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COMMON FIXED POINT THEOREMS IN MENGER SPACE FOR SIX SELF MAPS USING AN IMPLICIT RELATION

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ABSTRACT. The aim of this paper is to prove, mainly, a common fixed point theorem for six self mappings of a Menger space using two weakly compatible pairs satisfying an implicit relation. This generalizes several known results including those of Kohli et.al [**2**] and Sastry et.al [**7**].

1. Introduction

The pursuit of fixed point theorems in Menger space is an active area of research in the present days. Menger [**4**] introduced the concept of probabilistic Menger space. Singh et.al [**10**] introduced the notion of weakly commuting mappings on Menger spaces. Kohli et. al [**2**] established a common fixed point theorem for six self mappings using pointwise R-weakly commuting mappings with a contractive type implicit relation. This generalizes the results of Kumar and Pant [**3**]. Sastry et. al [**7**] made some modifications to the results of Kohli et. al [**2**].

In this paper, we further generalized the results of [**2**] and [**7**]. As usual $\mathbb{R}$ stands for the set of all real numbers, $\mathbb{R}^+$ stands for the set of all non-negative real numbers, $\mathbb{Q}$ stands for the set of rational numbers and $\mathbb{N}$ stands for the set of natural numbers.

2. Preliminaries

We take the standard definitions given in Schweizer and Sklar [**8**].

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Key words and phrases. Menger space, weakly compatible mappings, pointwise R-weakly commuting mappings, common fixed point.

We hereunder give the following definitions and the result required in subsequence section.

Definition 2.1. ([**10**]) Self mappings f and g of a probabilistic metric space (X, F) are said to be weakly commuting if and only if (iff) $F_{fgx,gfx}(t) \geqslant F_{fx,gx}(t)$ for each $x \in X$ and $t > 0$.

Definition 2.2. ([**1**]) Self mappings f and g of a probabilistic metric space (X, F) are said to be pointwise R-weakly commuting if given z in X, there exists $R > 0$(depending on x) such that $F_{fgx,gfx}(t) \geqslant F_{fx,gx}(\frac{t}{R})$ for $t > 0$.

Note 2.1. Weakly commuting mappings are pointwise R-weakly commuting with $R = 1$.

Definition 2.3. ([**3**]) Self mappings f and g of a probabilistic metric space (X, F) are said to be reciprocally continuous if $fgx_n \to fz$ and $gfx_n \to gz$, whenever $\{x_n\}$ is a sequence such that $fx_n, gx_n \to z$ for some z in X.

Note 2.2. Every pair of continuous mappings is reciprocally continuous.

Definition 2.4. Self mappings f and g of a probabilistic metric space (X, F) are said to be weakly compatible iff $fx = gx$ for some $x \in X$ implies $fgx = gfx$.

Definition 2.5. ([**5**]) Self mappings f and g of a probabilistic metric space (X, F) are said to be weakly compatible if $F_{fgx_n,gfx_n}(t) \to 1$ for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that $fx_n, gx_n \to z$ for some $z \in X$.

Note 2.3. Compatible implies weakly compatible but the converse is not true.

We, hereunder give a pair of self mappings on a Menger space that are weakly compatible but not compatible, R-weakly commuting and weakly commuting.

Example 2.1. Let $X = [0, \lambda]$ $(\lambda \geqslant 2)$, $a * b = min\{a, b\}$ for all $a, b \in [0, 1]$ and $F_{x,y}(t) = \frac{t}{t+|x-y|}$ for all $x, y \in X$ and for all $t > 0$. Then $(X, F, *)$ is a complete Menger space.

Define self mappings f and g on X by

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x < \frac{\lambda}{2}, \\ \lambda & \text{if } \frac{\lambda}{2} \leqslant x \leqslant \lambda, \end{cases}$$

$$g(x) = \begin{cases} \lambda - x & \text{if } 0 \leqslant x < \frac{\lambda}{2}, \\ \lambda & \text{if } \frac{\lambda}{2} \leqslant x \leqslant \lambda. \end{cases}$$

Claim 1: $\{f, g\}$ is weakly compatible.
For $x \in [0, \frac{\lambda}{2})$, $fx < \frac{\lambda}{2} < gx$. Hence, $fx \neq gx$, for every $x \in [0, \frac{\lambda}{2})$.
For every $x \in [\frac{\lambda}{2}, \lambda]$,
$fx = \lambda = gx$ and $fg(x) = f(\lambda) = \lambda = g(\lambda) = gf(\lambda)$.

Therefore, $\{f, g\}$ is weakly compatible.

Claim 2: $\{f, g\}$ is not compatible.
Take $x_n = \{\frac{\lambda}{2} - \frac{1}{n}\}$.
$fx_n = \{\frac{\lambda}{2} - \frac{1}{n}\} \to \frac{\lambda}{2}$ as $n \to \infty$ and $gx_n = \{\lambda - \frac{\lambda}{2} + \frac{1}{n}\} \to \frac{\lambda}{2}$ as $n \to \infty$.
$fg(x_n) = f(\frac{\lambda}{2} + \frac{1}{2}) = \lambda$ and $gf(x_n) = g(\frac{\lambda}{2} - \frac{1}{2}) = \frac{\lambda}{2}$.
$F_{fgx_n, gfx_n}(t) = F_{\lambda, \frac{\lambda}{2}}(t) \to \frac{t}{t+\frac{\lambda}{2}} < 1$ as $n \to \infty$.
Hence, $\{f, g\}$ is not compatible.

Claim 3: $\{f, g\}$ is not weakly commuting.
Take $x = \frac{3\lambda}{8}$.
$fx = \frac{3\lambda}{8}$ and $gx = \lambda - \frac{3\lambda}{8} = \frac{5\lambda}{8}$.
$fg(x) = f(\frac{5\lambda}{8}) = \lambda$ and $gf(x) = g(\frac{3\lambda}{8}) = \frac{5\lambda}{8}$.
Since $\frac{3\lambda}{8} > \frac{\lambda}{4}$, follows that $F_{fgx, gfx}(t) < F_{fx, gx}(t)$.
Therefore, $\{f, g\}$ is not weakly commuting.

Claim 4: $\{f, g\}$ is not R-weakly commuting.
Take $x \in [\frac{3\lambda}{8}, \frac{\lambda}{2})$.
$fx = x$ and $gx = \lambda - x$.
$fg(x) = f(\lambda - x) = \lambda$ and $gf(x) = g(x) = \lambda - x$.
Let $R > 0$.
$F_{fgx, gfx}(t) = F_{\lambda, \lambda - x}(t) = \frac{t}{t+x}$ and
$F_{fx, gx}(\frac{t}{R}) = F_{x, \lambda - x}(\frac{t}{R}) = \frac{\frac{t}{R}}{\frac{t}{R} + (\lambda - 2x)} = \frac{t}{t + R(\lambda - 2x)}$.
Now, $F_{fgx, gfx}(t) \geqslant F_{fx, gx}(\frac{t}{R}) \Leftrightarrow x \leqslant R(\lambda - 2x) \Leftrightarrow R \geqslant \frac{x}{(\lambda - 2x)}$.
Since, $\text{Sup}\{(\lambda - 2x) : x \in [\frac{3\lambda}{8}, \frac{\lambda}{2})\} = +\infty$, it follows that such R does not exist.
Therefore, $\{f, g\}$ is not R-weakly commuting.
(Observe that the pair $\{f, g\}$ is pointwise R-weakly commuting, since for any $x \in [\frac{3\lambda}{8}, \frac{\lambda}{2})$, we can select $R_x \geqslant \frac{x}{(\lambda - 2x)}$).

Definition 2.6. ([**6**]) A function $\phi : (\mathbb{R}^+)^4 \to \mathbb{R}$ is said to be an implicit relation if

(i.) ϕ is continuous,
(ii.) ϕ is Monotonic increasing in the first argument and
(iii.) ϕ satisfies the following conditions:
  (a) for $x, y \geqslant 0$, $\phi(x, y, x, y) \geqslant 0$ or $\phi(x, y, y, x) \geqslant 0$ implies $x \geqslant y$,
  (b) $\phi(x, x, 1, 1) \geqslant 0$ implies $x \geqslant 1$.

Example 2.2. Define $\phi : (\mathbb{R}^+)^4 \to \mathbb{R}$ by $\phi(x_1, x_2, x_3, x_4) = ax_1 + bx_2 + cx_3 + dx_4$ with $a + b + c + d = 0$,$a + b > 0$, $a + c > 0$ and $a + d > 0$.
Clearly, ϕ is an implicit relation.
In particular,

(i.) $\phi(x_1, x_2, x_3, x_4) = 6x_1 - 3x_2 - 2x_3 - x_4$,
(ii.) $\phi(x_1, x_2, x_3, x_4) = 5x_1 - 3x_3 - 2x_4$

are implicit relations.

Notation: Let Φ be the class of all implicit relations.

LEMMA 2.1. ([**9**]) *Let* $\{x_n\}(n = 0, 1, 2, \dots)$ *be a sequence in a Menger space* $(X, F, *)$. *If there is a* $k \in (0, 1)$ *such that*

$$F_{x_n, x_{n+1}}(kt) \geqslant F_{x_{n-1}, x_n}(t)$$

for all $t > 0$ *and* $n \in \mathbb{N}$, *then* $\{x_n\}$ *is a Cauchy sequence in* X.

3. Main theorem

Kohli et.al [**2**] proved the following:

THEOREM 3.1. ([**2**]) *Let* (X, F, T) *be a complete Menger space, where* T *denotes a continuous t-norm. Let* f, g, h, k, p *and* q *be self maps of* X. *Further, let* $\{p, hk\}$ *and* $\{q, fg\}$ *be pointwise R-weakly commuting mappings, satisfying:*

(3.1.1) $p(X) \subseteq fg(X)$, $q(X) \subseteq hk(X)$;
(3.1.2) $\phi(F_{px,qy}(\alpha t), F_{hkx,fgy}(t), F_{px,hkx}(t), F_{qy,fgy}(\alpha t)) \geqslant 0$,
(3.1.3) $\phi(F_{px,qy}(\alpha t), F_{hkx,fgy}(t), F_{px,hkx}(\alpha t), F_{qy,fgy}(t)) \geqslant 0$,
for all $x, y \in X$ & $t > 0$ *and for some* $\phi \in \Phi$ & $\alpha \in (0, 1)$;
(3.1.4) k *commutes with* p & h *and* g *commutes with* q & f;
(3.1.5) *one of the mappings in the* <u>*compatible pair*</u> $\{p, hk\}$ *or* $\{q, fg\}$ *is continuous.*

Then f, g, h, k, p *and* q *have a unique common fixed point in* X.

The concepts of compatibility and the reciprocal continuity are used in obtaining this result.

Sastry et.al [**7**] made the modification of replacing 'pointwise R-weakly commuting' by 'weakly compatible' and deduced the result using the concepts 'compatibility' and 'reciprocal continuity'.

Now, we modify and generalize their results and establish the following:

THEOREM 3.2. *Let* (X, F, T) *be a Menger space, where* T *denotes a continuous t-norm and* f, g, h, k, p *and* q *be self maps of* X. *Further, let* $\{p, hk\}$ *and* $\{q, fg\}$ *be weakly compatible mappings, satisfying:*

(3.2.1) $p(X) \subseteq fg(X)$, $q(X) \subseteq hk(X)$;
(3.2.2) $\phi(F_{px,qy}(\alpha t), F_{hkx,fgy}(t), F_{px,hkx}(t), F_{qy,fgy}(\alpha t)) \geqslant 0$,
(3.2.3) $\phi(F_{px,qy}(\alpha t), F_{hkx,fgy}(t), F_{px,hkx}(\alpha t), F_{qy,fgy}(t)) \geqslant 0$,
for all $x, y \in X$ & $t > 0$ *and for some* $\phi \in \Phi$ & $\alpha \in (0, 1)$;
(3.2.4) $fg = gf$ *and 'either* $qg = gq$ *or* $qf = fq$*';*
(3.2.5) $hk = kh$ *and 'either* $pk = kp$ *or* $hp = ph$*';*
(3.2.6) *one of* $p(X)$, $q(X)$, $hk(X)$, $fg(X)$ *is a complete subspace of* X.

Then f, g, h, k, p *and* q *have a unique common fixed point in* X *say* z. *Also* z *is the unique common fixed point* h, k & p *as well as* f, g & q.

Proof:
Let $x_0 \in X$. *By (3.2.1) we construct sequences* $\{x_n\}$ *and* $\{y_n\}$ *in* X *such that*

$$px_{2n} = fgx_{2n+1} = y_{2n}(say)$$
$$and\ \ qx_{2n+1} = hkx_{2n+2} = y_{2n+1}(say), for \quad n = 0, 1, 2, \dots$$

By putting $x = x_{2n}(n \geqslant 1)$ *and* $y = x_{2n+1}$ *in* (3.2.2), *we get that*

$$\phi(F_{px_{2n},qx_{2n+1}}(\alpha t), F_{hkx_{2n},fgx_{2n+1}}(t), F_{px_{2n},hkx_{2n}}(t), F_{qx_{2n+1},fgx_{2n+1}}(\alpha t)) \geqslant 0$$
$$i.e, \phi(F_{y_{2n},y_{2n+1}}(\alpha t), F_{y_{2n-1},y_{2n}}(t), F_{y_{2n},y_{2n-1}}(t), F_{y_{2n+1},y_{2n}}(\alpha t)) \geqslant 0.$$

So, by the property of ϕ,

$$F_{y_{2n},y_{2n+1}}(\alpha t) \geqslant F_{y_{2n-1},y_{2n}}(t).$$

By putting $x = x_{2n+2}$ *and* $y = x_{2n+1}$ *in* (3.2.3), *we get that*

$$\phi(F_{px_{2n+2},qx_{2n+1}}(\alpha t), F_{hkx_{2n+2},fgx_{2n+1}}(t), F_{px_{2n+2},hkx_{2n+2}}(\alpha t), F_{qx_{2n+1},fgx_{2n+1}}(t)) \geqslant 0$$
$$i.e, \phi(F_{y_{2n+2},y_{2n+1}}(\alpha t), F_{y_{2n+1},y_{2n}}(t), F_{y_{2n+2},y_{2n+1}}(\alpha t), F_{y_{2n+1},y_{2n}}(t)) \geqslant 0$$
$$\Rightarrow F_{y_{2n+1},y_{2n+2}}(\alpha t) \geqslant F_{y_{2n},y_{2n+1}}(t).$$

Thus for all $n \in \mathbb{N}$,

$$F_{y_n,y_{n+1}}(\alpha t) \geqslant F_{y_{n-1},y_n}(t).$$

By Lemma(2.12), $\{y_n\}$ *is a Cauchy sequence in* X.
$\Rightarrow \{y_{2n}\}$ *and* $\{y_{2n\pm1}\}$ *are Cauchy sequences in* X.

Case I: *Suppose* $p(X)$ *or* $fg(X)$ *is a complete subspace of* X.
Since $\{y_n\} \subseteq p(X)(\subseteq fg(X))$, *there is a* $z \in X$ *such that* $y_{2n} \to z$ *as* $n \to \infty$.

Since $p(X) \subseteq fg(X)$, *by our supposition, there is a* $v \in X$ *such that* $fgv = z$.

By putting $x = x_{2n}(n \geqslant 1)$ *and* $y = v$ *in (3.2.2), we get that*

$$\phi(F_{px_{2n},qv}(\alpha t), F_{hkx_{2n},fgv}(t), F_{px_{2n},hkx_{2n}}(t), F_{qv,fgv}(\alpha t)) \geqslant 0$$
$$i.e, \phi(F_{y_{2n},qv}(\alpha t), F_{y_{2n-1},z}(t), F_{y_{2n},y_{2n-1}}(t), F_{qz,z}(\alpha t)) \geqslant 0.$$

Since ϕ *is continuous, letting* $n \to \infty$, *we get that*

$$\phi(F_{z,qv}(\alpha t), F_{z,z}(t), F_{z,z}(t), F_{qz,z}(\alpha t)) \geqslant 0.$$

By the property of ϕ, *follows that* $F_{z,qv}(\alpha t) \geqslant F_{z,z}(t) = 1$. $\Rightarrow z = qv$.
Thus $fgv = qv = z$.

Since $\{q, fg\}$ *is weakly compatible,* $q(fg)v = fg(q)v$. $i.e, qz = fgz$.

By putting $x = x_{2n}(n \geqslant 1)$ *and* $y = z$ *in (3.2.2), we get that*

$$\phi(F_{px_{2n},qz}(\alpha t), F_{hkx_{2n},fgz}(t), F_{px_{2n},hkx_{2n}}(t), F_{qz,fgz}(\alpha t)) \geqslant 0$$
$$i.e, \phi(F_{y_{2n},qz}(\alpha t), F_{y_{2n-1},qz}(t), F_{y_{2n},y_{2n-1}}(t), F_{qz,qz}(\alpha t)) \geqslant 0.$$

Letting $n \to \infty$, we get that

$$\phi(F_{z,qz}(\alpha t), F_{z,qz}(t), F_{z,z}(t), F_{qz,qz}(\alpha t)) \geqslant 0.$$
$$i.e, \phi(F_{z,qz}(\alpha t), F_{z,qz}(t), 1, 1) \geqslant 0.$$

Since ϕ is non-deceasing in the first argument, we get that $qz = z$.
Thus $z = qz = fgz = gfz$ (since $fg = gf$).

Suppose $qg = gq$, then $qgz = gqz = gz$.
Since $fg = gf$, we have $fg(gz) = gf(gz) = g(fgz) = gz$.
By putting $x = x_{2n}(n \geqslant 1)$ and $y = gz$ in (3.2.2), we get that

$$\phi(F_{px_{2n},gv}(\alpha t), F_{hkx_{2n},gz}(t), F_{px_{2n},hkx_{2n}}(t), F_{gz,gz}(\alpha t)) \geqslant 0$$
$$i.e, \phi(F_{y_{2n},gz}(\alpha t), F_{y_{2n-1},gz}(t), F_{y_{2n},y_{2n-1}}(t), F_{gz,gz}(\alpha t)) \geqslant 0.$$

Letting $n \to \infty$, we get that

$$\phi(F_{z,gz}(\alpha t), F_{z,gz}(t), F_{z,z}(t), F_{gz,gz}(\alpha t)) \geqslant 0.$$

$\Rightarrow \phi(F_{z,gz}(t), F_{z,gz}(t), 1, 1) \geqslant 0 \Rightarrow F_{z,gz}(t) \geqslant 1 \Rightarrow gz = z.$
Since $fgz = z$, follows that $fz = z$. Thus $z = fz = gz = qz$.

Suppose $qf = fq$, so $qfz = fqz = fz$.
Since $fg = gf$, we have $fgfz = f(gfz) = fz$.
By putting $x = x_{2n}(n \geqslant 1)$ and $y = fz$ in (3.2.2) as above we get that $z = fz$. Hence $z = fz = gz = qz$.

Since $q(X) \subseteq hk(X)$, there is a $w \in X$ such that $z = hkw$.
By putting $x = w$ and $y = x_{2n+1}$ in (3.2.2), we get that $pw = z$. Thus $pw = z = hkw$.
Since $\{p, hk\}$ is weakly compatible, $phkw = hkpw$. $i.e, pz = hkz$.
By putting $x = z$ and $y = x_{2n+1}$ in (3.2.2), we get that $pz = z$. Thus $z = pz = hkz = khz$ (since $hk = kh$).

Suppose $pk = kp$, then $pkz = kpz = kz$. Since $hk = kh$, we have $hk(kz) = kh(kz) = k(hkz) = kz$.
By putting $x = kz$ and $y = x_{2n+1}$ in (3.2.2), we get that $kz = z$. Since $hkz = z$, follows that $hz = z$.
Thus $z = hz = kz = pz$.
Suppose $ph = hp$, so $phz = hpz = hz$. Since $hk = kh$, we have $hk(kz) = h(khz) = hz$.
By putting $x = hz$ and $y = x_{2n+1}$ in (3.2.2), we get that $hz = z$. Since $hkz = z$, follows that $kz = z$.
Thus $z = hz = kz = pz$.
Hence $z = fz = gz = hz = kz = pz = qz$.

Case II: *Suppose $q(X)$ or $hk(X)$ is a complete subspace of X.*
On similar lines, first we get that $z = hz = kz = pz$ and then $z = fz = gz = qz$.

Thus $z = fz = gz = hz = kz = pz = qz$. Hence z is a common fixed point of f, g, h, k, p and q.

Uniqueness: *if z^1 is also a common fixed point of f, g, h, k, p and q, then $z^1 = fz^1 = gz^1 = hz^1 = kz^1 = pz^1 = qz^1$.*
By putting $x = z$ and $y = z^1$ in (3.2.2), we get that $z^1 = z$.
Hence z is the unique common fixed point of f, g, h, k, p and q.

We now prove that z is the unique common fixed point of h, k & p.
Suppose z^1 is also a common fixed point of h, k & p.
By putting $x = z^1$ and $y = z$ in (3.2.2), we get that $z^1 = z$.
Hence z is the unique common fixed point of h, k & p.
So is the case with f, g & q.
This completes the proof of the theorem.

Corollary 3.1. ([**7**] *Theorem 3.2*) *Let (X, F, T) be a complete Menger space, where T denotes a continuous t-norm. Let f, h, p and q be self maps of X. Further, let $\{p, h\}$ and $\{q, f\}$ be weakly compatible mappings, satisfying:*

(3.3.1) $p(X) \subseteq f(X)$, $q(X) \subseteq h(X)$;
(3.3.2) $\phi(F_{px,qy}(\alpha t), F_{hx,fy}(t), F_{px,hx}(t), F_{qy,fy}(\alpha t)) \geqslant 0$,
(3.3.3) $\phi(F_{px,qy}(\alpha t), F_{hx,fy}(t), F_{px,hx}(\alpha t), F_{qy,fy}(t)) \geqslant 0$,
for all $x, y \in X$ & $t > 0$ and for some $\phi \in \Phi$ & $\alpha \in (0, 1)$;
(3.3.4) *Suppose $\{p, h\}$ and $\{q, f\}$ are compatible pairs;*
(3.3.5) *one of the mappings in the compatible pairs $\{p, h\}$ or $\{q, f\}$ is continuous.*

Then f, h, p and q have a common fixed point in X.

Proof: *This can be deduced from our Theorem by taking $f = r$, $h = s$ and $g = k = I$(the identity map).*

Now we give the following example in support of our Theorem (3.2).

Example 3.1. Let $X = \mathbb{Q}$, $a * b = min\{a, b\}$ for all $a, b \in [0, 1]$ and $F_{x,y}(t) = \frac{t}{t+|x-y|}$ for all $x, y \in X$ and for all $t > 0$. Then $(X, F, *)$ is a Menger space.

Define self mappings f, g, h, k, p and q on X by $px = qx = l(> 1)$,

$$h(x) = \begin{cases} 0 & \text{if } x \leqslant 1, \\ l & \text{if } x > 1, \end{cases}$$

$$f(x) = \begin{cases} 2 - x & \text{if } x \leqslant 1, \\ l & \text{if } x > 1, \end{cases}$$

$kx = gx = x$, for all $x \in X$.
Define $\phi : (\mathbb{R}^+)^4 \to \mathbb{R}$ by $\phi(x_1, x_2, x_3, x_4) = 6x_1 - 3x_2 - 2x_3 - x_4$ then ϕ is an implicit relation.

For $x, y \leqslant 1$,

$$\phi(F_{l,l}(\alpha t), F_{0,(2-y)}(t), F_{l,0}(t), F_{l,(2-y)}(\alpha t)) > 6 - 3 - 2 - 1 = 0.$$

For $x, y > 1$,

$$\phi(F_{l,l}(\alpha t), F_{l,l}(t), F_{l,l}(t), F_{l,l}(\alpha t)) = 6 - 3 - 2 - 1 = 0.$$

For $x \leqslant 1$ and $y > 1$,

$$\phi(F_{l,l}(\alpha t), F_{0,l}(t), F_{l,0}(t), F_{l,l}(\alpha t)) > 6 - 3 - 2 - 1 = 0.$$

For $x > 1$ and $y \leqslant 1$,

$$\phi(F_{l,l}(\alpha t), F_{l,(2-y)}(t), F_{l,l}(t), F_{l,(2-y)}(\alpha t)) \geqslant 6 - 3 - 2 - 1 = 0.$$

The other conditions of the Theorem are trivially satisfied. Clearly $'l'$ is the unique common fixed point of f, g, h, k, p and q in X as well as f, g & p and h, k & q. (Observe that X is not complete.)

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