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Application of Homotopy Perturbation Method to study wave propagation in Transversely Isotropic Thermoelastic Three Dimensional Plate

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ABSTRACT: Wave motion in an infinite transversely isotropic, thermoelastic plate in the context of conventional coupled thermoelasticity (CT), Lord-Shulman (LS) and Green-Lindsay (GL) theories of generalized thermoelasticity has been studied by using Homotopy perturbation method (HPM). The expressions for displacement components and temperature are derived. Finally, the numerical solution is carried out for transversely isotropic plate. The dispersion curves of displacements with thickness and time are presented graphically for coupled and generalized theories of thermoelasticity.

Keywords: Thermal Relaxation; Transversely Isotropic; HPM.

INTRODUCTION

He [1] has studied few problems with or without small parameters with the homotopy perturbation technique and the proposed method does not require small parameters in the equations, so the limitations of the traditional perturbation methods can be eliminated. The initial approximation can be freely selected with possible unknown constants. The approximations obtained by this method are valid not only for small parameters, but also for very large parameters.

In the numerical method, stability and convergence should be considered, to avoid divergent or inappropriate results. Therefore, approximate analytical solutions were introduced, among which HPM, He [1-6] are the most effective and convenient ones for heat equation. Developing the perturbation method for different usage is very difficult because this method has some limitations and based on the existence of a small parameter. Therefore, many different new methods have recently introduced some ways to eliminate the small parameter such as artificial parameter method. One of the semi-exact methods is HPM, introduced by He [1-6] has successfully been applied to solve many types of linear and nonlinear functional equations. The methods have a useful feature in that it provides the solution in a rapid convergent power series with elegantly computable convergence of the solution. This method, which is a combination of homotopy in topology and classic perturbation techniques, provides us with a convenient way to obtain analytic or approximate solutions to a wide variety of problems arising in different fields. He has studied few problems with or without small parameters with the homotopy perturbation technique and the proposed method does not require small parameters in the equations, so the limitations of the traditional perturbation methods can be eliminated. The initial approximation can be freely selected with possible unknown constants. The approximations obtained by this method are valid not only for small parameters, but also for very large parameters. Chun et al. [7] solved the wave equation, where the domain of the space variable is unbounded, and a modified homotopy perturbation method to some nonlinear diffusion equations to obtain exact solutions without any restrictive assumption that may change the physical behavior of the solutions. Biazar and Ghazvini [8] studied the problem of convergence of HPM and presented sufficient condition for convergence of method. Babolian et al. [9] proposed some guidelines for beginners who intend to solve some problems using HPM.

The present work is an attempt to find a displacement and temperature relation from three-dimensional analog of the Rayleigh-Lamb frequency equation that would be sufficient for wave motion in generalized thermoelastic plates. The analysis is based on the approach and HPM used in Refs [1-6].

FORMULATION OF THE PROBLEM:

We consider wave motion in a transversely isotropic coupled thermoelastic plate of thickness $2h$ initially at uniform temperature T_0. The origin of Cartesian co-ordinate system $oxyz$ is taken at any point o in the middle plane of the plate and z-axis is pointed along the thickness of the plate. We assume that the plate is infinite in x and y directions which thus occupies the region

$$\Omega = \{-\infty < x, y < \infty, -h \le z \le h\}$$

In the region Ω, the corresponding basic non dimensional governing equations for homogenous transversely isotropic linear thermoelasticity in the absence of body forces and heat sources are given by

$$\left\{\frac{\partial^2}{\partial x^2} + c_4 \frac{\partial^2}{\partial y^2} + c_2 \frac{\partial^2}{\partial z^2}\right\} u + (1-c_4)\frac{\partial^2 v}{\partial x \partial y} + c_3 \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial}{\partial x}\left(1 + \delta_{2k} t_1 \frac{\partial}{\partial t}\right) T = \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\left\{c_4 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + c_2 \frac{\partial^2}{\partial z^2}\right\} v + (1-c_4)\frac{\partial^2 u}{\partial x \partial y} + c_3 \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial}{\partial y}\left(1 + \delta_{2k} t_1 \frac{\partial}{\partial t}\right) T = \frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$\left\{c_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + c_1 \frac{\partial^2}{\partial z^2}\right\} w + c_3\left\{\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z}\right\} - \overline{\beta}\frac{\partial}{\partial z}\left(1 + \delta_{2k} t_1 \frac{\partial}{\partial t}\right) T = \frac{\partial^2 w}{\partial t^2} \tag{3}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \overline{K}\frac{\partial^2}{\partial z^2}\right) T - \frac{\partial}{\partial t}\left(1 + t_0 \frac{\partial}{\partial t}\right) T - \varepsilon \frac{\partial}{\partial t}\left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \overline{\beta}\frac{\partial w}{\partial z}\right) = 0 \tag{4}$$

where $u_i (i = 1, 2, 3)$ are the displacement components, c_{ij} are isothermal elastic parameters, $T = T(x, y, z, t)$ is temperature change; ρ is mass density and C_e is the specific heat at constant strain, This type of medium has only one axis of elastic symmetry that which is also an axis of thermal symmetry and is taken along $z-$axis. So that $\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$, $\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$, α_1, K_1 are the coefficients of linear thermal expansion and thermal conductivity, in the direction orthogonal to axis of symmetry ,α_3, K_3 are the corresponding quantities along the axis of symmetry, δ_{ik}, $i = 1,2$ is Kronecker delta, where $k = 1$ corresponds to Lord –Shulman (LS) and $k = 2$ corresponds to Green Lindsay (GL) theory of generalized thermoelasticity, t_0, t_1 are thermal relaxation times, moreover $t_0 = 0 = t_1$ leads to coupled (CT) theory of thermoelasticity and further $\beta_1 = 0 = \beta_3$ leads to uncoupled theory of thermoelasticity.

It can be shown thermodynamically that $K_1 > 0$, $K_3 > 0$ and also $\rho > 0$, $T_0 > 0$. In addition, we assume that $C_e > 0$ and isothermal linear elasticities are components of positive definite fourth-order tensor. The necessary and sufficient conditions for satisfaction of this requirement are

$$c_{11} > 0, \quad c_{11}^{\ 2} > c_{12}^{\ 2}, \quad c_{44} > 0, \quad c_{33}(c_{11} + c_{12}) > c_{13}^{\ 2}.$$

The non-dimensional quantities

$$\left(x', y', z'\right)=\frac{\omega^*}{v_p}\left(x\ ,\ y\ ,\ z\ \right),\ \left(u', v', w'\right)=\frac{\rho\omega^* v_p}{\beta_1 T_0}\left(u\ ,\ v\ ,\ w\ \right),$$

$$T' = \frac{T}{T_0},\ t' = \omega^* t,\ t_1' = \omega^* t_1,\ t_0' = \omega^* t_0,\ h' = \frac{\omega^* h}{v_p},\ c_1 = \frac{c_{33}}{c_{11}},\ c_2 = \frac{c_{44}}{c_{11}},\ c_3 = \frac{c_{13}+c_{44}}{c_{11}},$$

$$c_4 = \frac{c_{11}-c_{12}}{2c_{11}},\ c_5 = \frac{c_3 c_2^{-1}-1}{c_1},\ c_6 = \frac{c_{13}}{c_{11}}, v_p = \sqrt{\frac{c_{11}}{\rho}},\quad \overline{K} = \frac{K_3}{K_1},\ \overline{\beta} = \frac{\beta_3}{\beta_1},\ \varepsilon = \frac{\beta_1^2 T_0}{\rho c_e c_{11}},$$

$$\omega' = \frac{\omega}{\omega^*},\ \omega^* = \frac{c_e c_{11}}{K_1},\ \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_1 T_0}. \tag{5}$$

where ω^* is characteristics frequency, ε is thermoelastic-coupling constant and v_p are the velocity of longitudinal wave.

Initial conditions

$$u(x, y, z, 0) = v(x, y, z, 0) = w(x, y, z,0) = T(x, y, z, 0) = 0\ ,$$

$$\dot{u}(x, y, z, 0) = \dot{v}(x, y, z, 0) = \dot{w}(x, y, z, 0) = \dot{T}(x, y, z, 0) = 0\,, \tag{6}$$

Boundary conditions

$$u(x, y, \pm h, t) = v(x, y, \pm h, t) = w(x, y, \pm h, t) = T(x, y, \pm h, t) = 0 \tag{7}$$

SOLUTION OF THE PROBLEM:

We assume harmonic wave solution of the form

$$\left(u, T, v, w\right)\left(x, y, z, t\right) = \vec{u}\left(z, t\right)\exp\left\{-i(\vec{r}.\vec{n})\right\} \tag{8}$$

Where $\vec{u}(z, t) = \left(U(z, t), \theta(z, t), V(z, t), W(z, t)\right)$ is amplitude vector, $\vec{r} = \left(x, y\right)$ is position vector and $\vec{n} = (l, m) = (\sin\alpha, 0)$ is wave number, where α is angle of incidence with axis of symmetry (z -axis).

On applying solution (8) to governing Eqs. (1) - (4) and initial and boundary conditions Eqs. (6) and (7),

$$\left\{l^2 - c_2\frac{\partial^2}{\partial z^2}\right\}U + ilc_3\frac{\partial W}{\partial z} - il\left(1+\delta_{2k}t_1\frac{\partial}{\partial t}\right)\theta + \frac{\partial^2 U}{\partial t^2} = 0 \tag{9}$$

$$\left\{c_4 l^2 - c_2\frac{\partial^2}{\partial z^2}\right\}V + ilc_3\frac{\partial W}{\partial z} + \frac{\partial^2 V}{\partial t^2} = 0 \tag{10}$$

$$\left\{c_2 l^2 - c_1\frac{\partial^2}{\partial z^2}\right\}W + ic_3 l\frac{\partial U}{\partial z} + \overline{\beta}\frac{\partial}{\partial z}\left(1+\delta_{2k}t_1\frac{\partial}{\partial t}\right)\theta + \frac{\partial^2 W}{\partial t^2} = 0 \tag{11}$$

$$\left(l^2 - \overline{K}\frac{\partial^2}{\partial z^2}\right)\theta + \frac{\partial}{\partial t}\left(1+t_0\frac{\partial}{\partial t}\right)\theta - \varepsilon i\frac{\partial}{\partial t}\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\left(lU - i\overline{\beta}\frac{\partial W}{\partial z}\right) = 0 \tag{12}$$

Initial conditions

$$U(z, 0) = V(z, 0) = W(z,0) = a\sin\left(\frac{z\pi}{h}\right)\ ,\ \theta(z, 0) = T_0$$

$$\dot{U}(z,0)=\dot{V}(z,0)=\dot{W}(z,0)=\dot{\theta}(z,0)=0 \tag{13}$$

Boundary conditions

$$U(\pm h,t)=V(\pm h,\ t)=W(\pm h,\ t)=\theta(\pm h,t)=0 \tag{14}$$

BASIC IDEA OF HOMOTOPY PERTURBATION METHOD:

To convey an idea of HPM, we consider a general equation of the type

$$L(u)=0, \quad (15$$

In equation (15), L is an integral or differential operator. We define a convex homotopy $H(u,p)$

by

$$H(u,p)=(1-p)F(u)+pL(u) \tag{16}$$

$F(u)$ is functional operator with known solution v_0, which can be easily obtained. It is clear that

$$H(u,p)=0 \tag{17}$$

From which we have $H(u,0)=F(u)$ and $H(u,1)=L(u)$.

This shows that $H(u,\ p)$ continuously traces an implicitly defined curve from a starting point $H(v_0,0)$ to a solution $H(f,1)$. The embedding parameter increases monotonically from zero to unity as the problem $F(u)=0$ continuously deforms the original problem $L(u)=0$. The embedding parameter can be considered as an expanding parameter. The HPM uses the homotopy parameter 'p' as an expanding parameter to obtain

$$u=\sum_{i=0} p^i u_i = u_0+pu_1+p^2u_2+\cdots \tag{18}$$

If $p\to 1$, then equation (18) corresponds to (16) and becomes the approximate solution of the form

$$u=\lim_{p\to 1} u=\sum_{i=0}^{\infty} u_i \tag{19}$$

It is well know that the series (19) is convergent for most of the cases and also the rate of convergence is dependent on $L(u)$.

APPLICATIONS:

In this section we present the homotopy perturbation method for solving linear partial differential coupled Eqs. (9) - (12), and initial and boundary conditions (13) and (14). According to the homotopy perturbation, we construct the following homotopy:

$$(1-p)\left[\frac{\partial^2 U^*}{\partial t^2}-\frac{\partial^2 U_0^*}{\partial t^2}\right]+p\left\{\left(l^2-c_2\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial t^2}\right)U^*+ilc_3\frac{\partial W^*}{\partial z}-il\partial_t\theta^*\right\}=0$$

$$(1-p)\left[\frac{\partial^2 V^*}{\partial t^2}-\frac{\partial^2 V_0^*}{\partial t^2}\right]+p\left\{\left(c_4 l^2-c_2\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial t^2}\right)V^*+ilc_3\frac{\partial W^*}{\partial z}\right\}=0$$

$$(1-p)\left[\frac{\partial^2 W^*}{\partial t^2}-\frac{\partial^2 W_0^*}{\partial t^2}\right]+p\left\{\left(c_2 l^2-c_1\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial t^2}\right)W^*+ic_3 l\frac{\partial U^*}{\partial z}+\bar{\beta}\frac{\partial}{\partial z}\partial_t\theta^*\right\}=0$$

$$(1-p)\left[\frac{\partial\theta^*}{\partial t}-\frac{\partial\theta_0^*}{\partial t}\right]+p\left\{\left(l^2-\bar{K}\frac{\partial^2}{\partial z^2}\right)\theta^*+\frac{\partial}{\partial t}\left(1+t_0\frac{\partial}{\partial t}\right)\theta^*+i\varepsilon\frac{\partial}{\partial t}\partial_t\left(lU^*+i\bar{\beta}\frac{\partial W^*}{\partial z}\right)\right\}=0$$

or equivalently;

$$\frac{\partial^2 U^*}{\partial t^2}-\frac{\partial^2 U_0^*}{\partial t^2}+p\left\{\frac{\partial^2 U_0^*}{\partial t^2}+\left(l^2-c_2\frac{\partial^2}{\partial z^2}\right)U^*+ilc_3\frac{\partial W^*}{\partial z}-il\partial_t\theta^*\right\}=0 \tag{20}$$

$$\frac{\partial^2 V^*}{\partial t^2}-\frac{\partial^2 V_0^*}{\partial t^2}+p\left\{\frac{\partial^2 V_0^*}{\partial t^2}+\left(c_4 l^2-c_2\frac{\partial^2}{\partial z^2}\right)V^*+ilc_3\frac{\partial W^*}{\partial z}\right\}=0 \tag{21}$$

$$\frac{\partial^2 W^*}{\partial t^2}-\frac{\partial^2 W_0^*}{\partial t^2}+p\left\{\frac{\partial^2 W_0^*}{\partial t^2}+\left(c_2 l^2-c_1\frac{\partial^2}{\partial z^2}\right)W^*+ic_3 l\frac{\partial U^*}{\partial z}+\bar{\beta}\frac{\partial}{\partial z}\partial_t\theta^*\right\}=0 \tag{22}$$

$$\frac{\partial\theta^*}{\partial t}-\frac{\partial\theta_0^*}{\partial t}+p\left\{\frac{\partial\theta_0^*}{\partial t}+\left(l^2-\bar{K}\frac{\partial^2}{\partial z^2}\right)\theta^*+t_0\frac{\partial^2\theta^*}{\partial t^2}+i\varepsilon\frac{\partial}{\partial t}\partial_t^*\left(lU^*+i\bar{\beta}\frac{\partial W^*}{\partial z}\right)\right\}=0 \tag{23}$$

Where $\partial_t=1+\delta_{2k}t_1\frac{\partial}{\partial t}$, $\partial_t^{\ *}=1+\delta_{1k}t_0\frac{\partial}{\partial t}$ and U_0^*, V_0^*, W_0^*, θ_0^* are initial solutions.
Suppose the solutions of system of Eqs. (20) – (23) has the form

$$U^*=\sum_{i=0}p^i U_i=U_0+pU_1+p^2U_2+\cdots \tag{24}$$

$$V^*=\sum_{i=0}p^i V_i=V_0+pV_1+p^2V_2+\cdots \tag{25}$$

$$W^*=\sum_{i=0}p^i W_i=W_0+pW_1+p^2W_2+\cdots \tag{26}$$

$$\theta^*=\sum_{i=0}p^i\theta_i=\theta_0+p\theta_1+p^2\theta_2+\cdots \tag{27}$$

Substituting solutions (24) – (27) into Equations (20) – (23), and comparing coefficients of terms with identical powers of p, leads to:

$$p^0:\frac{\partial^2 U_0}{\partial t^2}-\frac{\partial^2 U_0^*}{\partial t^2}=0 \tag{28.1}$$

$$p^0:\frac{\partial^2 V_0}{\partial t^2}-\frac{\partial^2 V_0^*}{\partial t^2}=0 \tag{28.2}$$

$$p^0 : \frac{\partial^2 W_0}{\partial t^2} - \frac{\partial^2 W_0^*}{\partial t^2} = 0 \tag{28.3}$$

$$p^0 : \frac{\partial \theta_0}{\partial t} - \frac{\partial \theta_0^*}{\partial t} = 0 \tag{28.4}$$

$$p^i : \frac{\partial^2 U_i}{\partial t^2} + \delta_{1i} \frac{\partial^2 U_0^*}{\partial t^2} + \left(l^2 - c_2 \frac{\partial^2}{\partial z^2} \right) U_{i-1} + ilc_3 \frac{\partial W_{i-1}}{\partial z} - il\partial_t \theta_{i-1} = 0 \tag{29.1}$$

$$p^i : \frac{\partial^2 V_i}{\partial t^2} + \delta_{1i} \frac{\partial^2 V_0^*}{\partial t^2} + \left(c_4 l^2 - c_2 \frac{\partial^2}{\partial z^2} \right) V_{i-1} + ilc_3 \frac{\partial W_{i-1}}{\partial z} = 0 \tag{29.2}$$

$$p^i : \frac{\partial^2 W_i}{\partial t^2} + \delta_{1i} \frac{\partial^2 W_0^*}{\partial t^2} + \left(c_2 l^2 - c_1 \frac{\partial^2}{\partial z^2} \right) W_{i-1} + ic_3 l \frac{\partial U_{i-1}}{\partial z} + \overline{\beta} \frac{\partial}{\partial z} \partial_t \theta_{i-1} = 0 \tag{29.3}$$

$$p^i : \frac{\partial \theta_i}{\partial t} + \delta_{1i} \frac{\partial \theta_0^*}{\partial t} + \left(l^2 - \overline{K} \frac{\partial^2}{\partial z^2} \right) \theta_{i-1} + t_0 \frac{\partial^2 \theta_{i-1}}{\partial t^2} + i\varepsilon \frac{\partial}{\partial t} \partial_t^{\,*} \left(lU_{i-1} + i\overline{\beta} \frac{\partial W_{i-1}}{\partial z} \right) = 0 \tag{29.4}$$

$i = 1,\ 2,\ 3, \cdots$

Initial conditions

$p^0 : U_0(z,\ 0) = V_0(z,\ 0) = W_0(z,0) = a \sin(\delta\, z)\ ,\ \theta_0(z,\ 0) = T_0$

$p^i : U_i(z,\ 0) = V_i(z,\ 0) = W_i(z,0) = \theta_i(z,0)\ ,\ i = 1,\ 2,\ 3, \cdots$

$$p^i : \dot{U}_i(z,\ 0) = \dot{V}_i(z,\ 0) = \dot{W}_i(z,0) = \dot{\theta}_i(z,0) = 0\ ,\ i = 0,\ 1,\ 2,\ 3, \cdots \tag{30}$$

Boundary conditions

$$p^i : U_i(\pm h,\ t) = V_i(\pm h,\ t) = W_i(\pm h,\ t) = \theta_i(\pm h,\ t) = 0\ ,\ i = 0,\ 1,\ 2,\ 3, \cdots \tag{31}$$

Suppose initial solutions

$U_0^* = V_0^* = a \cos(\omega_T t) \sin(\delta\, z)$, $W_0^* = a \cos(\omega_L t) \sin(\delta\, z)$,

$$\theta_0^* = \frac{4T_0}{\pi} \sin(\delta\, z) \exp\left(-\sqrt{\overline{K}} \delta^2\, t \right) \tag{32}$$

Where $\omega_T = \sqrt{c_2}\delta$, $\omega_L = \sqrt{c_1}\delta$, $\delta = \pi / h$.

Solving equations (28) and (29), by using (30) – (32), we get

$U_0 = V_0 = a \cos(\omega_T t) \sin(\delta\, z)$,

$W_0 = a \cos(\omega_L t) \sin(\delta\, z)$,

$$\theta_0 = \frac{4T_0}{\pi}\sin(\delta z)\exp\left(-\sqrt{\overline{K}}\delta^2 t\right),$$

$$U_1 = \sin(\delta z)\left\{\phi_1^{(0)}\cos(\omega_T t) + \xi_1^{(0)}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \psi_1^{(0)}\cos(\omega_L t)\cos(\delta z),$$

$$V_1 = \phi_1^{(1)}\cos(\omega_T t)\sin(\delta z) + \psi_1^{(1)}\cos(\omega_L t)\cos(\delta z),$$

$$W_1 = \cos(\delta z)\left\{\psi_1^{(2)}\cos(\omega_T t) + \xi_1^{(2)}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \phi_1^{(2)}\cos(\omega_L t)\sin(\delta z),$$

$$\theta_1 = \cos(\delta z)\left[\phi_1^{(3)}\cos(\omega_L t) + \psi_1^{(3)}\sin(\omega_L t)\right] - \sin(\delta z)\left[\phi_2^{(3)}\cos(\omega_T t) + \psi_2^{(3)}\sin(\omega_T t) + \xi_1^{(2)}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right],$$

$$U_2 = \sin(\delta z)\left\{A_{11}\cos(\omega_T t) + A_{14}\sin(\omega_T t) + A_{15}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \cos(\delta z)\left\{A_{12}\cos(\omega_L t) + A_{13}\sin(\omega_L t)\right\},$$

$$V_2 = \sin(\delta z)\left[B_{11}\cos(\omega_T t) + B_{13}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right] + B_{12}\cos(\omega_L t)\cos(\delta z),$$

$$W_2 = \sin(\delta z)\left\{D_{11}\cos(\omega_L t) - D_{13}\sin(\omega_L t)\right\} + \cos(\delta z)\left\{D_{12}\cos(\omega_T t) + D_{14}\sin(\omega_T t) + D_{15}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\},$$

$$\theta_2 = \sin(\delta z)\left\{E_{14}\cos(\omega_T t) + E_{13}\sin(\omega_T t) + E_{15}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \cos(\delta z)\left\{E_{11}\sin(\omega_L t) + E_{12}\cos(\omega_L t)\right\},$$

$$U_3 = \sin(\delta z)\left\{F_{11}\cos(\omega_T t) + F_{12}\sin(\omega_T t) + F_{13}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \cos(\delta z)\left\{F_{14}\cos(\omega_L t) + F_{15}\sin(\omega_L t)\right\},$$

$$V_3 = \sin(\delta z)\left[G_{11}\cos(\omega_T t) - G_{12}\sin(\omega_T t) - G_{13}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right] + \cos(\delta z)\left[G_{14}\cos(\omega_L t) - G_{15}\sin(\omega_L t)\right],$$

$$W_3 = \sin(\delta z)\left\{H_{11}\cos(\omega_L t) + H_{12}\sin(\omega_L t)\right\} + \cos(\delta z)\left\{H_{13}\cos(\omega_T t) + H_{14}\sin(\omega_T t) + H_{15}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\},$$

$$\theta_3 = \sin(\delta z)\left\{J_{11}\sin(\omega_T t) + J_{12}\cos(\omega_T t) + J_{13}\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\} + \cos(\delta z)\left\{J_{14}\sin(\omega_L t) + J_{15}\cos(\omega_L t)\right\},$$

Where

$$\phi_1^{(0)} = a\left[\left(s^2 + c_2\delta^2\right)\omega_T^{-2} - 1\right], \psi_1^{(0)} = ic_3 sa\delta\omega_L^{-2},\ \xi_1^{(0)} = 4iT_0\overline{K}^{-1}\delta^{-4}\pi^{-1}s(1 - t_1\delta_{2k}\delta^2\sqrt{\overline{K}}),$$

$$\phi_1^{(1)} = a\left[\left(c_4 s^2 + c_2\delta^2\right)\omega_T^{-2} - 1\right], \psi_1^{(1)} = ic_3 sa\delta\omega_L^{-2}, \phi_1^{(2)} = a\left[\left(c_2 s^2 + c_1\delta^2\right)\omega_L^{-2} - 1\right],$$

$$\psi_1^{(2)} = ic_3 sa\delta\omega_T^{-2}, \xi_1^{(2)} = -4\overline{\beta}T_0\overline{K}^{-1}\delta^{-3}\pi^{-1}(1 - t_1\delta_{2k}\delta^2\sqrt{\overline{K}}), \phi_1^{(3)} = -a\overline{\beta}\delta\varepsilon, \quad \psi_1^{(3)} = a\varepsilon t_0\delta_{1k}\delta\overline{\beta}\omega_L,$$

$$\phi_2^{(3)} = -is\varepsilon a, \psi_2^{(3)} = ia\varepsilon s t_0\delta_{1k}\omega_T, \xi_1^{(3)} = 4T_0\pi^{-1}\left\{(s^2 + \overline{K}\delta^2)\delta^{-2}\overline{K}^{-1/2} + t_0\sqrt{\overline{K}}\delta^2 - 1\right\},$$

$$A_{11} = \delta^{-2}c_2^{-1}\left[\left(s^2 + c_2\delta^2\right)\phi_1^{(0)} + is\phi_2^{(3)}\right] + is\delta^{-1}c_2^{-1}\left[c_3\psi_1^{(2)} + t_1\delta_{2k}\sqrt{c_2}\psi_2^{(3)}\right],$$

$$A_{12}=\delta^{-2}c_1^{-1}\left[\left(s^2+c_2\delta^2\right)\psi_1^{(0)}-is\phi_1^{(3)}\right]+is\delta^{-1}c_1^{-1}\left[c_3\phi_1^{(2)}-t_1\delta_{2k}\sqrt{c_1}\psi_1^{(3)}\right],$$

$$A_{13}=-is\delta^{-1}c_1^{-1}\left[\delta^{-1}\psi_1^{(3)}-t_1\delta_{2k}\sqrt{c_1}\phi_1^{(3)}\right],\ A_{14}=is\delta^{-2}c_2^{-1},$$

$$A_{15}=-\delta^{-4}\overline{K}^{-1}\left[\left(s^2+c_2\delta^2\right)\xi_1^{(0)}-is\xi_1^{(3)}\right]-\delta^{-2}\overline{K}^{-1}\left[\sqrt{\overline{K}}\xi_1^{(3)}-ic_3s\delta^{-1}\xi_1^{(2)}\right],$$

$$B_{11}=\delta^{-2}c_2^{-1}\left(c_4s^2-c_2\delta^2\right)\phi_1^{(1)}+is\delta\, c_3\psi_1^{(2)},\ B_{12}=\delta^{-2}c_1^{-1}\left(c_4s^2-c_2\delta^2\right)\psi_1^{(1)}+is\delta\, c_3\phi_1^{(2)},$$

$$B_{13}=isc_3\delta\xi_1^{(2)},\ D_{11}=\delta^{-2}c_1^{-1}\left[\left(c_2s^2+c_1\delta^2\right)\phi_1^{(2)}-isc_3\delta\psi_1^{(0)}-\overline{\beta}\delta\phi_1^{(3)}\right]-\overline{\beta}t_1\delta_{2k}\psi_1^{(3)}c_1^{-1/2},$$

$$D_{12}=\delta^{-2}c_2^{-1}\left[\left(c_1\delta^2-c_2s^2\right)\psi_1^{(2)}+isc_3\delta\phi_1^{(0)}-\overline{\beta}\delta\phi_2^{(3)}\right]-\overline{\beta}t_1\delta_{2k}\psi_2^{(3)}c_2^{-1/2},$$

$$D_{13}=\overline{\beta}c_1^{-1}\left[\delta^{-1}\psi_1^{(3)}-t_1\delta_{2k}c_1^{-1/2}\phi_1^{(3)}\right],\ D_{14}=-\overline{\beta}c_2^{-1}\left[\delta^{-1}\psi_2^{(3)}-t_1\delta_{2k}c_1^{-1/2}\phi_2^{(3)}\right],$$

$$D_{15}=-\delta^{-4}\overline{K}^{-1}\left[\left(c_2s^2+c_1\delta^2\right)\xi_1^{(2)}+isc_3\delta\xi_1^{(0)}+\overline{\beta}\delta\left(\xi_1^{(3)}+t_1\delta_{2k}\delta^2\xi_1^{(3)}\right)\right],$$

$$E_{11}=-\delta^{-1}c_1^{-1/2}\left(s^2+\overline{K}\delta^2\right)\phi_1^{(3)}+\delta\sqrt{c_1}\left[t_1\phi_1^{(3)}-\varepsilon\, t_0\delta_{1k}\left(is\psi_1^{(0)}-\overline{\beta}\delta\phi_1^{(2)}\right)\right],$$

$$E_{12}=\delta^{-1}c_1^{-1/2}\left(s^2+\overline{K}\delta^2\right)\psi_1^{(3)}-\delta\, t_1\sqrt{c_1}\psi_1^{(3)}+\varepsilon\left(is\psi_1^{(0)}-\overline{\beta}\delta\phi_1^{(2)}\right),$$

$$E_{13}=\delta^{-1}c_2^{-1/2}\left(s^2+\overline{K}\delta^2\right)\phi_2^{(3)}-\delta\, t_1\sqrt{c_2}\phi_2^{(3)}+\varepsilon\, t_0\delta\delta_{1k}\sqrt{c_2}\left(is\phi_1^{(0)}+\overline{\beta}\delta\psi_1^{(2)}\right),$$

$$E_{14}=\delta^{-1}c_2^{-1/2}\left(s^2+\overline{K}\delta^2\right)\psi_2^{(3)}+\delta\, t_1\sqrt{c_2}\psi_2^{(3)}+\varepsilon\left(is\phi_1^{(0)}+\overline{\beta}\delta\psi_1^{(2)}\right),$$

$$E_{15}=\delta^{-2}\overline{K}^{-1/2}\left(s^2+\overline{K}\delta^2\right)\xi_1^{(3)}+\delta^2t_1\sqrt{\overline{K}}\xi_1^{(3)}+\varepsilon\left[is\xi_1^{(0)}+\overline{\beta}\delta\xi_1^{(2)}+t_0\delta_{1k}\delta^2\sqrt{\overline{K}}\left(is\xi_1^{(0)}+\overline{\beta}\delta\xi_1^{(2)}\right)\right].$$

$$F_{11}=\omega_T^{-2}\left[\left(s^2+c_2\delta^2\right)A_{11}-is\left(c_3D_{12}+E_{14}+t_1\delta_{2k}\omega_TE_{13}\right)\right],$$

$$F_{12}=\omega_T^{-2}\left[\left(s^2+c_2\delta^2\right)A_{14}-is\left(c_3D_{14}+E_{13}-t_1\delta_{2k}\omega_TE_{14}\right)\right],$$

$$F_{13}=-\overline{K}^{-1}\delta^{-4}\left[\left(s^2+c_2\delta^2\right)A_{15}-is\left(c_3D_{15}+E_{15}-t_1\delta_{2k}\delta^2\sqrt{\overline{K}}E_{15}\right)\right],$$

$$F_{14}=\omega_L^{-2}\left[\left(s^2+c_2\delta^2\right)A_{12}+is\left(c_3D_{11}-E_{12}-t_1\delta_{2k}\omega_LE_{11}\right)\right],$$

$$F_{15}=\omega_L^{-2}\left[\left(s^2+c_2\delta^2\right)A_{13}-is\left(c_3D_{13}+E_{11}-t_1\delta_{2k}\omega_TE_{12}\right)\right],$$

$$G_{11}=\omega_T^{-2}\left[\left(c_4s^2+c_2\delta^2\right)B_{11}-isc_3\delta D_{12}\right],\ G_{12}=isc_3\delta\, D_{14}\omega_T^{-2},$$

$$G_{13} = \delta^{-4}\overline{K}^{-1}\left[\left(c_4 s^2 + c_2\delta^2\right)B_{13} - isc_3\delta D_{15}\right],\ G_{14} = \omega_L^{-2}\left[\left(c_4 s^2 + c_2\delta^2\right)B_{12} + isc_3\delta D_{11}\right],$$

$$G_{12} = isc_3\delta\, D_{13}\omega_L^{-2},\ H_{11} = \omega_L^{-2}\left[\left(c_2 s^2 - c_1\delta^2\right)D_{11} - is\delta\, c_3 A_{12} - \overline{\beta}\delta\left(E_{12} + t_1\delta_{2k}\omega_L E_{11}\right)\right],$$

$$H_{12} = -\omega_L^{-2}\left[\left(c_2 s^2 - c_1\delta^2\right)D_{13} + is\delta\, c_3 A_{13} + \overline{\beta}\delta\left(E_{11} - t_1\delta_{2k}\omega_L E_{12}\right)\right],$$

$$H_{13} = \omega_T^{-2}\left[\left(c_2 s^2 - c_1\delta^2\right)D_{12} + is\delta\, c_3 A_{11} + \overline{\beta}\delta\left(E_{14} + t_1\delta_{2k}\omega_T E_{13}\right)\right],$$

$$H_{14} = \omega_T^{-2}\left[\left(c_2 s^2 - c_1\delta^2\right)D_{14} + is\delta\, c_3 A_{14} + \overline{\beta}\delta\left(E_{13} - t_1\delta_{2k}\omega_T E_{14}\right)\right],$$

$$H_{15} = -\delta^{-4}\overline{K}^{-1}\left[\left(c_2 s^2 - c_1\delta^2\right)D_{15} + is\delta\, c_3 A_{15} + \overline{\beta}\delta\, E_{15}\left(1 - t_1\delta_{2k}\delta^2\sqrt{\overline{K}}\right)\right],$$

$$J_{11} = -\omega_T^{-1}\left(s^2 + \overline{K}\delta^2\right)E_{14} + t_0\omega_T\left[E_{14} + \varepsilon\delta_{1k}\left(isA_{11} + \delta\overline{\beta}D_{12}\right)\right] - \varepsilon\left(isA_{14} + \delta\overline{\beta}D_{14}\right),$$

$$J_{12} = \omega_T^{-1}\left(s^2 + \overline{K}\delta^2\right)E_{13} - t_0\omega_T\left[E_{13} - \varepsilon\delta_{1k}\left(isA_{14} + \delta\overline{\beta}D_{14}\right)\right] - \varepsilon\left(isA_{11} + \delta\overline{\beta}D_{12}\right),$$

$$J_{13} = \delta^{-2}\overline{K}^{-1/2}\left(s^2 + \overline{K}\delta^2\right)E_{15} + t_0\delta^2\sqrt{\overline{K}}\left[E_{15} + \varepsilon\delta_{1k}\left(isA_{15} + \delta\overline{\beta}D_{15}\right)\right] - \varepsilon\left(isA_{15} + \delta\overline{\beta}D_{15}\right),$$

$$J_{14} = -\omega_L^{-1}\left(s^2 + \overline{K}\delta^2\right)E_{12} + t_0\omega_L\left[E_{12} + \varepsilon\delta_{1k}\left(isA_{12} - \delta\overline{\beta}D_{11}\right)\right] - \varepsilon\left(isA_{13} + \delta\overline{\beta}D_{13}\right),$$

$$J_{15} = \omega_L^{-1}\left(s^2 + \overline{K}\delta^2\right)E_{11} - t_0\omega_L\left[E_{11} + \varepsilon\delta_{1k}\left(isA_{13} - \delta\overline{\beta}D_{13}\right)\right] - \varepsilon\left(isA_{12} - \delta\overline{\beta}D_{11}\right),$$

If $p \to 1$, then equations (24) to (27) becomes the approximate solutions of the form

$$U = \lim_{p\to 1} U^* = U_0 + U_1 + U_2 + \cdots$$

$$= \sin(\delta z)\left\{\left(A_{11} + F_{11} + \phi_1^{(0)} + a\right)\cos(\omega_T t) + \left(A_{14} + F_{12}\right)\sin(\omega_T t) + \left(A_{15} + F_{13} + \xi_1^{(0)}\right)\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\}$$

$$+ \cos(\delta z)\left\{\left(A_{12} + F_{14} + \psi_1^{(0)}\right)\cos(\omega_L t) + \left(A_{13} + F_{15}\right)\sin(\omega_L t)\right\},$$

$$V = \lim_{p\to 1} V^* = V_0 + V_1 + V_2 + \cdots$$

$$= \sin(\delta z)\left[\left(\phi_1^{(1)} + B_{11} + G_{11} + a\right)\cos(\omega_T t) - G_{12}\sin(\omega_T t) + \left(B_{13} - G_{13}\right)\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right]$$

$$+ \cos(\delta z)\left[\left(\psi_1^{(1)} + B_{12} + G_{14}\right)\cos(\omega_L t) - G_{15}\sin(\omega_L t)\right],$$

$$W = \lim_{p\to 1} W^* = W_0 + W_1 + W_2 + \cdots$$

$$= \sin(\delta z)\left\{\left(\phi_1^{(2)} + D_{11} + H_{11} + a\right)\cos(\omega_L t) + \left(H_{12} - D_{13}\right)\sin(\omega_L t)\right\}$$

$$+ \cos(\delta z)\left\{\left(\psi_1^{(2)} + D_{12} + H_{13}\right)\cos(\omega_T t) + \left(D_{14} + H_{14}\right)\sin(\omega_T t) + \left(\xi_1^{(2)} + D_{15} + H_{15}\right)\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\}$$

$$\theta = \lim_{p\to 1}\theta^* = \theta_0 + \theta_1 + \theta_2 + \cdots$$

$$= \sin(\delta z)\left\{\left(E_{14} + J_{12} - \phi_2^{(3)}\right)\cos(\omega_T t) + \left(E_{13} - \psi_2^{(3)} + J_{11}\right)\sin(\omega_T t) + \left(E_{15} + J_{13} - \xi_1^{(2)} + 4T_0/\pi\right)\exp\left(-\sqrt{\overline{K}}\delta^2 t\right)\right\}$$

$$+ \cos(\delta z)\left\{\left(\psi_1^{(3)} + E_{11} + J_{14}\right)\sin(\omega_L t) + \left(\phi_1^{(3)} + E_{12} + J_{15}\right)\cos(\omega_L t)\right\}.$$

CONCLUSION

The homotopy perturbation method has an advantage in comparison to the traditional perturbation methods. The approximate analytic solution obtained applying the homotopy perturbation method for solving coupled complex-valued second-order differential equations is in very good agreement with the exact closed form analytic solution. In this paper, we have shown that the HP method can be used successfully for finding the solution of the linear-boundary value problem.. The HP method is not affected by round off errors and the solution is found without taking a long time and a large amount of computer memory. Therefore, it may be concluded that this technique is very powerful and efficient in finding the analytical solutions for system of differential equations.

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