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Nonexistence of positive solutions for a system of higher-order nonlinear boundary value problems

Sabbavarapu Nageswara Rao

Abstract. We determine intervals for two eigenvalues for which there exists no positive solution of a system of nonlinear higher-order ordinary differential equations

$$(-1)^m u^{(2m)} = \lambda f(t, u(t), v(t)) = 0, \ t \in [a, b],$$
$$(-1)^n v^{(2n)} = \mu g(t, u(t), v(t)) = 0, \ t \in [a, b],$$

subject to the two-point boundary conditions

$$u^{(2i)}(a) = u^{(2i)}(b) = 0, \ 0 \leqslant i \leqslant m-1,$$
$$v^{(2j)}(a) = v^{(2j)}(b) = 0, \ 0 \leqslant j \leqslant n-1,$$

where $\lambda, \mu > 0, \ m, n \in \mathbb{N}$.

1. Introduction

Boundary value problem (BVPs) play a major role in many fields of engineering design and manufacturing. Major established industries such as automobile, aerospace, chemical, pharmaceutical, petroleum, electronics and communications, as well as emerging technologies such as nanotechnology and biotechnology rely on the BVPs to simulate complex phenomena at different scales for design and manufactures of high-technology products. In these applied settings, positive solutions are meaningful [**1, 5**]. Due to their important role in both theory and applications, the BVPs have generated a great deal of interest in recent years.

In the last decades, nonlocal boundary value problems for ordinary differential or difference equations\system have become a rapidly growing area of research. Several phenomena in engineering, physics, and life sciences can be modelled in this way. These problems have been studied by many authors by using different

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methods, such as fixed point theorems in cones, the Leray-Schauder continuation theorem, nonlinear alternatives of Leray-Schauder and coincidence degree theory.

In this paper, we consider the system of nonlinear higher-order ordinary differential equations

$$\begin{aligned} (-1)^m u^{(2m)} &= \lambda f(t, u(t), v(t)) = 0,\ t \in [a, b], \\ (-1)^n v^{(2n)} &= \mu g(t, u(t), v(t)) = 0,\ t \in [a, b], \end{aligned} \tag{1.1}$$

satisfying the two-point boundary conditions

$$\begin{aligned} u^{(2i)}(a) &= u^{(2i)}(b) = 0,\ 0 \leqslant i \leqslant m-1, \\ v^{(2j)}(a) &= v^{(2j)}(b) = 0,\ 0 \leqslant j \leqslant n-1, \end{aligned} \tag{1.2}$$

where $\lambda, \mu > 0,\ m, n \in \mathbb{N}$.

The aim of this paper, we establish intervals for the eigenvalues λ and μ such that there exists no positive solutions of problem (1.1)-(1.2). By a positive solution of (1.1)-(1.2) we mean a pair of function $(u, v) \in C^m([a, b]) \times C^n([a, b])$ satisfying (1.1) and (1.2) with $u(t) \geqslant 0,\ v(t) \geqslant 0$ for all $t \in [a, b]$ and $(u, v) \neq (0.0)$. The existence of positive solutions for the above problem was investigated in [**24**] by using the Guo-Krasnosel'skii fixed point theorem. Some particular cases of the problem from [**24**] have been studied in [**9, 11, 18, 22, 23**].

The following assumptions are made to establish our results:
$(A1)$ The functions $f, g \in C[[a, b] \times [0, \infty) \times [0, \infty), [0, \infty)]$,
$(A2)$ The limits

$$\begin{aligned} f_0^s &= \lim_{u+v \to 0^+} \sup_{t \to [a,b]} \frac{f(t,u,v)}{u+v},\ g_0^s = \lim_{u+v \to 0^+} \sup_{t \to [a,b]} \frac{g(t,u,v)}{u+v}, \\ f_0^i &= \lim_{u+v \to 0^+} \inf_{t \to [a,b]} \frac{f(t,u,v)}{u+v},\ g_0^i = \lim_{u+v \to 0^+} \inf_{t \to [a,b]} \frac{g(t,u,v)}{u+v}, \\ f_\infty^s &= \lim_{u+v \to \infty} \sup_{t \to [a,b]} \frac{f(t,u,v)}{u+v},\ g_\infty^s = \lim_{u+v \to \infty} \sup_{t \to [a,b]} \frac{g(t,u,v)}{u+v}, \\ f_\infty^i &= \lim_{u+v \to \infty} \inf_{t \to [a,b]} \frac{f(t,u,v)}{u+v},\ g_\infty^i = \lim_{u+v \to \infty} \inf_{t \to [a,b]} \frac{g(t,u,v)}{u+v}, \end{aligned}$$

exist.

The rest of the paper is organized as follows. In Section 2, we present some preliminaries and lemmas that will be used to prove our main results. In Section 3, we will consider the conditions of the nonexistence of a positive solution.

2. Preliminary results

In this section, we present some notations and lemmas that will be to prove our results. Here we consider the Banach space $C[a, b] \times C[a, b]$ equipped with the standard norm

$$\| (u,v) \| = \| u \| + \| v \| = \max_{t\in[a,b]} | u(t) | + \max_{t\in[a,b]} | v(t) |, \ (u,v) \in C[a,b] \times C[a,b].$$

Let $G_n(t,s)$ be the Green's function of a homogeneous boundary value problem:

$$(-1)^n w^{(2n)}(t) = 0, \ t \in [a,b]$$
$$w^{(2i)}(a) = w^{(2i)}(b) = 0, \ 0 \leqslant i \leqslant n-1.$$

By induction, the Green's function $G_n(t,s)$ can be expressed as (see [1])

$$G_i(t,s) = \int_a^b G(t,u)G_{i-1}(u,s)du, \ 2 \leqslant i \leqslant n, \tag{2.1}$$

where

$$G_1(t,s) = G(t,s) = \begin{cases} \frac{(t-a)(b-s)}{b-a}, & a \leqslant t \leqslant s \leqslant b, \\ \frac{(s-a)(b-t)}{b-a}, & a \leqslant s \leqslant t \leqslant b. \end{cases} \tag{2.2}$$

It is clear that

$$G_n(t,s) > 0, \ (t,s) \in (a,b) \times (a,b). \tag{2.3}$$

Lemma 2.1. **[23]** *For any* $(t,s) \in [a,b] \times [a,b]$,

$$G_n(t,s) \leqslant \Big(\frac{b-a}{6}\Big)^{n-1} \frac{(s-a)(b-s)}{b-a}. \tag{2.4}$$

Proof. For $(t,s) \in [a,b] \times [a,b]$, it is clear from (2.2) that

$$G(t,s) \leqslant \frac{(s-a)(b-s)}{b-a}. \tag{2.5}$$

i.e (2.4) is true for $n = 1$. Assume that (2.4) holds for $n = k (k \geqslant 1)$. Then, for $(t,s) \in [a,b] \times [a,b]$, it follows from (2.1), (2.3) and (2.5) that

$$\begin{aligned} G_{k+1}(t,s) &= \int_a^b G(t,u)G_k(u,s)du \\ &\leqslant \int_a^b \frac{(u-a)(b-u)}{b-a}\Big(\frac{b-a}{6}\Big)^{k-1}\frac{(s-a)(b-s)}{b-a}du \\ &= \Big(\frac{b-a}{6}\Big)^{k}\frac{(s-a)(b-s)}{b-a}. \end{aligned}$$

Thus (2.4) is true for $n = k+1$. □

Lemma 2.2. *Let* $\delta \in (a, \frac{a+b}{2})$, *then for all* $(t,s) \in [\delta, b-\delta] \times [a,b]$, *we have*

$$G_n(t,s) \geqslant \theta_n(\delta)\frac{(s-a)(b-s)}{b-a} \geqslant \Big(\frac{6}{b-a}\Big)^{n-1}\theta_n(\delta) \max_{t\in[a,b]} G_n(t,s), \tag{2.6}$$

where $0 < \theta_n(\delta) < 1$ *is a constant given by*

$$\theta_n(\delta) = (\delta - a)^n \left(\frac{4\delta^3 - 6b\delta^2 + 6ab\delta - 3ab^2 + b^3}{6(b-a)}\right)^{n-1}$$

PROOF. For $(t,s) \in [\delta, b-\delta] \times [a,b]$, from (2.2) we find

$$G(t,s) = \begin{cases} \frac{(t-a)(b-s)}{b-a}, & t \leqslant s \\ \frac{(s-a)(b-t)}{b-a}, & s \leqslant t \end{cases}$$

$$\geqslant \begin{cases} \frac{(\delta-a)(b-s)}{b-a}, & t \leqslant s \\ \frac{(s-a)(b-(b-\delta))}{b-a}, & s \leqslant t \end{cases} \tag{2.7}$$

$$\geqslant \frac{(\delta-a)(s-a)(b-s)}{b-a}.$$

Hence (2.6) is true for $n = 1$. Suppose now that (2.6) holds for $n = k (k \geqslant 1)$. Then, using (2.1), (2.3) and (2.7), we get for $(t,s) \in [\delta, b-\delta] \times [a,b]$

$$\begin{aligned} G_{k+1}(t,s) &= \int_a^b G(t,u)G_k(u,s)du \\ &\geqslant \int_b^{b-\delta} G(t,u)G_k(u,s)du \\ &\geqslant \int_\delta^{b-\delta} \frac{(\delta-a)(u-a)(b-u)}{b-a}\theta_k(\delta)\frac{(s-a)(b-s)}{b-a}du \\ &= \theta_{k+1}(\delta)\frac{(s-a)(b-s)}{b-a}. \end{aligned}$$

So, (2.6) is true for $n = k+1$.
In Lemma 2.2, let

$$\begin{aligned} \gamma_m &= \Big(\frac{6}{b-a}\Big)^{m-1}\theta_m\Big(\frac{3a+b}{4}\Big) = \frac{\big(11b^3+27a^3-51ab^2+45a^2b\big)^{m-1}}{2^{6m-4}(b-a)^{m-2}}, \\ \gamma_m &= \Big(\frac{6}{b-a}\Big)^{n-1}\theta_n\Big(\frac{3a+b}{4}\Big) = \frac{\big(11b^3+27a^3-51ab^2+45a^2b\big)^{n-1}}{2^{6n-4}(b-a)^{n-2}}, \\ \gamma &= \min\{\gamma_m, \gamma_n\}. \end{aligned}$$

According to Lemma 2.1 and Lemma 2.2, one obviously has $0 < \gamma < 1$.

It is well know that the system (1.1)-(1.2) is equivalent to the equation

$$\begin{cases} u(t) = \lambda \int_a^b G_m(t,s)f(s,u(s),v(s))ds, & a \leqslant t \leqslant b, \\ v(t) = \mu \int_a^b G_n(t,s)g(s,u(s),v(s))ds, & a \leqslant t \leqslant b. \end{cases}$$

For $\lambda, \mu > 0$, we define the operators $Q_\lambda, Q_\mu : C[a,b] \times C[a,b] \to C[a,b]$ as

$$\begin{aligned} Q_\lambda(u,v)(t) &= \lambda \int_a^b G_m(t,s)f(s,u(s),v(s))ds, \quad a \leqslant t \leqslant b, \\ Q_\mu(u,v)(t) &= \mu \int_a^b G_n(t,s)g(s,u(s),v(s))ds, \quad a \leqslant t \leqslant b, \end{aligned}$$

and an operator $Q : C[a,b] \times C[a,b] \to C[a,b] \times C[a,b]$ as

$$Q(u,v) = \Big(Q_\lambda(u,v), Q_\mu(u,v)\Big),\ (u,v) \in C[a,b] \times C[a,b]. \tag{2.8}$$

It is clear that the existence of a positive solution to the system (1.1)-(1.2) is equivalent to the existence of a fixed point of Q in $C[a,b] \times C[a,b]$.

We define a cone κ in $C[a,b] \times C[a,b]$ by

$$\kappa = \Big\{(u,v) : C[a,b] \times C[a,b] : u(t) \geqslant 0,\ v(t) \geqslant 0,$$
$$\min_{t\in\ [\frac{3a+b}{4}, \frac{a+3b}{4}]} (u(t) + v(t)) \geqslant \gamma \parallel (u,v) \parallel \Big\}.$$

□

Lemma 2.3. *$Q : \kappa \to \kappa$ is completely continuous.*

Proof. Since the proof of the completely continuous is standard, we need only to prove $Q(\kappa) = \kappa$.

In fact, for any $(t,s) \in\ [\frac{3a+b}{4}, \frac{a+3b}{4}] \times [a,b]$, we have

$$\begin{aligned}
&Q_\lambda(u,v)(t) + Q_\mu(u,v)(t)\\
&\quad= \lambda \int_a^b G_m(t,s) f(s,u(s),v(s))ds + \mu \int_a^b G_n(t,s) g(s,u(s),v(s))ds\\
&\quad\geqslant \lambda\theta_m \Big(\frac{3a+b}{4}\Big)\Big(\frac{6}{b-a}\Big)^{m-1} \max_{t\in[a,b]} \int_a^b G_m(t,s) f(s,u(s),v(s))ds\\
&\quad\ + \mu\theta_n \Big(\frac{3a+b}{4}\Big)\Big(\frac{6}{b-a}\Big)^{n-1} \max_{t\in[a,b]} \int_a^b G_n(t,s) g(s,u(s),v(s))ds\\
&\quad= \gamma_m \parallel Q_\lambda(u,v) \parallel + \gamma_n \parallel Q_\mu(u,v) \parallel \geqslant \gamma \parallel Q(u,v) \parallel,
\end{aligned}$$

hence,

$$\min_{t\in[\frac{3a+b}{4}, \frac{a+3b}{4}]} [Q_\lambda(u,v)(t) + Q_\mu(u,v)(t)] \geqslant \gamma \parallel Q(u,v) \parallel .$$

Therefore, $Q(\kappa) \subset \kappa$. □

3. Main Results

In this section, we give some sufficient conditions for the nonexistence of positive solution to the BVP (1.1)-(1.2).

Theorem 3.1. *Assume that (A1) − (A2) hold. If $f_0^s, f_\infty^s, g_0^s, g_\infty^s < \infty$, then there exists positive constants λ_0, μ_0 such that for every $\lambda \in (0, \lambda_0)$ and $\mu \in (0, \mu_0)$, the boundary value problem (1.1)-(1.2) has no positive solution.*

Proof. Since $f_0^s, f_\infty^s < \infty$, we deduce that there exist $M_1^{'}, M_1^{''}, r_1, r_1^{'} > 0, r_1 < r_1^{'}$ such that

$$f(t,u,v) \leqslant M_1^{'}(u+v),\ \forall\ u,v \geqslant 0,\ u+v \in [0, r_1]$$
$$f(t,u,v) \leqslant M_1^{''}(u+v), \forall\ u,v \geqslant 0,\ u+v \in [r_1^{'}, \infty).$$

We consider $M_1 = \max\left\{M_1^{'}, M_1^{''}, \max_{r_1 \leqslant u+v \leqslant r_1^{'}} \frac{f(t,u,v)}{u+v}\right\} > 0$. Then, we obtain $f(t,u,v) \leqslant M_1(u+v),\ \forall\ u,v \geqslant 0$. Since $g_0^s, g_\infty^s < \infty$, we deduce that there exist $M_2^{'}, M_2^{''}, r_2, r_2^{'} > 0, r_2 < r_2^{'}$ such that

$$g(t,u,v) \leqslant M_2^{'}(u+v),\ \forall\ u,v \geqslant 0,\ \ u+v \in [0, r_2]$$
$$g(t,u,v) \leqslant M_2^{''}(u+v), \forall\ u,v \geqslant 0,\ \ u+v \in [r_2^{'}, \infty).$$

We consider $M_2 = \max\left\{M_2^{'}, M_2^{''}, \max_{r_2 \leqslant u+v \leqslant r_2^{'}} \frac{g(t,u,v)}{u+v}\right\} > 0$. Then, we obtain $g(t,u,v) \leqslant M_2(u+v),\ \forall\ u,v \geqslant 0$. We define $\lambda_0 = \frac{1}{2M_1B}$ and $\mu_0 = \frac{1}{2M_2D}$, where

$$B = \left(\frac{b-a}{6}\right)^{m-1} \int_a^b \frac{(s-a)(b-s)}{b-a} ds \text{ and}$$
$$D = \left(\frac{b-a}{6}\right)^{n-1} \int_a^b \frac{(s-a)(b-s)}{b-a} ds.$$

We shall show that for every $\lambda \in (0, \lambda_0)$ and $\mu \in (0, \mu_0)$, the problem (1.1)-(1.2) has no positive solution.

Let $\lambda \in (0, \lambda_0)$ and $\mu \in (0, \mu_0)$. We suppose that (1.1)-(1.2) has a positive solution $(u(t), v(t)), t \in [a, b]$. Then, we have

$$\begin{aligned} u(t) = Q_\lambda(u,v)(t) &= \lambda \int_a^b G_m(t,s) f(s, u(s), v(s)) ds \\ &\leqslant \lambda\Big(\frac{b-a}{6}\Big)^{m-1} \int_a^b \frac{(s-a)(b-s)}{b-a} f(s, u(s), v(s)) ds \\ &\leqslant \lambda\Big(\frac{b-a}{6}\Big)^{m-1} \int_a^b \frac{(s-a)(b-s)}{b-a} M_1(u(s)+v(s)) ds \\ &\leqslant \lambda M_1\Big(\frac{b-a}{6}\Big)^{m-1} \int_a^b \frac{(s-a)(b-s)}{b-a} (\parallel u \parallel + \parallel v \parallel) ds \\ &= \lambda M_1 B \parallel (u,v) \parallel,\ \forall\ t \in [a,b]. \end{aligned}$$

Therefore, we conclude

$$\parallel u \parallel \leqslant \lambda M_1 B \parallel (u,v) \parallel < \lambda_0 M_1 B \parallel (u,v) \parallel = \frac{1}{2} \parallel (u,v) \parallel .$$

In a similar manner,

$$\begin{aligned}
v(t) = Q_\mu(u,v)(t) &= \mu \int_a^b G_n(t,s) f(s,u(s),v(s))ds \\
&\leqslant \mu\Big(\frac{b-a}{6}\Big)^{n-1} \int_a^b \frac{(s-a)(b-s)}{b-a} f(s,u(s),v(s))ds \\
&\leqslant \mu\Big(\frac{b-a}{6}\Big)^{n-1} \int_a^b \frac{(s-a)(b-s)}{b-a} M_2(u(s)+v(s))ds \\
&\leqslant \mu M_2\Big(\frac{b-a}{6}\Big)^{n-1} \int_a^b \frac{(s-a)(b-s)}{b-a} (\| u \| + \| v \|)ds \\
&= \mu M_2 D \| (u,v) \|, \ \forall\, t \in [a,b].
\end{aligned}$$

Therefore, we conclude

$$\| v \| \leqslant \mu M_2 D \| (u,v) \| < \mu_0 M_2 D \| (u,v) \| = \frac{1}{2} \| (u,v) \| .$$

Hence, $\| (u,v) \| = \| u \| + \| v \| < \frac{1}{2} \| (u,v) \| + \frac{1}{2} \| (u,v) \| = \| (u,v) \|$, which is a contradiction. So, the boundary value problem (1.1)-(1.2) has no positive solution. ☐

THEOREM 3.2. *Assume that* $(A1)-(A2)$ *hold.*
(i) If $f_0^i, f_\infty^i > 0$, *then there exists a positive constant* $\tilde{\lambda_0}$ *such that for every* $\lambda > \tilde{\lambda_0}$ *and* $\mu > 0$, *the boundary value problem* (1.1)-(1.2) *has no positive solution.*
(ii) If $g_0^i, g_\infty^i > 0$, *then there exists a positive constant* $\tilde{\mu_0}$ *such that for every* $\mu > \tilde{\mu_0}$ *and* $\lambda > 0$, *the boundary value problem* (1.1)-(1.2) *has no positive solution.*
(iii) If $f_0^i, f_\infty^i, g_0^i, g_\infty^i > 0$, *then there exist positive constants* $\tilde{\tilde{\lambda_0}}$ *and* $\tilde{\tilde{\mu_0}}$ *such that for every* $\lambda > \tilde{\tilde{\lambda_0}}$ *and* $\mu > \tilde{\tilde{\mu_0}}$, *the boundary value problem* (1.1)-(1.2) *has no positive solution.*

PROOF. (i) Since $f_0^i, f_\infty^i > 0$, we deduce that there exist $m_1^{'}, m_1^{''}, r_3, r_3^{'} > 0, r_3 < r_3^{'}$ such that

$$\begin{aligned}
f(t,u,v) &\geqslant m_1^1(u+v), \ \forall\, u,v \geqslant 0, \ u+v \in [0,r_3] \\
f(t,u,v) &\geqslant m_1^{11}(u+v), \forall\, u,v \geqslant 0, \ u+v \in [r_3^{'},\infty).
\end{aligned}$$

We introduce $m_1 = \min\Big\{m_1^{'}, m_1^{''}, \min_{r_3 \leqslant u+v \leqslant r_3^{'}} \frac{f(t,u,v)}{u+v}\Big\} > 0$. Then, we obtain $f(t,u,v) \geqslant m_1(u+v), \ \forall\, u,v \geqslant 0$. We define $\tilde{\lambda_0} = \frac{1}{\gamma m_1 A} > 0$, where

$$A = \frac{11b^3 - 11a^3 + 33a^2b - 33ab^2}{96(b-a)} \theta_m\Big(\frac{3a+b}{4}\Big).$$

We shall show that for every $\lambda > \tilde{\lambda_0}$ and $\mu > 0$ the problem (1.1)-(1.2) has no positive solution.

Let $\lambda > \tilde{\lambda_0}$ and $\mu > 0$. We suppose that (1.1)-(1.2) has a positive solution $(u(t), v(t)), t \in [a, b]$. Then, we obtain

$$\begin{aligned}
u(t) = Q_\lambda(u,v)(t) &= \lambda \int_a^b G_m(t,s) f(s,u(s),v(s))ds \\
&\geqslant \lambda \theta_m\Big(\frac{3a+b}{4}\Big) \int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a} f(s,u(s),v(s))ds \\
&\geqslant \lambda \theta_m\Big(\frac{3a+b}{4}\Big) \int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a} m_1(u(s)+v(s))ds \\
&\geqslant \lambda m_1 \theta_m\Big(\frac{3a+b}{4}\Big) \int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a} \gamma \parallel (u,v) \parallel ds \\
&= \frac{11b^3 - 11a^3 + 33a^2b - 33ab^2}{96(b-a)} \theta_m\Big(\frac{3a+b}{4}\Big) \lambda\gamma m_1 \parallel (u,v) \parallel \\
&= \lambda\gamma m_1 A \parallel (u,v) \parallel .
\end{aligned}$$

Therefore, we deduce

$$\parallel u \parallel \geqslant u(t) \geqslant \lambda\gamma m_1 A \parallel (u,v) \parallel > \tilde{\lambda_0}\gamma m_1 A \parallel (u,v) \parallel = \parallel (u,v) \parallel .$$

and so,$\parallel (u,v) \parallel = \parallel u \parallel + \parallel v \parallel \geqslant \parallel u \parallel > \parallel (u,v) \parallel$, which is a contradiction. Therefore, the boundary value problem (1.1)-(1.2) has no positive solution.

(ii) Since $g_0^i, g_\infty^i > 0$, we deduce that there exist $m_2^{'}, m_2^{''}, r_4, r_4^{'} > 0, r_4 < r_4^{'}$ such that

$$\begin{aligned}
&g(t,u,v) \geqslant m_2^{'}(u+v),\ \forall\ u,v \geqslant 0,\ u+v \in [0, r_4] \\
&g(t,u,v) \geqslant m_2^{''}(u+v),\ \forall\ u,v \geqslant 0,\ u+v \in [r_4^{'}, \infty).
\end{aligned}$$

We introduce $m_2 = \min\Big\{m_2^{'}, m_2^{''}, \min_{r_4 \leqslant u+v \leqslant r_4^{'}} \frac{g(t,u,v)}{u+v}\Big\} > 0$. Then, we obtain $g(t,u,v) \geqslant m_2(u+v),\ \forall\ u,v \geqslant 0$. We define $\tilde{\mu_0} = \frac{1}{\gamma m_2 C} > 0$, where

$$C = \frac{11b^3 - 11a^3 + 33a^2b - 33ab^2}{96(b-a)} \theta_n\Big(\frac{3a+b}{4}\Big).$$

We shall show that for every $\mu > \tilde{\mu_0}$ and $\lambda > 0$ the problem (1.1)-(1.2) has no positive solution.

Let $\mu > \tilde{\mu_0}$ and $\lambda > 0$. We suppose that (1.1)-(1.2) has a positive solution $(u(t), v(t)), t \in [a, b]$. Then, we obtain

$$\begin{aligned}
v(t) = Q_\mu(u,v)(t) &= \mu \int_a^b G_n(t,s)g(s,u(s),v(s))ds \\
&\geqslant \mu\theta_n\Big(\frac{3a+b}{4}\Big)\int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a}g(s,u(s),v(s))ds \\
&\geqslant \mu\theta_n\Big(\frac{3a+b}{4}\Big)\int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a}m_2(u(s)+v(s))ds \\
&\geqslant \mu m_2\theta_n\Big(\frac{3a+b}{4}\Big)\int_{\frac{3a+b}{4}}^{\frac{a+3b}{4}} \frac{(s-a)(b-s)}{b-a}\gamma \parallel (u,v) \parallel ds \\
&= \frac{11b^3-11a^3+33a^2b-33ab^2}{96(b-a)}\theta_n\Big(\frac{3a+b}{4}\Big)\mu\gamma m_2 \parallel (u,v) \parallel \\
&= \mu\gamma m_2 C \parallel (u,v) \parallel .
\end{aligned}$$

Therefore, we deduce

$$\parallel v \parallel \geqslant v(t) \geqslant \mu\gamma m_2 C \parallel (u,v) \parallel > \tilde{\mu_0}\gamma m_2 C \parallel (u,v) \parallel = \parallel (u,v) \parallel .$$

and so,$\parallel (u,v) \parallel = \parallel u \parallel + \parallel v \parallel \geqslant \parallel v \parallel > \parallel (u,v) \parallel$, which is a contradiction. Therefore, the boundary value problem (1.1)-(1.2) has no positive solution.

(iii) Because $f_0^i, f_\infty^i, g_0^i, g_\infty^i > 0$, we deduce as above, that there exist $m_1, m_2 > 0$ such that

$$f(t,u,v) \geqslant m_1(u+v),\ g(t,u,v) \geqslant m_2(u+v), \forall\ u,v \geqslant 0.$$

We define

$$\tilde{\tilde{\lambda}}_0 = \frac{1}{2\gamma m_1 A}\Big(= \frac{\tilde{\lambda}_0}{2}\Big) \text{ and } \tilde{\tilde{\mu}}_0 = \frac{1}{2\gamma m_2 C}\Big(= \frac{\tilde{\mu_0}}{2}\Big).$$

Then for every $\lambda > \tilde{\tilde{\lambda}}_0$ and $\mu > \tilde{\tilde{\mu}}_0$, the problem (1.1)-(1.2) has no positive solution.

Indeed, let $\lambda > \tilde{\tilde{\lambda}}_0$ and $\mu > \tilde{\tilde{\mu}}_0$. We suppose that (1.1)-(1.2) has a positive solution $(u(t), v(t)), t \in [a, b]$. Then in a similar manner as above, we deduce

$$\parallel u \parallel \geqslant \lambda\gamma m_1 A \parallel (u,v) \parallel,\ \parallel v \parallel \geqslant \mu\gamma m_2 C \parallel (u,v) \parallel,$$

and so,

$$\begin{aligned}
\parallel (u,v) \parallel &= \parallel u \parallel + \parallel v \parallel \\
&\geqslant \lambda\gamma m_1 A \parallel (u,v) \parallel + \mu\gamma m_2 C \parallel (u,v) \parallel \\
&> \tilde{\tilde{\lambda}}_0\gamma m_1 A \parallel (u,v) \parallel + \widetilde{\tilde{\mu_0}}\gamma m_2 C \parallel (u,v) \parallel \\
&= \frac{1}{2} \parallel (u,v) \parallel + \frac{1}{2} \parallel (u,v) \parallel = \parallel (u,v) \parallel
\end{aligned}$$

which is a contradiction. Therefore, the boundary value problem (1.1)-(1.2) has no positive solution. □

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Department of Mathematics, Jazan University, Jazan, Kingdom of Saudi Arabia
E-mail address: `snrao@jazanu.edu.sa`