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Fixed Point Results for Rational type contraction in Partially ordered Complex Valued Metric Spaces

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Abstract. In this paper we present some results of fixed point theory in a recently introduced generalization of the metric space, that is, complex valued metric space where the metric assumes values in the set of complex number. The notion of inequality here is the rational type order inequality in a partial ordering of the complex numbers. The results are illustrated with examples. The work is a part of the rapid extension of metric fixed point theory to more generalized structures beyond the boundary of metric spaces.

1. Introduction and Preliminaries

Metric fixed point theory is widely recognized to have been originated in the work of S. Banach in 1922 [**4**] where he proved the famous contraction mapping principle. Banach's contraction mapping principle has very few parallels in modern science in terms of the influence it has exerted in the developments of different branches of mathematics and of physical science in general. Over the years metric fixed point theory has developed in different directions. A comprehensive account of this development is provided in the handbook entitled by Kirk and Sims [**23**].

Also there are large efforts for generalizing metric spaces by changing the form and interpretation of the metric function. Ghaler [**15**] introduced 2-metric spaces where a real number is assigned to any three points of the space. Probabilistic metric spaces were introduced by Schweizer et al ([**26**], [**27**]) in which any pair of points is assigned to a suitable distribution function making possible a probabilistic sense of distance. Fuzzy metric spaces were introduced in more than one ways by various means of fuzzification as, for example, in [**16**] by assigning any pair of points to a suitable fuzzy set and spelling out the triangular inequality by using a t-norm. Another example is in the work of Kaleva et al [**21**] where any pair of points is

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assigned to a fuzzy number. G-metric space [**25**] is another generalization in which every triplet of points is assigned to a non-negative real number but in a different way than in 2-metric spaces. There are also other extensions of the metric which are not mentioned above. It can be seen that in recent times efforts of extending the concept of metric space has continued in a rapid manner. Simultaneously, metric fixed point theory has been extended rapidly in these spaces over the recent years.

Complex valued metric spaces are generalization of metric space where the metric function takes values from the set of complex numbers, thus opening the scope of the concepts from complex analysis for incorporation in the metric space structure. The space was originally introduced by Azam et al [**2**]. Fixed point theory has been studied in this space in a suitable number of papers, some of which we mention in ([**7**], [**28**] - [**30**]). We begin with a description of complex valued metric space.

Let $\mathcal{C}$ be the set of complex numbers and $z_1,\ z_2 \in \mathcal{C}$. Define a partial order $\precsim$ on $\mathcal{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leqslant Re(z_2) \text{ and } Im(z_1) \leqslant Im(z_2).$$

It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

$$(i) \quad Re(z_1) = Re(z_2), \quad Im(z_1) < Im(z_2)$$

$$(ii) \quad Re(z_1) < Re(z_2), \quad Im(z_1) = Im(z_2)$$

$$(iii) \quad Re(z_1) < Re(z_2), \quad Im(z_1) < Im(z_2)$$

$$(iv) \quad Re(z_1) = Re(z_2), \quad Im(z_1) = Im(z_2).$$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied. Note that

$$z_1 \precsim z_2, \quad z_2 \prec z_3 \implies z_1 \prec z_3.$$

Definition 1.1 ([**2**]). Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathcal{C}$ satisfies:

(i) $0 \precsim d(x,\ y)$, for all $x,\ y \in X$ and $d(x, y) = 0$ if and only if $x = y$

(ii) $d(x,\ y) = d(y,\ x)$, for all $x,\ y \in X$

(iii) $d(x,\ y) \precsim d(x,\ z) + d(z,\ y)$, for all $x, y, z \in X$.

Then d is called a complex valued metric on X and $(X,\ d)$ is called a complex valued metric space.

Definition 1.2 ([**2**]). Let $(X,\ d)$ be a complex valued metric space, $\{x_n\}$ be a sequence in X and $x \in X$.
(i) If for every $c \in \mathcal{C}$ with $0 \prec c$ there is $n_0 \in \mathbf{N}$ such that for all $n > n_0$, $d(x_n,\ x) \prec c$, then $\{x_n\}$ said to be convergent and $\{x_n\}$ converges to x. We denote this by $\lim\limits_{n \longrightarrow \infty} x_n = x$, or $x_n \longrightarrow x$ as $n \longrightarrow \infty$.
(ii) If for every $c \in \mathcal{C}$ with $0 \prec c$ there is $n_0 \in \mathbf{N}$ such that for all $n,\ m > n_0$, $d(x_n,\ x_m) \prec c$, then $\{x_n\}$ is said to be a Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent, then $(X,\ d)$ is a complete complex valued metric space.

Lemma 1.1. Let $(X,\ d)$ be a complex valued metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ converges to x if and only if $d(x_n,\ x) \longrightarrow 0$ as $n \longrightarrow \infty$.

Note 1.1 We can also replace the limit in lemma 1.1 by the equivalent limiting condition $|d(x_n,\ x)| \longrightarrow 0$ as $n \longrightarrow \infty$.

Lemma 1.2. *Let $(X,\ d)$ be a complex valued metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n,\ x_m) \longrightarrow 0$ as $n, m \longrightarrow \infty$.*

Note 1.2 We can also replace the limit in lemma 1.2 by the equivalent limiting condition $|d(x_n,\ x_m)| \longrightarrow 0$ as $n,\ m \longrightarrow \infty$.

Definition 1.3. Let $(X,\ d)$ be a complex valued metric space, $T : X \longrightarrow X$ and $x \in X$. Then the function T is continuous at x if for any sequence $\{x_n\}$ in X,

$$x_n \longrightarrow x \quad \Longrightarrow \quad Tx_n \longrightarrow Tx.$$

In [**14**], Dass and Gupta generalized Banach contraction principle using a rational type contraction.

Theorem 1.1 ([**14**]). *Let (X, d) be a complete metric space and $T : X \to X$ a mapping such that there exist $\alpha, \beta \geqslant 0$ with $\alpha + \beta < 1$ satisfying*

$$d(Tx, Ty) \leqslant \alpha \frac{d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)} + \beta d(x, y), \text{ for all } x, y \in X. \tag{1.1}$$

Then T has a unique fixed point.

In [**8**], Cabrera, Harjani and Sadarangani proved the above theorem in the context of partially ordered metric spaces.

Definition 1.4. Let $(X,\ \preceq)$ be a partially ordered set and $T : X \longrightarrow X$. The mapping T is said to be nondecreasing if for all $x_1,\ x_2 \in X$, $x_1 \preceq x_2$ implies $Tx_1 \preceq Tx_2$ and nonincreasing if for all $x_1,\ x_2 \in X$, $x_1 \preceq x_2$ implies $Tx_1 \succeq Tx_2$.

Theorem 1.2 ([**8**]). *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Let $T : X \to X$ be a continuous and nondecreasing mapping such that (1.1) is satisfied for all $x, y \in X$ with $x \preceq y$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$ then T has a fixed point.*

Theorem 1.3 ([**8**]). *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Assume that if $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \to x$, then $x_n \preceq x$, for all $n \in N$. Let $T : X \to X$ be a nondecreasing mapping such that (1.1) is satisfied for all $x, y \in X$ with $x \preceq y$. If there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$ then T has a fixed point.*

Fixed point theorems for contractive type conditions satisfying rational inequalities in metric spaces have been developed in a number of works ([**9**], [**10**], [**17**], [**19**], [**20**], [**24**]).

Rational inequalities cannot be considered in cone metric spaces [**18**], but can be constructed in complex valued metric spaces. We discuss this elaborately in remark 2.3.

The concept of almost contractions were introduced by Berinde ([**5**],[**6**]).

Definition 1.5 ([**5**]). Let (X, d) be a metric space. A mapping $T : X \longrightarrow X$ is called an almost contraction if there exist $k \in (0, 1)$ and some $L \geqslant 0$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leqslant kd(x, y) + Ld(y, Tx).$$

It was shown in [**5**] that any strict contraction, the Kannan [**22**] and Zamfirescu [**31**] mappings, as well as a large class of quasi-contractions, are all almost contractions. Almost contractions and its generalizations were further considered in several works like ([**1**], [**3**], [**11**], [**12**], [**13**]).

The purpose of this paper is to study fixed points of a class of mappings satisfying a rational type almost contraction in the frame-work of a complex valued ordered metric space. Our results are supported with examples.

2. Main Results

Theorem 2.1. *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there exists a complex valued metric d on X such that $(X,\ d)$ is complete complex valued metric space. Let $T : X \longrightarrow X$ be a continuous and nondecreasing mapping. Suppose there exist nonnegative real numbers $\alpha, \beta, \gamma, \delta, \lambda, \mu$ and L with $\alpha + 2\beta + 2\gamma + \delta + \mu < 1$ such that for all $x,\ y \in X$ with $x \preceq y$,*

(2.1)
$$\begin{aligned} d(Tx,\ Ty) \precsim \alpha\ d(x,\ y) + \beta\ [d(x,\ Tx) + d(y,\ Ty)] + \gamma\ [d(y,\ Tx) + d(x,\ Ty)] \\ + \delta\ \frac{d(y,\ Ty)\ [1 + d(x,\ Tx)]}{1 + d(x,\ y)} + \lambda\ \frac{d(y,\ Tx)\ [1 + d(x,\ Ty)]}{1 + d(x,\ y)} \\ + \mu\ \frac{d(x,\ y)\ [1 + d(x,\ Tx) + d(y,\ Tx)]}{1 + d(x,\ y)} + L\ d(y,\ Tx). \end{aligned}$$

If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point.

Proof. If $x_0 = Tx_0$, then we have the result. Suppose that $x_0 \prec Tx_0$. Then we construct a sequence $\{x_n\}$ in X such that

$$x_{n+1} = Tx_n, \quad \text{for every} \quad n \geqslant 0. \tag{2.2}$$

Since T is a nondecreasing mapping, we obtain by induction that

$$x_0 \prec Tx_0 = x_1 \preceq Tx_1 = x_2 \preceq \ldots \preceq Tx_{n-1} = x_n \preceq Tx_n = x_{n+1} \preceq \ldots \tag{2.3}$$

If there exists some $N \geqslant 1$ such that $x_{N+1} = x_N$, then from (2.2), $x_{N+1} = Tx_N = x_N$, that is, x_N is a fixed point of T and the proof is finished. So, we suppose that

$x_{n+1} \neq x_n$, for all $n \geqslant 1$. Since $x_n \prec x_{n+1}$, for all $n \geqslant 1$, applying (2.1) we have

$$\begin{aligned} d(Tx_n,\ Tx_{n+1}) &\precsim \alpha\ d(x_n,\ x_{n+1}) + \beta\ [d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2})] \\ &\quad +\gamma\ [d(x_{n+1},\ x_{n+1}) + d(x_n,\ x_{n+2})] \\ &+\delta\ \frac{d(x_{n+1},\ x_{n+2})\ [1 + d(x_n,\ x_{n+1})]}{1 + d(x_n,\ x_{n+1})} + \lambda\ \frac{d(x_{n+1},\ x_{n+1})\ [1 + d(x_n,\ x_{n+2})]}{1 + d(x_n,\ x_{n+1})} \\ &+\mu\ \frac{d(x_n,\ x_{n+1})}{1 + d(x_n,\ x_{n+1})}\ [1 + d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+1})] + L\ d(x_{n+1},\ x_{n+1}), \end{aligned}$$

that is,

$$\begin{aligned} d(x_{n+1},\ x_{n+2}) &\precsim \alpha\ d(x_n,\ x_{n+1}) + \beta\ [d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2})] \\ &\quad + \gamma\ d(x_n,\ x_{n+2}) + \delta\ d(x_{n+1},\ x_{n+2}) + \mu\ d(x_n,\ x_{n+1}) \\ &\precsim \alpha\ d(x_n,\ x_{n+1}) + \beta\ [d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2})] \\ &\quad + \gamma\ [d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2})] + \delta\ d(x_{n+1},\ x_{n+2}) + \mu\ d(x_n,\ x_{n+1}), \end{aligned}$$

which implies that

$$d(x_{n+1},\ x_{n+2}) \precsim \frac{\alpha+\beta+\gamma+\mu}{1-\beta-\gamma-\delta}\ d(x_n,\ x_{n+1}). \tag{2.4}$$

Now, $\alpha+2\beta+2\gamma+\delta+\mu < 1$ implies that $\frac{\alpha+\beta+\gamma+\mu}{1-\beta-\gamma-\delta} < 1$. Put $\frac{\alpha+\beta+\gamma+\mu}{1-\beta-\gamma-\delta} = k$. Then by repeated application (2.4), we have

$$d(x_{n+1},\ x_{n+2}) \precsim k\ d(x_n,\ x_{n+1}) \precsim k^2\ d(x_{n-1},\ x_n) \precsim ... \precsim k^{n+1}\ d(x_0,\ x_1). \tag{2.5}$$

For any $m > n$,

$$\begin{aligned} d(x_m,\ x_n) &\precsim d(x_n,\ x_{n+1}) + d(x_{n+1},\ x_{n+2}) + ... + d(x_{m-1},\ x_m) \\ &\precsim [k^n + k^{n+1} + k^{n+2} +\ ...\ + k^{m-1}]\ d(x_0,\ x_1) \\ &\precsim \frac{k^n}{1-k}\ d(x_0,\ x_1) \longrightarrow 0 \text{ as } n \longrightarrow \infty, \end{aligned}$$

which implies that, $\{x_n\}$ is a Cauchy sequence. From the completeness of X, there exists $z \in X$ such that

$$x_n \longrightarrow z \quad \text{as} \quad n \longrightarrow \infty. \tag{2.6}$$

The continuity of T implies that $Tz = \lim_{n\longrightarrow\infty} Tx_n = \lim_{n\longrightarrow\infty} x_{n+1} = z$, that is, z is a fixed point of T. □

In our next theorem we relax the continuity assumption of the mapping T in Theorem 2.1 by imposing the following order condition of the complex valued metric space X:

If $\{x_n\}$ is a non-decreasing sequence in X such that $x_n \longrightarrow x$, then $x_n \preceq x$, for all $n \in \mathbf{N}$.

THEOREM 2.2. *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there exists a complex valued metric d on X such that $(X,\ d)$ is complete complex valued metric space. Assume that if $\{x_n\}$ is a nondecreasing sequence in X such that*

$x_n \longrightarrow x$, then $x_n \preceq x$, for all $n \in \mathbf{N}$. Let $T : X \longrightarrow X$ be a nondecreasing mapping such that for all $x,\ y \in X$ with $x \preceq y$, (2.1) is satisfied, where the condition on $\alpha, \beta, \gamma, \delta, \lambda, \mu$ and L are same as in theorem 2.1. If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point.

Proof. We take the same sequence $\{x_n\}$ as in the proof of theorem 2.1. Arguing like in the proof of theorem 2.1, we prove that $\{x_n\}$ is a nondecreasing sequence which satisfies (2.6), that is, $x_n \longrightarrow z$ as $n \to \infty$. Then, by the conditions of the theorem $x_n \preceq z$, for all $n \in \mathbf{N}$. Applying (2.1), we have

$$\begin{aligned} & d(x_{n+1},\, Tz) = \\ & d(Tx_n,\, Tz) \precsim \alpha\ d(x_n,\, z) + \beta\ [d(x_n,\, x_{n+1}) + d(z,\, Tz)] + \gamma\ [d(z,\, x_{n+1}) + d(x_n,\, Tz)] \\ & \qquad + \delta\ \frac{d(z,\, Tz)\ [1 + d(x_n,\, x_{n+1})]}{1 + d(x_n,\, z)} + \lambda\ \frac{d(z,\, x_{n+1})\ [1 + d(x_n,\, Tz)]}{1 + d(x_n,\, z)} \\ & \qquad + \mu\ \frac{d(x_n,\, z)}{1 + d(x_n,\, z)}\ [1 + d(x_n,\, x_{n+1}) + d(z,\, x_{n+1})] + L\ d(z,\, x_{n+1}). \end{aligned}$$

Taking the limit as $n \longrightarrow \infty$ in the above inequality and using (2.6), we have

$$d(z,\, Tz) \precsim (\beta + \gamma + \delta) d(z,\, Tz).$$

Since $(\beta + \gamma + \delta) < 1$, it is a contradiction unless $d(z,\ Tz) = 0$, that is, $Tz = z$, that is, z is a fixed point of T. $\square$

Example 2.1. Let $X = [a,\ b]$ where $1 < a < b$. Partial order $'\preceq'$ is defined as $x \preceq y$ iff $x \geqslant y$. Let the complex valued metric d be given as

$$d(x,\ y) = |x - y|\ \sqrt{2}\ e^{i\frac{\pi}{4}} = |x - y|(1 + i),\ \text{ for } x,\ y \in X.$$

Let $T : X \longrightarrow X$ be defined as follows:

$$Tx = x + \frac{1}{x} - \frac{1}{b}.$$

Let $1 - \dfrac{1}{b^2} \leqslant \alpha < 1$ and $\beta, \gamma, \delta, \lambda, \mu$ and L are arbitrary non-negetive real numbers such that $\alpha + 2\beta + 2\gamma + \delta + \mu < 1$. Here all the conditions of Theorems 2.1 and 2.2 are satisfied and it is seen that b is a fixed point of T.

Example 2.2. Let $X = [1.5,\ 2]$ with usual prtial order $'\leqslant'$. Let us consider the complex valued metric d as defined in example 2.1.

Let $T : X \longrightarrow X$ be defined as follows:

$$Tx = \begin{cases} 1.81, & \text{if } \ 1.5 \leqslant x < 1.75, \\ x + \frac{1}{x} - \frac{1}{2}, & \text{if } \ 1.75 \leqslant x \leqslant 2. \end{cases}$$

Let $\dfrac{3}{4} \leqslant \alpha < 1,\ \beta = \gamma = \delta = \mu = \lambda = 0$ and $L \geqslant 3$.

Here all the conditions of Theorem 2.2 are satisfied and $x = 2$ is a fixed point of T.

Remark 2.1. In the above example the function T is not continuous and hence it is not applicable to theorem 2.1.

Remark 2.2. By restricting the metric values to real numbers, we have the usual metric space in which case the results obtained in [**8**] which are also stated here as theorems 1.4 and 1.5, are obtained by an application of our theorems.

Remark 2.3. Complex valued metric spaces have close similarities with cone metric spaces in its structure, although conceptually they are very different. In cone metric spaces the metric takes up values in linear spaces over the real field where the linear space may be infinite dimensional, whereas in the case of complex valued metric spaces the metric values are in the set of complex number which is a one dimensional vector space over the complex field. The type of rational inequality we consider here is not meaningful in a cone metric space. This is an instance which implies why fixed point theory should be pursued independently in a complex valued metric space.

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