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EXCELLENT-JUST EXCELLENT DEGREE EQUITABLE DOMINATION IN FUZZY GRAPHS

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Abstract. Let G be a fuzzy graph. Let u and v be two vertices of G. A subset D of V is called a fuzzy equitable dominating set if every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $| deg(u) - deg(v) | \leqslant 1$ where $deg(u)$ denotes the degree of vertex u and $deg(v)$ denotes the degree of vertex v and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$. The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{ef} set of G. G is said to be γ^{ef}-excellent if every vertex of G belongs to a γ^{ef}-set of G. This paper aims at the study of the new concept namely fuzzy equitable excellent graphs. Just γ^{ef}-excellence is also defined and studied. Result of these new concepts are compared with those with respect to domination excellence and just γ^{ef}-excellence in graphs.

1. Introduction

It is well known graphs are simply models of relations. A graph is a convenient way of representing informations iinvolving relationship between objects. The objects are represented by vertices and relation by edges.

L.A.Zadeh(1965) introduced the concepts of a fuzzy subset of a set as a way for representing uncertainity. His idea have been applied to a wide range of scientific area.

Fuzzy concepts is also introduced in graph theory. Formally, fuzzy graph $G = (v, \sigma, \mu)$ is a nonempty set V together with a pair of functions $\sigma : v \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(u, v) \leqslant \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. σ is called fuzzy vertex set of G and μ is called the fuzzy edge set of G. The concept of **Excellent and Just-excellent degree equitable domination in fuzzy graph** introduced by K.M. Dharmalingam, V. Swaminathan [**9**]. In this paper, we are introducing "Excellent and just excellent fuzzy graphs in equitable domination".

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2. Definitions

Definition 2.1. A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ where for all $u, v \in V$, we have $\mu(u, v) \leqslant \sigma(u) \wedge \sigma(v)$.

Definition 2.2. The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum\limits_{x \in V} \sigma(x)$ and $q = \sum\limits_{xy \in E} \mu(xy)$.

Definition 2.3. The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $d(u)$.

Definition 2.4. Let $G = (\sigma, \mu)$ be a fuzzy graph and a vertex $u \in V$ is said to be fuzzy pendant vertex of G if $\sigma(u) = 1$.

Definition 2.5. A subset D of V is called an fuzzy equitable dominating set if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $\mid deg(u) - deg(v) \mid \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$. The minimum cardinality of such a dominating set is denoted by γ^{ef} and is called the fuzzy equitable domination number of G.

Definition 2.6. A vertex $u \in V$ is said tobe fuzzy equitable with a vertex $v \in V$ if $\mid deg(u) - deg(v) \mid \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$

Remark 2.1. If D is an equitable dominating set then any superset of D is an equitable dominating set.

Definition 2.7. An equitable dominating set D is said to be minimal equitable dominating set if no proper subset D is an equitable dominating set.

Definition 2.8. A minimal equitable dominating set of maximum cardinality is called a Γ^{ef}-set and its cardinality is denoted by Γ^{ef}.

Remark 2.2. If a vertex $u \in V$ be such that $\mid deg(u) - deg(v) \mid \geqslant 2$ for all $v \in N(u)$,and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$, then u is in every equitable dominating set. Such points are called equitable isolates. Let I_e denote the set of all equitable isolates. Vacuously isolated points are equitable isolated points. Hence $I_s \subseteq I_e \subseteq D$ for every equitable dominating set D where I_s is the set of all isolated points of G.

Definition 2.9. Let $u \in V$. Then the number of points which are fuzzy equitable with u, is called the fuzzy equitable number u and is denoted by $def(u)$. That is

$$def(u) = |\{v \in V - \{u\} \backslash |d(u) - d(v)| \leqslant 1\}|$$

Remark 2.3.

$$\begin{aligned} i) d_G^{ef}(u) + d_{\bar{G}}^{ef} &= def(u) \\ ii) def_G(u) + def_{\bar{G}} &\leqslant n - 1 \end{aligned}$$

Definition 2.10. Let $u \in V$. The equitable neighborhood of u is denoted by $N^{ef}(u)$ is defined as $N^{ef}(u) = \{v \in V / v \in N(u), |d(u) - d(v)| \leqslant 1\}$ and $u \in I_e \Leftrightarrow N^{ef}(u) = \varnothing$. The cardinality of $N^{ef}(u)$ is denoted by $d_G^{ef}(u)$.

Definition 2.11. The minimum and maximum fuzzy equitable of a point G are denoted respectively by $\delta^{ef}(G)$ and $\Delta^{ef}(G)$. That is

$$\Delta^{ef}(G) = max_{u\in V}|N^{ef}(u), \delta^{ef}(G) = min_{u\in V}|N^{ef}(u)|$$

Definition 2.12. Let G be a given fuzzy graph. Let H be the graph constructed from G as follows $V(H) = V(G)$, two points u and v are adjacent and fuzzy equitable in G. H is called the fuzzy equitable associate graph G and is denoted by $e^{ef}(G)$.

Remark 2.4. (i) $e = uv \in E(e^{ef}(G))$. Then u and v are adjacent and fuzzy equitable in G. Therefore $e^{ef} \in E(G)$, $E(e^{ef}(G)) \subseteq E(G)$.
(ii) An edge $e = uv \in E(e^{ef}(G))$issaidto be fuzzy equitable if $|d(u)-d(v)| \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$. Let $E^{ef}(G)$ be the set of all fuzzy equitable edges of G. Then clearly $E(e^{ef}(G)) = E^{ef}(G)$.

Definition 2.13. Let G be a simple fuzzy graph. A vertex $u \in V$ is called γ^{ef}-good if u belongs to a γ^{ef}-set of G.(A γ^{ef}-set of G is a minimum cardinality is a fuzzy equitable dominating set.)

Definition 2.14. Let G be a fuzzy simple graph. G is said to be γ^{ef}-excellent if every vertex of G belongs to a γ^{ef}-set of G. That is γ^{ef}-excellent if every vertex of G is γ^{ef}-good.

Definition 2.15. A graph G is just γ^{ef}-excellent if every vertex of G belongs to exactly one γ^{ef}-set of G.

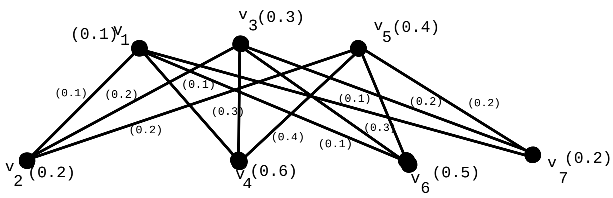


Figure 1

γ^{ef} sets are $\{v_1, v_2\}, \{v_1, v_4\}, \{v_1, v_6\}, \{v_1, v_7\}, \{v_3, v_2\}, \{v_3, v_4\}, \{v_3, v_6\},\{v_3, v_7\}, \{v_5, v_2\}, \{v_5, v_4\}, \{v_5, v_6\}, \{v_5, v_7\}$ Every vertex in figure(1) are belongs to γ^{ef}-sets. Hence fig(1) is γ^{ef} excellent.

3. Mine results

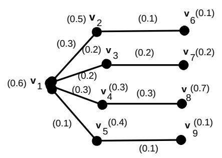
Theorem 3.1. *Let G be a non γ^{ef}-excellent graph. Then G can be embedded in a $\gamma^{ef}-$excellent graph.*

Proof. Since G be a non γ^{ef} excellent graph. Let $B = \{u_1, u_2, ..., u_t\}$ be the set of all γ^{ef}-bad vertices of G. Let $m_1 = max\{deg_G(v) : v \in N[u_1]\}$. Attach $m_1 - deg_G(u_1) + 2$ fuzzy pendant vertices at u_1. Let H_1 be the resulting graph. Let $m_2 = max\{deg_{H_1}(v_1) : v_1 \in N[u_2]\}$. Attach $m_2 - deg_G(u_2) + 2$ fuzzy pendant vertices at u_2. Let H_2 be the resulting graph. Proceeding this manner, we get the graph $H_3, H_4, ..., H_i$,such that H_i is an induced subgraph of $H_i + 1$, $1 \leqslant i \leqslant t-1$. Also G is an induced subgraph of $H_i (1 \leqslant i \leqslant t)$. hence G is an induced subgraph of H_t. Clearly, $u_1, u_2, ..., u_t$ and the fuzzy pendant vertices attached at these vertices are equitable ioslates in H_t. Therefore H_t is the γ^{ef}-excellent since any γ^{ef} set of H_t is obtained from γ^{ef} of G by adding $u_1, u_2, \ldots, u_t$ and the fuzzy pendant vertices attached at $u_1, u_2, \ldots, u_t$.

$$\gamma^{ef}(H) = \gamma^{ef}(G) + \lfloor m_1 - deg_G(u_1) + 2 + m_2 - deg_G(u_2) + 2 + \ldots \rfloor .$$

□

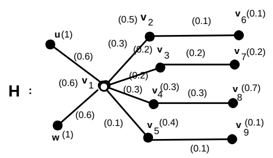
Illustration: Here v_1 is a γ^{ef}-bad vertex of fig(1). Let $m_1 = max\{deg_{Gf}(v) : v \in N[v_1]\} = 0.9$. Add $m_1 - deg_{Gf}(v_1) + 2 = \lfloor 0.9 - 0.9 + 2 \rfloor = 2$ fuzzy pendant vertices at v_1. Let H be the resulting graph.



v

Figure 2

The γ^{ef} sets of H are $\{v_1, u, w, v_2, v_3, v_4, v_5\}$, $\{v_1, u, w, v_6, v_7, v_8, v_9\}$. Therefore fig(2) H is γ^{ef} fuzzy excellent.



v

Figure 3

Theorem 3.2. *Every vertex of a transitive graph is γ^{ef}-excellent.*

PROOF. Let G be a transitive graph. Let $S = \{u_1, u_2, \ldots, u_{\gamma^{ef}(G)}\}$ be a γ^{ef}-set of G. Let $v \in V - S$. Since G is vertex transitive, there exists an automorphism $\phi : G \to G$ such that $\phi(u_1) = v$. Let $S_1 = \phi(S) = \{\phi(u_1), \phi(u_2), \ldots, \phi(\gamma^{ef}(G))\}$. Let $w \in V - S_1$. Consider $x \in \phi^{-1}(w)$. Then $\phi(x) = w \notin S_1$. Therefore $x \notin \phi^{-1}(S_1) = S$. Therefore there exists $y \in S$ such that x and y are adjacent and $|deg(x) - deg(y)| \leqslant 1$ and $\mu(uv) \leqslant \sigma(x) \wedge \sigma(y)$. Therefore $\phi(x)$ and $\phi(y)$ are adjacent and $|deg\phi(x) - deg\phi(y)| \leqslant 1$ and $\mu(uv) \leqslant \sigma\phi(x) \wedge \sigma\phi(y)$. (i.e) w and $\phi(y)$ are adjacent and $|deg(w) - deg\phi(y)| \leqslant 1$ $\mu(uv) \leqslant \sigma(w) \wedge \sigma\phi(y)$. Also $\phi(y) \in \phi(S)$. There $\phi(S)$ is a fuzzy equitable dominating set of G. $|\phi(S)| = |S| = \gamma^{ef}(G)$. Since $\phi(u_1) = v, v \in \phi(S)$. Therefore G is γ^{ef}-excellent. □

DEFINITION 3.1. A graph G is just γ^{ef}-excellent if every vertex of G belongs to exactly one γ^{ef}-set of G.

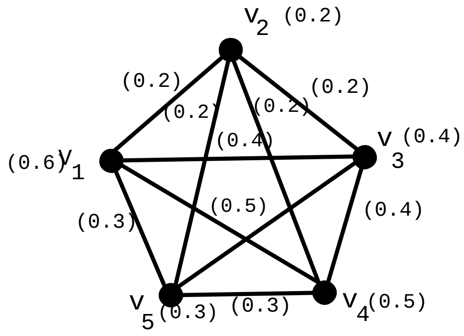


FIGURE 4

γ^{ef} sets are $\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}$ Every vertex in figure(4) is belongs to exactly one γ^{ef} set. Hence fig(4) is γ^{ef} just-excellent.

DEFINITION 3.2. A graph G is fuzzy equitably complete if every pair u, v of its vertices satisfies $|deg(u) - deg(v)| \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$

REMARK 3.1. A graph G is fuzzy equitably complete if and only if G^{ef} is complete.

PROPOSITION 3.1. *Let G be a just γ^{ef}-excellent graph and let $G \neq K_n$. Then $N[u]$ is not fuzzy equitably complete for any $u \in V(G)$.*

PROOF. Let G be a just γ^{ef}-excellent graph. Let $G \neq K_n$. Suppose $N[u]$ is fuzzy equitably complete (i.e) for every v, w in $N[u]$,v and w are fuzzy equitable.

Suppose $N[u] = \{u\}$. Then u is an isolate and hence G is not just γ^{ef}-excellent. Therefore $|N[u] \geqslant 2$. Since G is just γ^{ef}-excellent,there exists a unique γ^{ef}- set say S of G such that $u \in G$. Let $v \in N[u]$,$v \neq u$. Then $S - \{u\})$ is not empty. Therefore any vertex of $S - \{u\})$ belongs to atleast two γ^{ef} sets, namely S and $(S - \{u\}) \cup \{v\}$ of G, a contradiction. Therefore $N[u]$ is not fuzzy equitably complete. □

REMARK 3.2. There exists fuzzy graphs in which $N[u]$ is not fuzzy equitably complete for every $u \in V(G)$ but the graph may not be just-γ^{ef} excellent.

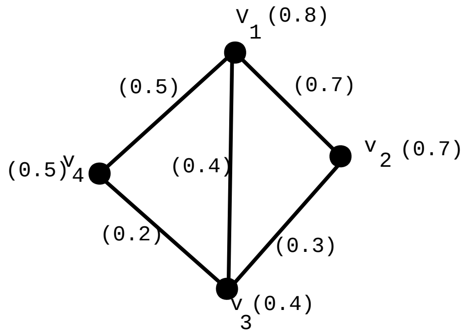


FIGURE 5

γ^{ef}-sets are $\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\}$. Therefore fig(5) is γ^{ef} fuzzy excellent but not γ^{ef} fuzzy just excellent. Note that $N[v_4]$ is equitably complete.

REMARK 3.3. $N[u]$ is fuzzy complete but not equitably fuzzy complete, just γ^{ef}-excellent.

REMARK 3.4. A just γ^{ef}-excellent graph need not be just γ^{ef}-excellent.

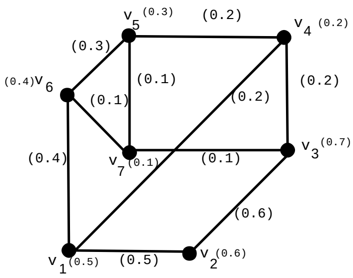


FIGURE 6

Every vertex in fig(6) is belongs to more than one γ^{ef} set. Hence fig(6)is not γ^{ef} just-excellent.

PROPOSITION 3.2. *If* $G \neq K_2$, $e(G) \neq \bar{K_n}$ *is just* γ^{ef}*-excellent then* $\delta^{ef}(G) \geqslant 2$.

PROOF. Suppose $\delta^{ef}(G) = 1$. Let u be a δ^{ef} vertex of G. Since G is just γ^{ef}-excellent,there exists a γ^{ef}-set S of G containing u. Since $deg^{ef}(u) = 1$ and $G \neq K_2$, $S - \{u\} = \emptyset$.

Let $S_1 = (S - \{u\}) \cup \{v\}$ where v is the unique neighbor of u which is fuzzy equitable with u. Then S_1 is a γ^{ef}-set of G. Therefore every vertex of $S - \{u\}$ lies in two γ^{ef}-sets of G namely S and S_1 a contradiction, Since G is just γ^{ef}-excellent. Therefore $\delta^{ef}(G) \neq 1$.

Suppose $\delta^{ef}(G) = 0$.Then there exists a vertex $u \in V(G)$ such that $N^{ef}(u) = \emptyset$. Therefore u is a fuzzy equitable isolates of G. Therefore u belongs to every γ^{ef}-set of G. Suppose there exists a unique γ^{ef}-set of G. Since G is just γ^{ef} excellent, every vertex of G belongs to the unique γ^{ef}-set of G. Therefore the unique $\gamma^{ef}-$set of G is $V(G)$. Therefore $\gamma^{ef}(G) = n$. But $\gamma^{ef}(G) = \gamma(e(G)) = n$. Therefore $e^{ef}(G) = \bar{K_n}$, a contradiction. Therefore there exists atleast two γ^{ef}-sets and of G and u belongs to every one of them,a contradiction. Since G is just γ^{ef}-excellent. Therefore $\delta^{ef}(G) \geqslant 0$ and $\delta^{ef}(G) \neq 1$. Therefore $\delta^{ef}(G) \geqslant 2$

□

PROPOSITION 3.3. *Let G be a γ^{ef}-just excellent graph. Then*

$$\delta^{ef}(G) \geqslant \frac{n}{\gamma^{ef}(G)} - 1$$

.

PROOF. Let G be a γ^{ef}-just excellent graph. Let $V(G) = S_1 \cup S_2 \cup \ldots \cup S_m$ be the partition of $V(G)$ into γ^{ef}-sets of G. Let $u \in V(G)$. Then u belongs to exactly one $S_j(1 \leqslant j \leqslant m)$. Therefore each $S_i(1 \leqslant i \leqslant m, i \neq j)$ being a γ^{ef}-sets of G, equitably dominates u. Therefore $N^{ef}(u) \geqslant m - 1$ and hence $deg^{ef}(u) \geqslant m - 1$. Therefore $\delta^{ef}(G) \geqslant m - 1$. Since $|V(G) = n$ and $|S_i = \gamma^{ef}(G)|$ for every i, $1 \leqslant i \leqslant mm, n = m\gamma^{ef}(G)$. Therefore $\delta^{ef}(G) \geqslant \frac{n}{\gamma^{ef}(G)} - 1$.

□

DEFINITION 3.3. A graph G is said to be fuzzy equitably connected if any two distinct vertices of G can be joined by a path in which degrees of adjacent vertices differ by atmost 1 and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$.

REMARK 3.5. A graph G is fuzzy equitably connected if and onlyif G^{ef} is connected. Indeed, Suppose $e^{ef}(G) \neq \bar{K_n}$. Suppose G is not equitably connected. Let G_1 be a equitably connected component of e^{ef} containing more than one vertex. Since G is γ^{ef}-just excellent,each component of $e^{ef}(G)$ is γ^{ef}-just excellent. Since G_1 is equitable connected, G_1 has atleast two γ^{ef}-sets say S_1, S_2. considering S_1 as a γ^{ef}-sets of G_1 and a γ^{ef}-sets T of $G - G_1$. $S_1 \cup T$ is a γ^{ef}-set of G. Also $S_2 \cup T$ is a γ^{ef}-set of G. Thus every vertex in T lies in $2\gamma^{ef}$-sets of G, a contradiction. Therefore G is equitably connected.

4. Applications

The Fuzzy relations are wide spread and important, especially in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expect system and also widely used in the fields of sociology, economics, ecology, meteorlogy, biology and others.

The applications of information theory are more extensive, they cover virtually the whole spectrum of science.

5. Conclusion

For graphical research, excellent and just-excellent fuzzy equitable domination graphs are useful represent persons in society with nearly equal status, tends to be friendly. This concepts are useful for solving wide range problems. In this paper we are introducing the excellent and just-excellent fuzzy graphs. Additional concepts, we are introduce a new class of excellent fuzzy graphs.

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