Moment Generating Function Based Performance Analysis of Network Coding Two-way Relaying Using Alamouti Scheme on Fading Channels

Zuhaib Ashfaq Khan, Muhammad Hasanain Chaudary and Juinn-Horng Deng

Abstract- This article discusses the performance analysis of a network coded relay network. The relay nodes operate in decode-and-forward (DF) mode. The channels are modeled as Rayleigh, Nakagami-m and Rician fading. The overall system performance is improved by using the Alamouti coding scheme. The closed-form expressions of moment generating function (MGF) are obtained over various fading channels. The system performance is analyzed in terms of SER and outage probability. MGF based approach is followed to derive the closed-form expressions of SER for MPSK modulation schemes. The derived expressions also present the diversity order. The simulation and theoretical results are produced to authenticate accuracy of the system.

Index Terms: Moment Generating Function, Symbol Error Rate, Decode and Forward, Rayleigh, Nakagami-m, Rician, Performance Analysis.

I. INTRODUCTION

The future of the wireless communication networks are attribute by high speed high reliability and large user capacity. However, the phenomena of interference and fading of wireless communication networks causes the design objectives to be the challenging gone. The co-operative communications emerge as an efficient and inexpensive technique to overcome the wireless fading challenges, that opens new horizons to attract a lot of research effort and practical implementations [1],[2]. Most of these systems keep separate the information of the various users. This, infect is a physical-layer routing (replicating detecting and forward) technique. The improvement in spectral efficiency is achieved by applying the core concept of network coding as received increasing attention, although the concept of network coding (NC) was first introduced almost a decade ago[3]. NC as a novel approach of information transmit formulate- hop networks, permits the mixing of information messages from different source nodes at an intermediate node, has shown a significant potential to improve the network throughput[4].

As a consequence, performance gains are achieved in network flow and energy efficiency, a significant increase in the transmission efficiency is anticipated as the amount of information transmitted in the wireless communication networks is reduced and a greater degree of choice in the design is achieved [4],[5]. Network coding can be applied in different relay networks scenarios, for example broadcasting, multicasting, two-way communication, and multiple access [6]-[12].

In [13], the performance enhancement of NC is proven with help of a three-node bi-directional model. To utilize this new approach in a better way and en courage its applications in wireless networks, various technical issues remain to here solved [13].

Relaying schemes are developed to achieve a robust and reliable wireless communication in the areas where source and destination cannot communicate in a proper manner with each other. Three relaying schemes are introduced in [14]; i.e. Decode-and-Forward (DF), Amplify-and-Forward (AF) and Coded Cooperation (CC). In DF, the signal received by the relay node first performs a completed encoding of the signal received by relay node and then transmits the re-encoded signal to destination node, where as in AF the signal received at relay node is amplified first and then re-transmits the scaled signal to the destination node.

Outage probability analysis of theoretical model for DF and AF relaying schemes is performed in [15]. The space-time block codes (STBCs) is among the most significant classes of space-time codes (STCs). The Alamouti code [16] is one paradigm of an STBC for a multiple-input multiple-output (MIMO) system consisting of two antennas. In a cooperative diversity, STC can be utilized to get the advantage similar to MIMO characteristics in order to achieve the higher spectral efficiency, spatial diversity, and coding gains. In [17]-[19], the performance analysis over various relay schemes is made using the Alamouti code over Rayleigh, Nakagami-m, and Rician

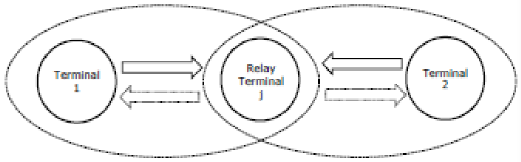

Fig. 1. System Model

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fading environment and presents that systems employing Alamouti code give much better performance in comparison to those systems that uses general repetition coding.

In [6] performance analysis is carried out for two-way relaying having multiple-antenna along with NC by utilizing Alamouti code over Rayleigh fading environment. In this article, we focus on NC based relay network addressing the performance issue. We analyzed the system performance under DF relaying schemes along with Alamouti coding. The key impact of this article includes, the derivations for closed form expressions of MGF over Rayleigh, Nakagami- m and Rician fading channels.

We use the MGF based approach to derive the SER close form expressions to analyze the system performance. Also the MGF based outage probability is obtained at various relay locations over Nakagami-m and Rician fading channels.

The rest of this article is structured as: Section 2 focuses on System model. Section 3 describes the transmission protocol. Section 4 discusses the MGF, while Section 5 gives the close form SER expression. Section 6 presents the outage probability. In Section 7 numerical simulation are provided and finally we concluded the article in Section 8.

II. SYSTEM MODEL

The system model consists of a source terminal, relay terminal and a destination terminal as shown in Figure 1. The relay is equipped with two antennas, where as the source and destination having a single antenna. The two-way relay system is working in such a way that, information from source to relays are termed as an uplink transmission and relays to destination as downlink transmission.

We evaluate the down link performance of our system with the assumption that a received signal at the relay is perfectly decoded. We investigate the performance for terminal 1. The performance of terminal 2 is the same due to symmetry.

A. Channel Model

The channel state information (CSI) is assumed to be known at the receiver. The flat-fading channel coefficients h_{ijk} follow Rayleigh, Nakagami-m and Rician distribution, where i $\in$ 1,2 indicates the i-th terminal, j $\in$ 1 indicates the relay terminal j and k $\in$ 1,2 indicates the k-th antenna. We assume aijk = 1 hijk 12 and PDF of aijk for Rayleigh Nakagami m and Rician fading channels are given in Table I, where $\Gamma(.)$ is a gamma function, $I_0(.)$ is the zero order modified Bessel function of the first kind, K is defined as the Rice factor of the fading channel and $\Omega ijk = E(|hijk|^2)$ is the mean channel power. The effect of path loss is captured by hijk, considering the mean channel power $\Omega ijk \propto dij^{-\beta}$ [20], where dij is the distance between i-th terminal and relay terminal j and β is the path loss exponent.

III. TRANSMISSIONPROTOCOL

For transmission purpose, the terminal 1 and terminal 2 are denoted by t1 and t2, respectively, will transmit their information signals st1 and st2 to the multiple relays. The relay terminal j will XOR the two received signal sand then transmits the XORed signals j to both the terminal nodes. The Alamouti coding is used to enhance the system performance.

A. Alamouti Coding

In this scheme, network coded symbols at time n and n+1 are buffered and encoded using Alamouti coding [16] to produce

$$s_j = \begin{bmatrix} s_j(n) & -s_j(n+1)^* \\ s_j(n+1) & s_j(n)^* \end{bmatrix} \qquad (1)$$

where sj represents the relay terminal j transmitted XORed signal. During the first time slot, the terminal nodes will transmit the information signals to the relay node, which uses the decode-and-forward (DF) scheme to decode the received signal along with the path loss component to show the practical implementation off adding effects. After decoding the received signals we apply network coding to XOR them.

$$s_j = s_{t_1} \oplus s_{t_2} \qquad (2)$$

where $\oplus$ is the bit-wise XOR operation In the second time slot relays forward the XOR bits in the form of network coded symbols. As we have employed the Alamouti coding scheme for our system, thus according to [16] the obtained diversity order is equal to applying maximal-ratio receiver combining (MRRC) with two antennas at the receiver.

B. Input /Output Equation:

The received signal at the relay terminal j is given by

$$y_j = H_{ijk}\sqrt{Ps} + n_j \qquad (3)$$

y_j is the received vector at the relay terminal j equipped with k antennas, where k =1,2. $s = [s_{t_1} \quad s_{t_2}]^T$ is the transmitted vector containing transmitted signal from s_{t1} to j and from $s_{t2}$to j $h_{1jk} = [h_{1j1} \quad h_{2j1}]^T$ is the vector channel from t1 to j ,$h_{2jk} = [h_{2j1} \quad h_{2j2}]^T$is the vector channel from node t2 to j &

$H_{ijk} = \begin{bmatrix} h_{1j1} & h_{2j1} \\ h_{1j2} & h_{2j2} \end{bmatrix} = [h_{1jk} \quad h_{2jk}]$ is a 2x2 channel matrix. P is the total transmitted power and hj is the additive white Gaussian noise (AWGN) vector containing elements with variance of σ_j^2After applying the Alamouti coding scheme, the resulting received signals at the node t1 is

$$[y_{1j}(n) \quad y_{1j}(n+1)] = \frac{h_{1jk}^T s_j}{\sqrt{2}} + n_{1j} \qquad (4)$$

where nij is the received noise vector at terminal node t1. The received signal is then linearly decoded to obtain an estimate of sj which is then XORed to recover the intended signal st1 or st2.

C. Signal to Noise Ratio (SNR)

For a system using Alamouti coding with two transmit antennas and one receive antenna, the instantaneous SNR (is given as in [21, pp.90, eq.6.3.18] and [6, eq.15]) at relay terminal j can be expressed as

$$\gamma_j = \frac{P||h_{1jk}||^2 E(|s_j|^2)}{2\sigma_{1j}^2} \qquad (5)$$

By taking the average energy E $(|sij|^2)$ equal to one and let $\zeta = \frac{P}{\sigma_{1j}^2}$, we get

$$\gamma_j = \frac{\varsigma||h_{1jk}||^2}{2} \qquad (6)$$

As both of the antennas will be used to transmit sj therefore we have

$$\gamma_j = \zeta\left(\frac{|h_{1j1}|^2+|h_{ij}|^2}{2}\right) = \zeta\left(\frac{\alpha_{1j1}+\alpha_{1j2}}{2}\right) \qquad (7)$$

In this section, we describe the moment generating function (MGF) over Rayleigh, Nakagami-m and Rician fading channels for our proposed system. The MGF for the relay terminal j can be given by

$$M_j(s) = E[e^{-\gamma_j s}] \qquad (8)$$

The expression to evaluate the MGF for our case becomes

$$M_j(s) = \int_0^\infty M_j(s)|_{\alpha_{1j2}} P_{\alpha_{1j2}}(\alpha_{1j2})d\alpha_{1j2} \qquad (9)$$

Case A: For Rayleigh fading Channel

For relay terminal j, given a_{ij2} , the MGF for Relay fading channel from (8) by taking average w. r. t. a_{1j1} is obtained as

$$M_j(s)|_{\alpha_{1j2}} = \frac{e^{-\left(\frac{\alpha_{1j2}}{2}\right)\zeta s}}{1+\frac{\zeta s\Omega_{1j1}}{2}} \qquad (10)$$

After substituting the value of Mj(s)|α_{1j2} and $P_{\alpha 1j2}\alpha_{1j2}$ in (9) and solving it we obtain the unconditional MGF for

TABLE 1
PDF of a_{ijk} For Rayleigh, Nakagami-m and Rician Fading

Fading Channel	Pa(a)
Rayleigh	$P_{\alpha_{ijk}}(\alpha_{ijk}) = \frac{1}{\Omega_{ijk}} exp\left\{-\frac{\alpha_{ijk}}{\Omega_{ijk}}\right\};\ \alpha_{ijk} \geq 0$
Nakagami-m	$P_{\alpha_{ijk}}(\alpha_{ijk}) = \frac{m^m}{\Omega_{ijk}^m\Gamma(m)}\alpha_{ijk}^{m-1} exp\left\{\frac{-m\alpha_{ijk}}{\Omega_{ijk}}\right\}\alpha_{ijk} \geq 0$
Rician	$P_{\alpha_{ijk}}(\alpha_{ijk}) = \left(\frac{1+K}{\Omega_{ijk}}\right)\exp\left\{-k - \left(\frac{1+K}{\Omega_{ijk}}\right)\alpha_{ijk}\right\} I_0\left(2\sqrt{\frac{K(K+1)}{\Omega_{ijk}}\alpha_{ijk}}\right);\ \alpha_{ijk} \geq 0$

Rayleigh fading distribution for our case (as given in [22, pp.45, eq.23-26] for n degree of freedom)

$$M_j(s) = \frac{1}{\left(1+\frac{\zeta s\Omega_{1j1}}{2}\right)\left(1+\frac{\zeta s\Omega_{1j2}}{2}\right)} \qquad (11)$$

Case B: For Nakagami-m fading Channel

Similarly the conditional MGF expression given α_{1j2} can be obtained for Nakagami-m fading channels as

$$M_j(s)|_{\alpha_{1j2}} = \frac{m^m e^{\left(\frac{\alpha_{1j2}}{2}\right)\zeta s}}{\Omega_{1j1}^m\left(\frac{\zeta s}{2}+\frac{m}{\Omega_{1j1}}\right)^m} \qquad (12)$$

By substituting the value of $M_j(s)|_{\alpha_{1j2}}$ and $P_{\alpha 1j2}(\alpha 1j)$ in (9) and solving it using [24,eq.3.381.4], we obtain the unconditional MGF as

$$M_j(s) = \frac{(2m)^{2m}}{(\Omega_{1j1}\zeta s+2m)^m(\Omega_{1j2}\zeta s+2m)^m} \qquad (13)$$

The MGF given α_{1j2} can be obtained from (8) by taking average w.r.t α_{1j2} as

$$M_j(s)|_{\alpha_{1j2}} = \left(\frac{1+K_{1j1}}{\Omega_{1j1}}\right) exp\left\{-\left(\frac{\alpha_{1j2}}{2}\right)\zeta s - K_{1j1}\right\} \frac{exp\left\{\frac{1}{2}\frac{\frac{K_{1j1}(1+K_{1j1})}{\Omega_{1j1}}}{\frac{\zeta s}{2}+\frac{1+K_{1j1}}{\Omega_{1j1}}}\right\}}{\sqrt{\left(\frac{\zeta s}{2}+\frac{1+K_{1j1}}{\Omega_{1j1}}\right)\frac{K_{1j1}(1+K_{1j1})}{\Omega_{1j1}}}} M_{-\frac{1}{2},0}\left(\frac{\frac{K_{1j1}(1+K_{1j1})}{\Omega_{1j1}}}{\frac{\zeta s}{2}+\frac{1+K_{1j1}}{\Omega_{1j1}}}\right) \qquad (14)$$

The unconditional MGF can be derived by substituting the value of Mj(s)|α_{1j2} and $P_{\alpha 1j2}\alpha_{1j2}$in (9) and solving it using [24, eq.6.614.3], [24,eq.9.220.2] and [24,eq 9.2151], we get

$$M_j(s) = \frac{(1+K_{1j1})(1+K_{1j2})\, exp\{-K_{1j1}-K_{1j2}\}}{\left(\frac{\Omega_{1j1}\zeta s}{2}+\ (1+K_{1j1})\right)\left(\frac{\Omega_{1j2}\zeta s}{2}+\ (1+K_{1j2})\right)} exp\left\{\frac{K_{1j1}(1+K_{1j1})}{\left(\frac{\Omega_{1j1}\zeta s}{2}+(1+K_{1j1})\right)}\frac{K_{1j2}(1+K_{1j2})}{\left(\frac{\Omega_{1j2}\zeta s}{2}+(1+K_{1j2})\right)}\right\} \qquad (15)$$

V. SER EXPRESSION

In this section, we evaluate the closed form symbol-error-rate (SER) over Rayleigh, Nakagami-m and Rician fading channels for our proposed system by using their corresponding MGF expression obtained earlier.

A. Closed form SER for BPSK

The SER for the proposed system using the MGF expressions can be obtained by using [23, pp.357, eq.9.249] is

$$\mathrm{SER} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{M}\left(\frac{1}{\sin^2\theta}\right) d\theta \tag{16}$$

Case A: For Rayleigh fading Channel

We obtain the closed form SER expression for BPSK modulation in a Rayleigh fading environment as

$$SER = \frac{1}{2\pi}\left(\frac{\zeta}{8m}\right)^{-2m}\left(\frac{1}{2}\right)^{2m} B\left(\frac{1}{2}, 2m+\frac{1}{2}\right)_2 F_1\left(2m, 2m+\frac{1}{2}; 2m+1; -\frac{4m}{\zeta}\right)$$

(17)

Proof: see Appendix A.1

Where B(…,…) is the beta function (Euler's integral of the first kind) [24, eq.8.380] and F_1(….,…) is the hyper geometric function of two variables [24 eq.9.180.1].

Case B: For Nakagami-m fading Channel

For Nakagami-m fading Channels, the closed form SER expression considering BPSK modulation is derived as

$$SER = \frac{1}{2\pi}\left(\frac{\zeta}{8m}\right)^{-2m}\left(\frac{1}{2}\right)^{2m} B\left(\frac{1}{2}, 2m+\frac{1}{2}\right)_2 F_1\left(2m, 2m+\frac{1}{2}; 2m+1; -\frac{4m}{\zeta}\right)$$

(18)

Proof: See Appendix A.2

Case C: For Rician fading Channel.

The SER expression considering BPSK modulation in a Rician fading environment, is given as

$$SER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(1+K_{1j1})(1+K_{1j2})\, exp\{-K_{1j1}-K_{1j2}\}}{\left(\frac{\Omega_{1j1}\zeta s}{2sin^2\theta}+(1+K_{1j1})\right)\left(\frac{\Omega_{1j2}\zeta s}{2sin^2\theta}+(1+K_{1j2})\right)} exp\left\{\frac{K_{1j1}(1+K_{1j1})}{\left(\frac{\Omega_{1j1}\zeta s}{2sin^2\theta}+(1+K_{1j1})\right)}\frac{K_{1j2}(1+K_{1j2})}{\left(\frac{\Omega_{1j2}\zeta s}{2sin^2\theta}+(1+K_{1j2})\right)}\right\} d\theta \tag{19}$$

B. Closed form SER for QPSK

For deriving the SER expressions of the proposed system, we use the earlier derived MFG expression in [23, pp.357, eq.9.249]

$$SER = \frac{1}{\pi}\int_0^{\frac{3\pi}{4}} M\left(\frac{1}{2sin^2\theta}\right) d\theta \tag{20}$$

Case A: For Rayleigh fading Channel

The derived SER closed form expression for QPSK modulation is a Rayleigh fading environment is

$$SER = \frac{1}{2\pi}\left(\frac{\zeta}{8}\right)^{-2}\left(\frac{1}{4}\right) B\left(\frac{1}{2},\frac{5}{2}\right)_2 F_1\left(2,\frac{5}{2}; 3; -\frac{4}{\zeta}\right) \tag{21}$$

Proof: See Appendix A.3

where 2F1(…,..) is the generalized hypergeometic series [24,eq.9.14.2]

Case B: For Nakagami-m fading Channel

For Nakagami-m fading environment, using QPSK modulation, the closed form SER expression is

$$SER = \frac{1}{2\pi}\left(\frac{\zeta}{8m}\right)^{-2m}\left(\frac{1}{2}\right)^{2m} B\left(\frac{1}{2}, 2m+\frac{1}{2}\right)_2 F_1\left(2m, 2m+\frac{1}{2}; 2m+1; -\frac{4m}{\zeta}\right) \tag{22}$$

Proof: See Appendix A.4

Case C: For Rician fading Channel

The SER expression for Rician fading environment over QPSK modulation is obtained as follows

$$SER = \frac{1}{\pi}\int_0^{\frac{3\pi}{4}} \frac{(1+K_{1j1})(1+K_{1j2})\, exp\{-K_{1j1}-K_{1j2}\}}{\left(\frac{\Omega_{1j1}\zeta s}{4sin^2\theta}+(1+K_{1j1})\right)\left(\frac{\Omega_{1j2}\zeta s}{4sin^2\theta}+(1+K_{1j2})\right)} exp\left\{\frac{K_{1j1}(1+K_{1j1})}{\left(\frac{\Omega_{1j1}\zeta s}{4sin^2\theta}+(1+K_{1j1})\right)}\frac{K_{1j2}(1+K_{1j2})}{\left(\frac{\Omega_{1j2}\zeta s}{4sin^2\theta}+(1+K_{1j2})\right)}\right\} d\theta \tag{23}$$

C. SER for 16-QAM

The SER for the proposed system using the derived MGF expressions can be obtained by using [25, eq.15] is

$$SER = \frac{3}{\pi}\int_0^{\frac{\pi}{2}} M\left(\frac{1}{10sin^2\theta}\right) d\theta - \frac{9}{\pi}\int_0^{\frac{\pi}{4}} M\left(\frac{1}{10sin^2\theta}\right) d\theta \tag{24}$$

Case A: For Rayleigh fading Channel

Employing16-QAM, the derived MGF can be employed to find the SER expression for Rayleigh fading environment as

$$SER = \frac{3}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{\left(1+\frac{\zeta\Omega_{1j1}}{20sin^2\theta}\right)\left(1+\frac{\zeta\Omega_{1j2}}{20sin^2\theta}\right)} d\theta - \frac{9}{\pi}\int_0^{\frac{\pi}{4}} \frac{1}{\left(1+\frac{\zeta\Omega_{1j1}}{20sin^2\theta}\right)\left(1+\frac{\zeta\Omega_{1j2}}{20sin^2\theta}\right)} d\theta \tag{25}$$

Case B: For Nakagami-m fading Channel

We obtain the SER expression for 16-QAM modulation in a Nakagami-m fading environment as

$$SER = \frac{3}{\pi}\int_0^{\frac{\pi}{2}} \frac{\left(20msin^2\theta\right)^{2m}}{\left(\Omega_{1j1}\zeta s+20msin^2\theta\right)^{m}\left(\Omega_{1j2}\zeta s+20msin^2\theta\right)^{m}}\, d\theta - \frac{9}{\pi}\int_0^{\frac{\pi}{4}} \frac{\left(20msin^2\theta\right)^{2m}}{\left(\Omega_{1j1}\zeta s+20msin^2\theta\right)^{m}\left(\Omega_{1j2}\zeta s+20msin^2\theta\right)^{m}} d\theta \qquad (26)$$

Case C: For Rician fading Channel

The SER expression for16-QAM modulation in a Rician fading environment is obtained as

$$SER = \frac{3}{\pi}\int_0^{\frac{\pi}{2}} \frac{(1+K_{1j1})(1+K_{1j2})\, exp\{-K_{1j1}-K_{1j2}\}}{\left(\frac{\Omega_{1j1}\zeta s}{20sin^2\theta}+(1+K_{1j1})\right)\left(\frac{\Omega_{1j2}\zeta s}{20sin^2\theta}+(1+K_{1j2})\right)} exp\left\{\frac{K_{1j1}(1+K_{1j1})}{\left(\frac{\Omega_{1j1}\zeta s}{20sin^2\theta}+(1+K_{1j1})\right)}\frac{K_{1j2}(1+K_{1j2})}{\left(\frac{\Omega_{1j2}\zeta s}{4sin^2\theta}+(1+K_{1j2})\right)}\right\} d\theta$$
$$- \frac{9}{\pi}\int_0^{\frac{\pi}{4}} \frac{(1+K_{1j1})(1+K_{1j2})\, exp\{-K_{1j1}-K_{1j2}\}}{\left(\frac{\Omega_{1j1}\zeta s}{20sin^2\theta}+(1+K_{1j1})\right)\left(\frac{\Omega_{1j2}\zeta s}{20sin^2\theta}+(1+K_{1j2})\right)} exp\left\{\frac{K_{1j1}(1+K_{1j1})}{\left(\frac{\Omega_{1j1}\zeta s}{20sin^2\theta}+(1+K_{1j1})\right)}\frac{K_{1j2}(1+K_{1j2})}{\left(\frac{\Omega_{1j2}\zeta s}{4sin^2\theta}+(1+K_{1j2})\right)}\right\} d\theta \qquad (27)$$

VI. OUTAGE PROBABILITY

Outage probability helps us to compute the probability of the event, when the SNR drops below a certain threshold, in other words when the system cannot perform. In [26, eq.11] the outage probability is given by

$$P_{out} = P_\gamma(\gamma_{th}; A, N, Q) = \frac{2^{-Q}e^{A/2}}{\gamma_{th}}\sum_{q=0}^{Q}\binom{Q}{q}\sum_{n=0}^{N+q}\frac{-1^n}{\beta_n}\, R\left\{\frac{M_r\left(-\frac{A+2\pi jn}{2\gamma_{th}}\right)}{\frac{A+2\pi jn}{2\gamma_{th}}}\right\} + E(A,N,Q \qquad (28)$$

Where $P_\gamma(.)$is the CDF of γ [26] and E(A,Q,N) denote the overall error terms, which is approximately bounded as

$$E(A,N,Q) \cong \frac{e^{-A}}{1+e^{-A}} + |\frac{2^{-Q}e^{A/2}}{\gamma_{th}}\sum_{q=0}^{Q}(-1)^{N+1+q}\binom{Q}{q}\, R\left\{\frac{M_r\left(-\frac{A+2\pi j(N+1+q)}{2\gamma_{th}}\right)}{\frac{A+2\pi j(N+1+q)}{2\gamma_{th}}}\right\} \qquad (29)$$

The outage probability of the system can be evaluated by substituting the MGF expressions given in equations (11), (13), and (15) respectively in equation (28).

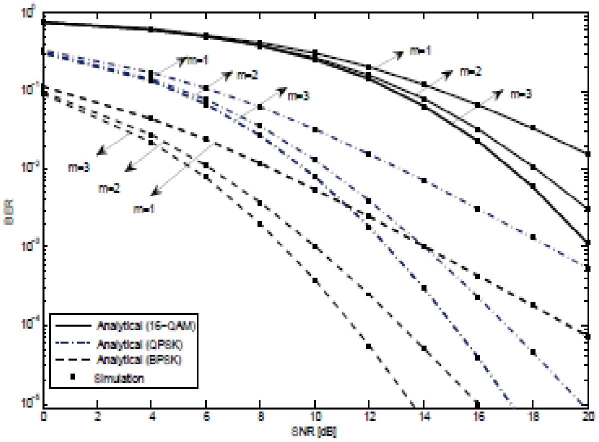

Fig. 2. SER for network coded relay network over Nakagami-m Fading Channel

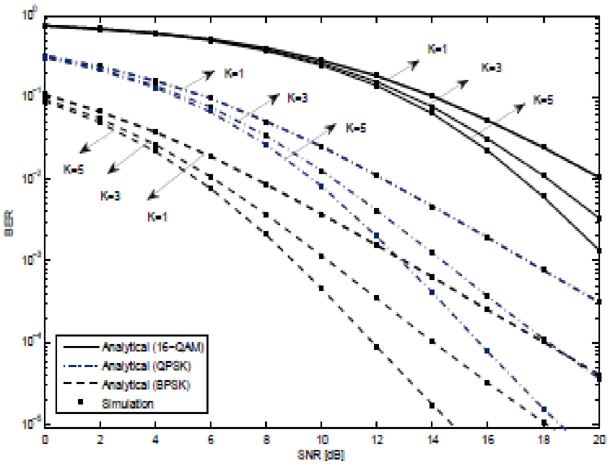

Fig. 3. SER for network coded relay network over Rician Fading Channel

VII. SIMULATION RESULTS AND DISCUSSION

In this section, we analyze the performance of the network coded relay cooperative network by obtaining the curves using analytical results and compare with the simulation results over Rayleigh, Nakagami-m and Rician fading channels. The performance is analyzed in terms of symbol error rate and outage probability. The fading channel gains are computed using the path loss model $d_{xy}^{-\beta}$, for $\beta = 3$ (suburban environment). Hence, the distance d∈ [0, 1] shows the location of relay with respect to source. 10^{-7} symbols are generated for the simulations. We plot the performance curves in terms of SER and P_{out} versus SNR (E_s/N_0 dB) of the transmitted signal. We build Mont- Carlo simulation using MATLAB for the system shown in Fig.1

Fig. 2 shows the variation of SER versus SNR of a single relay cooperative network with the relay location, d = 1. The SER curves are shown for BPSK, QPSK and 16-QAM schemes and different Nakagami-m factors. It is shown that the increase in m gives the improvement in the performance of the system in terms of SER, especially for high SNR values.

Fig. 3 presents the SER analysis of the system over Rician fading channels. The curves are obtained for K = 1; 3; 5 and various modulation schemes. The results show that the performance gets improved with the increase in K. Fig. 4 and Fig. 5 shows the outage probability performance over frequency flat Nakagami-m and Rician fading channels. These curves are generated using the derived MGF in (28), A set equal to 10ln10≅ 23.026 to guarantees a discretization error of less than 10^{-10}. The parameters N and Q were set to 21 and 15, respectively, in order that the resulting truncation error is less than 10^{-10}, thus negligible compared to the computed outage probability.

VIII. CONCLUSION

We analyze the performance of Network Coding based relay network over Rayleigh, Nakagami-m and Rician fading channels. We derive the closed-form MGF of the proposed system for Alamouti coding transmission protocol. The closed form SER is obtained using MGF-based approach to predict the performance of the proposed system. The analysis is carried out for BPSK, QPSK and 16-QAM modulation scheme.

It is clear from the simulation results that for higher values of m and K, we achieve better system performance. The result also shows that the derived expression can also be used to predict the performance of the proposed system. For higher SNR the simulation results clearly indicate QPSK gives better performance and for lower SNR BPSK gives better performance. Also the outage probability for different values of d over Nakagami-m and Rician fading channels is calculated. The outage probability gives a better performance with lower amount of fading (high m and K values) and a smaller distance from the source to the relay.

APPENDICES

A.1: Derivation of (17)

$$SER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{\left(1+\frac{\zeta\Omega_{1j1}}{2sin^2\theta}\right)\left(1+\frac{\zeta\Omega_{1j2}}{2sin^2\theta}\right)}\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(2sin^2\theta)^2}{(2sin^2\theta+\zeta\Omega_{1j1})(2sin^2\theta+\zeta\Omega_{1j1})}\,d\theta$$

(30)

Substituting $sin^2\theta{=}\upsilon$ and $d\theta = \frac{d\upsilon}{2\sqrt{\upsilon(1-\upsilon)}}$ into (30) yields

$$SER = \frac{1}{\pi}\int_0^1 \frac{(2\upsilon)^2}{(2\upsilon+\zeta\Omega_{1J1})(2\upsilon+\zeta\Omega_{1J1})}\frac{d\upsilon}{\sqrt{\upsilon(1-\upsilon)}} = \frac{2}{\pi\zeta^2\Omega_{1j1}\Omega_{1j2}}\int_0^1 \upsilon^{\frac{3}{2}}(1-\upsilon)^{-\frac{1}{2}}\left(1+\frac{2\upsilon}{\zeta\Omega_{1j1}}\right)^{-1} \quad (31)$$

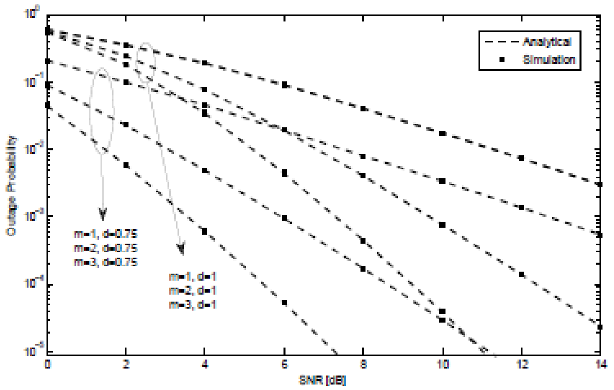

Fig. 4. Outage Probability for network coded relay network over Nakagami-m Fading Channel

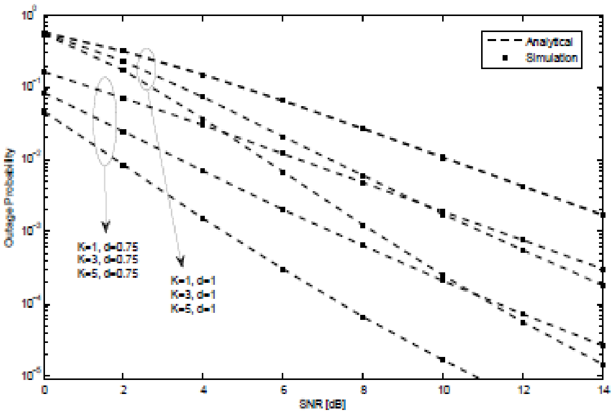

Fig. 5. Outage Probability for network coded relay network over Rician Fading Channel

By using [24, eq.3.211] gives the closed form SER expression.

$$SER = \frac{2}{\pi\zeta^2\Omega_{1j1}\Omega_{1j2}}B(\tfrac{1}{2},\tfrac{5}{2})F_1(\tfrac{5}{2},1,1,3;-\frac{2}{\zeta\Omega_{1j1}};-\frac{2}{\zeta\Omega_{1j2}}) \quad (32)$$

A.2: Derivation of (18)

$$SER = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{(2msin^2\theta)^{2m}}{(\Omega_{1j1}\zeta s+2msin^2\theta)^m(\Omega_{1j2}\zeta s+2msin^2\theta)^m}\,d\theta \qquad (33)$$

Substituting $sin^2\theta{=}\upsilon$ and $d\theta = \frac{d\upsilon}{2\sqrt{\upsilon(1-\upsilon)}}$ into (30) yields

$$SER = \frac{1}{\pi}\int_0^1 \frac{(2m\upsilon)^{2m}}{(2m\upsilon+\zeta\Omega_{1J1})^m(2m\upsilon+\zeta\Omega_{1J2})^m}\frac{d\upsilon}{2\sqrt{\upsilon(1-\upsilon)}} = \frac{2m^2}{\pi\zeta^2\Omega_{1j1}\Omega_{1j2}}\int_0^1 \upsilon^{\left(2m-\frac{1}{2}\right)}(1-\upsilon)^{-\frac{1}{2}}\left(1+\frac{2m\upsilon}{\zeta\Omega_{1j1}}\right)^{-m}\left(1+\frac{2m\upsilon}{\zeta\Omega_{1j2}}\right)^{-m}$$

(34)

By using [24, eq.3.211] gives the closed form SER expression.

$$SER = \frac{2m^2}{\pi\zeta^2\Omega_{1j1}\Omega_{1j2}}B(\tfrac{1}{2},2m+\tfrac{1}{2})F_1(2m+\tfrac{1}{2},m,m,2m+1;-\frac{2}{\zeta\Omega_{1j1}};-\frac{2}{\zeta\Omega_{1j2}})$$

(35)

A.3: Derivation of (21)

$$SER=\frac{1}{\pi}\int_0^{\frac{3\pi}{4}}\frac{1}{\left(1+\frac{\zeta\Omega_{1j1}}{4sin^2\theta}\right)\left(1+\frac{\zeta\Omega_{1j2}}{4sin^2\theta}\right)}\,d\theta=\frac{1}{\pi}\int_0^{\frac{3\pi}{4}}\frac{\left(\frac{sin^2\theta}{2}\right)^2}{\left(\frac{sin^2\theta}{2}+\frac{\zeta\Omega_{1j1}}{8}\right)\left(\frac{sin^2\theta}{2}+\frac{\zeta\Omega_{1j2}}{8}\right)}\,d\theta$$

(36)

Substituting $\frac{sin^2\theta}{2}=\frac{\upsilon}{2}$ and $d\theta=\frac{d\left(\frac{\upsilon}{2}\right)}{2\sqrt{\frac{\upsilon}{2}(\frac{1}{2}-\frac{\upsilon}{2})}}$ into (30) yields

$$SER=\frac{1}{\pi}\int_0^{\frac{1}{2}}\frac{\left(\frac{\upsilon}{2}\right)^2}{\left(\left(\frac{\upsilon}{2}\right)+\frac{\zeta\Omega_{1j1}}{8}\right)\left(\frac{\upsilon}{2}+\frac{\zeta\Omega_{1j2}}{8}\right)}\frac{d\left(\frac{\upsilon}{2}\right)}{2\sqrt{\frac{\upsilon}{2}(\frac{1}{2}-\frac{\upsilon}{2})}}$$
$$=\frac{1}{2\pi}\int_0^{\frac{1}{2}}\left(\frac{\upsilon}{2}\right)^{\frac{3}{2}}\left(\frac{1}{2}-\frac{\upsilon}{2}\right)^{-\frac{1}{2}}\left(\frac{\upsilon}{2}+\frac{\zeta}{8}\right)^{-2}d\left(\frac{\upsilon}{2}\right)$$

(37)

By using [24, eq.3.211] gives the closed form SER expression

$$SER=\frac{1}{2\pi}\left(\frac{\zeta}{8}\right)^{-2}\left(\frac{1}{4}\right)B\left(\frac{1}{2},\frac{5}{2}\right)_2F_1\left(2,\frac{5}{2};3;-\frac{4}{\zeta}\right) \qquad (38)$$

A.4: Derivation of (22)

$$SER=\frac{1}{\pi}\int_0^{\frac{3\pi}{4}}\frac{(4msin^2\theta)^{2m}}{\left(\Omega_{1j1}\zeta s+4msin^2\theta\right)^m\left(\Omega_{1j2}\zeta s+4msin^2\theta\right)^m}\,d\theta$$
$$=\frac{1}{\pi}\int_0^{\frac{3\pi}{4}}\frac{\left(\frac{sin^2\theta}{2}\right)^{2m}}{\left(\frac{\Omega_{1j1}\zeta s}{8m}+\frac{sin^2\theta}{2}\right)^m\left(\frac{\Omega_{1j2}\zeta s}{8m}+\frac{sin^2\theta}{2}\right)^m}\,d\theta$$

(39)

Substituting $\frac{sin^2\theta}{2}=\frac{\upsilon}{2}$ and $d\theta=\frac{d\left(\frac{\upsilon}{2}\right)}{2\sqrt{\frac{\upsilon}{2}(\frac{1}{2}-\frac{\upsilon}{2})}}$ into (30) yields

$$SER=\frac{1}{\pi}\int_0^{\frac{1}{2}}\frac{\left(\frac{\upsilon}{2}\right)^{2m}}{\left(\left(\frac{\upsilon}{2}\right)+\frac{\zeta\Omega_{1j1}}{8m}\right)\left(\frac{\upsilon}{2}+\frac{\zeta\Omega_{1j2}}{8m}\right)}\frac{d\left(\frac{\upsilon}{2}\right)}{2\sqrt{\frac{\upsilon}{2}(\frac{1}{2}-\frac{\upsilon}{2})}}=\frac{1}{2\pi}\int_0^{\frac{1}{2}}\left(\frac{\upsilon}{2}\right)^{\frac{3}{2}}\left(\frac{1}{2}-\frac{\upsilon}{2}\right)^{-\frac{1}{2}}\left(\frac{\upsilon}{2}+\frac{\zeta}{8}\right)^{-2}d\left(\frac{\upsilon}{2}\right)$$

(40)

By using [24, eq.3.211] gives the closed form SER expression

$$SER=\frac{1}{2\pi}\left(\frac{\zeta}{8m}\right)^{-2m}\left(\frac{1}{2}\right)^{2m}B\left(\frac{1}{2},2m+\frac{1}{2}\right)_2F_1\left(2m,2m+\frac{1}{2};2m+1;-\frac{4m}{\zeta}\right)$$

(41)

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