$

Corollary 2.7. $\gamma_{ccve}(C_p \circ K_1) = p$.

Proof. Since in $C_p \circ K_1$, each non pendant vertex is a support vertex, by Theorem 2.19,

$$\gamma_{ccve}(C_p \circ K_1) \leqslant p.$$

Also for any *ved - set* D with $|D| < p$, $< V - D >$ is disconnected. Hence the result follows. $\square$

Corollary 2.8. *If a graph G satisfies any of the conditions mentioned in Theorem 2.19, then*

$$\gamma'(G) \leqslant m.$$

Furthermore, the bound is sharp as it is attained in the case of P_5.

Proof. Since a graph satisfies any of the conditions mentioned in Theorem 2.19 has its pendant edges as its edge dominating set, hence the result follows.

Now we give the $ccved - numbers$ of some standard graphs. $\square$

Theorem 2.20. (1) *For any cycle C_p with $p \geqslant 4$, $\gamma_{ccve}(G) = p - 3$.*
(2) *For any complete graph $K_p (p \geqslant 2)$, $\gamma_{ccve}(K_p) = 1$.*
(3) *For any wheel graph W_p, $\gamma_{ccve}(W_p) = 1$.*
(4) *For a friendship graph F_p, $\gamma_{ccve}(F_p) = 2p - 1$.*

Theorem 2.21. *For a tree T having diameter atleast four, $\gamma_{cnve}(T) \leqslant \gamma_{ccve}(T)$. Furthermore, equality holds if and only if $diam(T) = 4$.*

Proof. Since for a tree having diameter atleast four, every ccved-set is a cnved-set the result follows. $\square$

THEOREM 2.22. *For a unicyclic graph G with internal vertices having degree atleast three,*

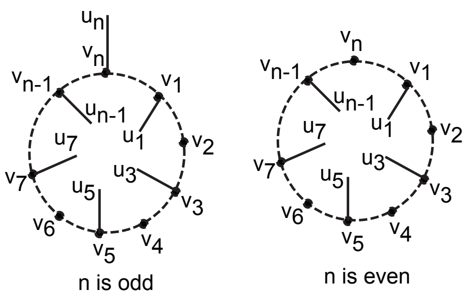
$$\gamma_{ccve}(G) \leqslant p - n.$$

where n is the length of the cycle in G.

Furthermore, the bound is sharp as it is attained in the case of $C_n \circ K_1$.

PROOF. By the hypothesis it is clear that each vertex in G is either a pendant vertex or a vertex of degree 3. Also the set of all pendant edges in G form an edge dominating set for G. Then by Theorem 2.19, the set of all pendant vertices form a ccved - set for G. Hence the result follows. □

THEOREM 2.23. *For a unicyclic graph G, $\gamma_{ccve}(G) = p - n$ if and only if G is isomorphic to one of the following :*



PROOF. The proof is trivial. □

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