ON EFFECTIVITY OF INDUCTIVE METHODS IN MATHEMATICAL EDUCATION AT SECONDARY SCHOOL

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Abstract

Effectivity and quality of mathematical education at secondary school is always very actual didactic topic. One way how to increase them is using unconvencional experimentally inductive methods. There are described the methods as well as process and results of research conected with this problem there. The aim of the research was to check possibilities of using of modern unconvencional methods for more effective mathematical education at secondary school. The inductive methods are compared with the traditional ones when students get completed knowledge without any derivation or proof. Using Student's t-test and F-test were verified two hypotheses. The first hypothesis that the experimentally inductive approach and methods in mathematical education at secondary school are more effective and interesting for students than the traditional methods was validated. The second hypothesis which supposed longer persistence of knowledge obtained by the inductive methods was not validated.
Key words: *deduction, induction, mathematical education, secondary school, pedagogical research.*

Introduction

Since mathematics represents very abstract discipline one should be careful about formalism during transfer of mathematical knowledge. Particularly, such danger is actual at university mathematical education (Kopka, 2000; Emanovský, 2001) as well as at secondary school (Kopka, 1999; Břehovský, Emanovský, 2009). The authors of the articles recommend to use some non-traditional methods based on experiment and induction.

Despite of traditional deductive approach in mathematics most mathematical theories have both an experimental and inductive character. Their beginnings arise out of tentative searching and speculative trial and error; they gain a deductive character only after their period of investigation. Investigations, as described in Kopka, J. (2004), is a method of teaching and learning mathematics which permits students to enter and penetrate more deeply into the world of mathematics that most other teaching approaches fail to do. If one wishes the students to have experiences of how mathematics evolves, then it should be respected how mathematical theories come into existence, how they develop and how they finally gain their form and nature. Too frequently students are only exposed to mathematics in its final and approved form. Using investigations is one method of teaching involved in the full range of the development of a mathematical theory. Investigations also provide students with insights into what it is like to be a mathematician and to experience mathematical thinking at work. Students should be able to investigate certain mathematical situations and consequently to formulate problems and hypotheses. This inductive approach should be completed by validation of the hypotheses, i.e. by return deduction. Clearly, the inductive way is much more time-consuming and difficult for teachers and students than traditional one. On the other hand, it contains very important and worth student activities which are all about making the students more active participants in the learning process – an observation, an investigation, formulation and solving of problems and formulation and validation of hypotheses.

Inductive Teaching and Learning Methods

The most commonly used inductive teaching and learning methods are inquiry learning, problem-based learning, project-based learning, case-based teaching, discovery learning and just-in-time teaching (Prince, Felder, 2006). The investigations according to Kopka, J. (2004) is possible to consider as a method of the first cathegory. The inquiry learning means that students are presented with questions to be answered, problems to be solved, or a set of observations to be explained (Bateman, 1990). If the method is implemented effectively, the students should learn to „formulate good questions, identify and collect appropriate evidence, present results systematically, analyze and interpret results, formulate conclusions, and evaluace the worth and importace those conclusions“ (Lee, 2004).

Research Problems

The experimental inductive methods are much more suitable for student at secondary school because of their age. The objective of this paper is to describe the inductive approach (in the sense of Kopka's investigations) as well as the process and results of research whose aim was to verify the effectivity of the non-traditional inductive methods for mathematical education at secondary school.

There were formulated the following problems for the research:

- Does the submission of the inductive methods to mathematical education guarantee more effective and permanent transfer of knowledge?
- Does the using of the inductive methods in mathematical education contribute to better understanding of learning?
- Does the using of the inductive methods in mathematical education contribute to better ability to apply new knowledge and skills?

Metodology of Research

There were set the following two hypotheses for the research:

H_1: The using of the inductive approach and methods in mathematical education raises the standard of student knowledge in given topic in comparison with the traditional methods?

H_2: The using of the inductive approach and methods in mathematical education leads to longer persistence of obtained knowledge and skills in comparison with the traditional methods?

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 22, 2010

Pedagogical experiment was chosen as research instrument for verification of the hypotheses. Linear functions were chosen as the topic suitable for using the experimental methods. The research was realized at two technical schools and two grammar schools because of comparison (Břehovský, 2010).

Pedagogical Experiment

The experiment was based on division of chosen student sample to two comparable groups – control and experimental. The division was done according to results of entrance test (pretest) for investigation of entrance knowledge and skills level. There were used the traditional teaching methods in the control group and the non-traditional ones in the experimental group during the experiment.

The traditional methods in this context mean that students are only exposed to mathematics in its final and approved form without any experiment, derivation or proof. The traditional teaching in the control group was realized in the common manner, i.e. by presentation of input information, speciment example, exercise and revision of knowledge.

In contrast of it, by the non-traditional approach is understood inquiry learning based on observation, investigation, formulation and solving of problems and formulation of hypotheses and their validation. Within the non-traditional teaching in the experimental group the students are presented with simple real problems to be solved. Consequently, the students should formulate and solve another more difficult and more general problems or formulate and validate a hypothesis. The students work on their solutions separately and then they show them to others. Each idea is scarified by others and consequently used for the following work or refused. The role of a teacher is to supervise all described activities, to help students with formulation of problems and hypotheses, with verification of hypotheses and putting conclusions more precise. At the close of the class there are summarized all essential results which were discovered by students and there are repeated all steps that anticipated the discoveries.

Testing of exit knowledge level (posttest) was worked out immediately after the experiment (verification of the hypothesis H_1). To check the persistence of the new knowledge was realized the retest a month later (verification of the hypothesis H_2). Using standardized didactic test (Břehovský, 2010) were obtained data for posttest (see Table 1.) and for retest (see Table 4.). Each test contains 15 problems intent on memorizing and understanding of knowledge and ability of their application in standard and problem situation. The standardization of the tests was realized before the experiment according to Chráska (1999) with 355 students at 5 secondary schools (Břehovský, 2010). Student′s t-test and F-test were used to validate the hypothese because of the type of data.

Results of Research

Verification of Hypothesis H_1

The following table shows results of the test which was done immediately after the experiment. Total number of students taking part in the experiment was 101 (50 students in experimental group and 51 in control group) and maximal number of points in the test was 15 (Břehovský, 2010).

Table 1. Results of posttest.

	Experimental group E			Control group C		
Points x_i	Frequency n_i	Cumulative frequency	Percentile order	Frequency n_i	Cumulative frequency	Percentile order
0	0	0	0,0	0	0	0,0
1	0	0	0,0	2	2	2,0
2	0	0	0,0	1	3	4,9
3	0	0	0,0	0	3	5,9
4	4	4	4,0	7	10	12,7
5	0	4	8,0	5	15	24,5
6	8	12	16,0	8	23	37,3
7	3	15	27,0	3	26	48,0
8	2	17	32,0	4	30	54,9
9	4	21	38,0	4	34	62,7
10	7	28	49,0	2	36	68,6
11	5	33	61,0	5	41	75,5
12	3	36	69,0	3	44	83,3
13	5	41	77,0	1	45	87,3
14	4	45	86,0	5	50	93,1
15	5	50	95,0	1	51	99,0

To verify the hypothesis H_1 was formulated the following zero hypothesis H_{01} and alternative hypothesis H_{A1} :

H_{01} : There is no statistically significant difference between average number of points obtained in posttest in the group E and in the group C.

H_{A1} : There is statistically significant difference between average number of points obtained in posttest in the group E and in the group C.

Using data from the posttest one can compute value of t-test criterion $t = 2{,}8018$ (for details see Table 2., Table 3. and e.g. Chráska (2007)). Since the table value of the criterion for significance level $\alpha = 0{,}05$ is equal to $1{,}984 < 2{,}8018$ the hypothesis H_{01} is refused. It means that for this significance level there exists statistically significant difference between average number of points obtained in posttest in the group E and in the group C and the hypothesis H_1 can be accepted.

Table 2. Partial results for computation of test criterion t (hypothesis H_1).

Experimental group	Control group
$n_E = 50$	$n_C = 51$
$\Sigma x_i = 494$	$\Sigma x_i = 403$
$\Sigma x_i^2 = 5442$	$\Sigma x_i^2 = 3869$
average $\Phi_E = 9{,}88$	average $\Phi_C = 7{,}9$
dispersion $s_E = 3{,}38$	dispersion $s_C = 3{,}7$

Table 3. Results of computation of test criterion t (hypothesis H_1).

computed test criterion	$t = 2{,}8018$
number of degrees of freedom	$f = 98$
table value of criterion t for $\alpha = 0{,}05$ and $f = 100$	$t_{0,05}(100) = 1{,}984$
table value of criterion t for $\alpha = 0{,}01$ and $f = 100$	$t_{0,01}(100) = 2{,}626$

Verification of Hypothesis H_2

The following table shows results of the retest which was done a month after the experiment for the same student sample and maximal number of points in the test (Břehovský, 2010).

Table 4. Results of retest.

	Experimental group E			Control group C		
Points x_i	Frequency n_i	Cumulative frequency	Percentile order	Frequency n_i	Cumulative frequency	Percentile order
0	0	0	0,0	0	0	0,0
1	1	1	1,0	1	1	1,0
2	2	3	4,2	3	4	4,8
3	1	4	7,3	1	5	8,7
4	2	6	10,4	4	9	13,5
5	4	10	16,7	3	12	20,2
6	2	12	22,9	6	18	28,8
7	7	19	32,3	6	24	40,4

8	6	25	45,8	3	27	49,0
9	0	25	52,1	3	30	54,8
10	1	26	53,1	4	34	61,5
11	5	31	59,4	3	37	68,3
12	3	34	67,7	5	42	76,0
13	2	36	72,9	2	44	82,7
14	3	39	78,1	4	48	88,5
15	9	48	90,6	4	52	96,2

To verify the hypothesis H_2 was formulated the following zero hypothesis H_{02} and alternative hypothesis H_{A2} :

H_{02} : There is no statistically significant difference between average number of points obtained in retest in the group E and in the group C.

H_{A2} : There is statistically significant difference between average number of points obtained in retest in the group E and in the group C.

Similarly, using data from the retest one can compute value of t-test criterion t = 0,99 (for details see Table 5. and Table 6.). Since the table value of the criterion for significance level α = 0,05 is equal to 1,984 > 0,99 the hypothesis H_{02} is accepted. It means that for this significance level there does not exist statistically significant difference between average number of points obtained in retest in the group E and in the group C and the hypothesis H_2 can not be accepted.

Table 5. Partial results for computation of test criterion t (hypothesis H_2).

Experimental group	Control group
$n_E = 48$ $\Sigma x_i = 449$ $\Sigma x_i^2 = 5037$ average $\Phi_E = 9{,}35$ dispersion $s_E = 4{,}22$	$n_K = 51$ $\Sigma x_i = 444$ $\Sigma x_i^2 = 4602$ average $\Phi_K = 8{,}54$ dispersion $s_K = 3{,}99$

Table 6. Results of computation of test criterion t (hypothesis H_2).

computed test criterion	***t* = 0,99**
number of degrees of freedom	***f* = 98**
table value of criterion *t* for α = 0,05 and *f* =100	**$t_{0,05}(100) = 1{,}984$**
table value of criterion *t* for α = 0,01 and *f* =100	**$t_{0,01}(100) = 2{,}626$**

The tables 3. and 6. show that 0,99 < $t_{0,01}(100)$ = 2,626 < 2,8018, i.e. the same conslusions for the hypotheses H_1 and H_2 can be pronounced for significance level α = 0,01.

Discussion

The results of the described research correspond with several published analyses which conclude that inquiry-based teaching is generally more effective than traditional instruction for achieving a variety of learning outcomes (Smith, 1996; Haury, 1993; Shymansky, Hedges, Woodworth, 1990).

Conslusions

- The results of the described research have validated the hypothesis H_1 which supposed that the experimentally inductive approach and methods in mathematical education at secondary school are more effective and interesting for students than the traditional methods.
- The hypothesis H_2 which supposed that the using of the inductive approach and methods in mathematical education at secondary school leads to longer persistence of obtained knowledge and skills in comparison with the traditional methods was not validated.

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