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Relations between capacity utilization, minimal bin size, and bin number

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Abstract. We consider the two-dimensional bin packing problem (BPP): given a set $I = \{1, ..., n\}$ of rectangular items L x w_i , find the minimal number of rectangular bins L x W needed to pack all items. Rotation of the items is not permitted.

Keywords: bin packing problem; two-dimensional problem; minimal number of rectangular bins.

Two-dimensional geometric bin packing, both with and without rotations, is one of the very classical problems in combinatorial optimization, and its study has begun several decades ago (cf. [4]). This is not only due to its theoretical appeal, but also to a large number of applications, ranging from print and web layout (printing all ads and articels onto the minimum number of pages) to office planning (putting a fixed number of office cubicles into a small number of floors), to transportation problems (packing goods into the minimum number of standard-sized containers) and VLSI design.

BPP is known to be NP-hard [3], so a lot of work is concerned with heuristic approaches (e. g. [1]). We are here interested in a worst-case analysis in dependence on the total area of items.

If every item of the BPP instance fits into the given bin (as general assumption) and if the total area of the items to be packed does not exceed the area of the bin then at most three bins are needed to pack all items. The bound three is tight.

Moreover, if the total area of items is not larger than 75 % of the bin area then all items can be arranged in at most two bins. The bound two is tight.

In case, the total area of items is bounded by $k/4$-times the bin area and $k > 2$ then all items can be packed into at most $k - 1$ bins This bound is also tight. This improves a theorem of [2].

The last statement can be strengthened when additional assumptions on the total area of smaller items are made. E.g., if the instance has no large item, i.e. there is no item I with $L_i > L/2$ and $w_i > W/2$, and the total area of items is bounded by $k/3$-times the bin area and $k > 2$ then all items can be packed into at most k bins.

Furthermore, we show, if the size parameters in one direction of all items are not larger than the half of that of the bin and if the size parameters in the other direction of all items are not larger than a third of that of the bin then only two bins are necessary to pack all items.

Further new related results are also given. The proofs of the statements are often based on a theorem of Steinberg [5] and use in general non-trivial case by case analysis.

Some extensions to three-dimensional orthogonal packing problems are also discussed.

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