# MATRIX-BASED PROJECT PLANNING METHODS 

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#### Abstract

Network planning methods for project planning and scheduling have been applied for more than fifty years including CPM, PDM, PERT and GERT (Fondahl, 1961; Fulkerson, 1962; Kelley \& Walker, 1959; PMI, 2006; Pritsker, 1966), which can be widely applied to project planning fromareas as diverse as construction and $R \& D$. However, these network planning methods are not very appropriate in cases where IT, innovation or product development are involved. There are some shortcomings when using network planning methods for scheduling these kinds of projects, because these methods cannot handle the importance of the task realizations. They cannot solve the problem, when some tasks have to be left out from the project because of the constraints, or when the completion order of tasks can be different. In this paper new matrix-based project planning methods are introduced which illustrate how all possible solutions can be determined in two steps based on the Project Expert Matrix. Firstly those tasks are selected, which have to be or can be realized during the project. Afterwards the dependencies (the sequence of the chosen tasks) are determined taking the project constraints into account. The possible solutions can be ranked and the most probable solution can be chosen which can be realized within the given constraints. This method can be a useful tool for project managers as a part of an expert system.


Key words: matrix-based project planning method, Project Expert Graph, Project Expert Matrix, Stochastic Network Planning Method.

## Introduction

In this paper the main focus is on applying matrix-based methods for planning and scheduling of projects; at the same time a comprehensive comparison of network planning (see e.g. in (Schwarze, 2006; Kerzner, 2009)) and matrix-based methods (introduced in this paper) is undertaken. The oldest and most popular methods, such as the Gantt chart and network planning methods including PDM, CPM (Kelley \& Walker, 1959; Fondahl, 1961), can be widely used for planning and scheduling projects. The DSM method (Steward, 1981) has also been shown to be appropriate for deterministic planning.

There are various project types (e.g. construction projects, IT projects, new product development projects, etc.) with different uncertainties. The uncertainty in task duration and consequently in the project lead time can be estimated by employing stochastic time planning methods like PERT (Fulkerson, 1962) or GERT (Pritsker, 1966). There could be iterations, repetitions during software and product development projects, however, from the previously mentioned methods only the GERT method can be used to represent cycles in the process. In contrast, there are some algorithms in connection with the DSM method for detecting and separating the circles and cycles.

As it appears from these reviews, matrix-based methods can be used not just in those cases, where "traditional" methods can be applied, but they can also be introduced to solve special problems, such as detecting and handling cycles. There are other advanced matrix-based methods that can be used during logic planning to handle the uncertainty of the realization
sequence of the project tasks and in addition, to choose which tasks can be carried out during the projects. These matrix-based methods constitute the basis of this paper. As can be seen in the following section on the Methodology of Research, matrix-based methods can be used to represent different logic plans depicted using the above mentioned methods together, allowing several solutions to be generated using the values of the matrix.

## Background of the Research

Matrix-based methods are also used for planning and scheduling mainly in the cases where the new product development projects. These matrix methods are based on the DSM (Design Structure Matrix/Dependency Structure Matrix) method published by Steward (1981). Tasks of the project are represented in the rows/columns of the matrix in the same order. The DSM method was originally used for system and architecture modeling. However it can be applied for project planning as well. Researchers at the Massachusetts Institute of Technology (Eppinger, Whitney, Smith \& Gebala, 1994) improved the original DSM method for modeling the relationship between two tasks and therefore this DSM method became useable for project planning. The DSM method specifies three basic relations between two tasks. These relations can be sequential, parallel and coupled (iterative) (dsmweb.org; Whitney, Xiao, Chen, Xiao \& Tao, 2007). Relations are shown in an adjacent matrix (see Table 1) where " $X$ " specifies the relation between two tasks.

Table 1. Basic relation between two tasks (dsmweb.org; Whitney, Xiao, Chen, Xiao \& Tao, 2007).

| Sequential relation |  |  |  | Parallel relation |  |  |  | Coupled (iterative) relations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 今 | A | B |  | $\dagger$ | A | B | A | 今 | A | B |  | A |
| A |  | X | $A \rightarrow B$ | A |  |  |  | A |  | X |  |  |
| B |  |  |  | B |  |  | B | B | X |  |  |  |

This method handles the iterations between the two tasks. The iterative relation between task A and task B means that the sequence of task A and task B has to be realized more than once. This relation is the simplest cyclic dependency. The cyclic dependencies are called "Circuits" or "Cycles". Cycles can contain more than two tasks. Detecting cycles is a very important challenge in project management, because the iteration can lead to the increase of the project's duration. When using matrix-based methods for project planning one of the most important functions is to determine the sequence of the tasks. If a project plan does not contain cycles, the matrix of project plan can be reordered into an upper triangular matrix (see Table 2) and the Activity-on-Node graph of the project plan can be topologically ordered. This method is called sequencing (Eppinger, Whitney, Smith \& Gebala, 1994; Danilovic \& Browning, 2007).

Table 2. Sequencing method and topologically ordered graph of a given project.

| $f$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ |  |  | X |  |  |
| $\mathbf{B}$ |  |  |  | X | X |
| $\mathbf{C}$ |  |  |  |  |  |
| $\mathbf{D}$ | X |  |  |  |  |
| $\mathbf{E}$ | X |  | X |  |  |
| Iritial DSM matrix |  |  |  |  |  |
|  |  |  |  |  |  |


| $f$ | $\mathbf{B}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{A}$ | $\mathbf{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{B}$ |  | X | X |  |  |
| $\mathbf{D}$ |  |  |  | X |  |
| $\mathbf{E}$ |  |  |  | X | X |
| $\mathbf{A}$ |  |  |  |  | X |
| $\mathbf{C}$ |  |  |  |  |  |
| Reordered upper-triangular DSM |  |  |  |  |  |
| matrix |  |  |  |  |  |



If a project plan contains cycles, the DSM matrix cannot be reordered into the upper triangular matrix. Therefore, the analyst's objective changes from eliminating the feedback marks to moving them as close as possible to the diagonal (this form of the matrix is known as 'block triangular') (Chen \& Lin, 2002). There are several partitioning algorithms that can be used to detect cycles. Some researchers explain how to summarize the cycle into a new task (Gebala \& Eppinger, 1991). Applying the DSM in cases where the tasks are reordered allows the modules of the projects to be detected. In this case minimally connected sub-graphs of the project network will be identified, which is known as clustering (Meehan, Duffy \& Whitfield, 2007; Thebeau, 2001).

The formerly introduced methods show how to use matrix-based methods for project planning. However, the DSM matrices can also be applied for scheduling (Huang \& Chen, 2006) and resource allocation (Yan, Wang \& Jiang, 2002) as well as for reorganizing projects (Khoo, Chen \& Jiao, 2003; Rick, Márk \& Bercsey, 2006). This allows the time, cost or resource demands of tasks can be represented in the diagonal or in additional columns. Numbers in the off-diagonal cells can show also the lags of successors/predecessors.

## Handling Uncertain Relations

One of the deficiencies of the previously introduced DSM method is that this procedure is not able to handle the uncertain relations between two tasks. There is only a single strict rule of dependence: task A depends on task B or not, hence the method introduced can also be called the binary DSM method. However, one of the most advanced network planning methods (GERT) can handle the decision points and the different kinds of probable project variations. The (binary) DSM method cannot handle probabilities/uncertainties of relations and therefore cannot model the R\&D projects.

Many researchers (Tang, Zhu, Tang, Xu \& He, 2009; Yassine, Falkenburg \& Chelst, 1999) showed that in the case of project planning, there could be uncertain relations between two tasks. They suggested the introduction of a new method called Numerical DSM (NDSM), which can be applied to handle the strength of the relation between the two tasks. When using NDSM matrices the level of the dependency of the relations between the two tasks can be represented. With numbers instead of " X "-es NDSM can represent "Dependency Strength", "Probability of Repetition" and "Impact Strength" (Browning \& Eppinger, 2002). In our approach the values stand for "Dependency Strength". This can be a measure between 0 and 1, where 1 represents an extremely strong dependency. The matrix can now be partitioned by minimizing the sum of the dependency strengths below the diagonal.

Uncertain relations between the two tasks means that a relationship between the tasks can have a probability or importance value to express how probable or how important that the relation will be realized or occur. One of the most important questions is how to determine or estimate the uncertainty of the relationship between the two tasks. The dependency strength can be estimated based on the experience of any prior similar projects (Tang, Zhu, Tang, Xu \& He, 2009), or by taking the opinion of different project management experts into account (Yassine, Falkenburg \& Chelst, 1999). However, some authors did not show that different kinds of project structures exist, if an uncertain relation between the two tasks occurs (see Table 3). In case of IT and innovation projects some tasks would not be realized within the given project. Despite the NDSM method's ability to appropriately handle uncertain relations, it cannot be used to handle the priorities in task realization. Nevertheless, when planning IT or innovation projects it is very important to handle which tasks must be realized, which tasks should be realized, and which tasks can be omitted from the projects if there is not enough time or resources. Working out a method to be able to categorize the project tasks was a very important goal of this research.

## Methodology of Research

In this section several matrix-based methods are introduced. Some of them (i.e. SNPM, PEM, ePEM) can be applied for project planning and generating different kinds of project structures or project variables. There are some methods, which can be used for scheduling and resource allocation; however, they would be the subject of another paper.

## Generating Possible Project Structures

The Stochastic Network Planning Method (SNPM) has already been published (Kosztyán, Fejes \& Kiss, 2008) for generating all possible project networks and project structures, including the different ways of realizing the tasks. The acronym of SNPM alludes to an uncertain project network. If there is an uncertain (successive) relation between task A and task B (denoted as "?" in Table 3) there are two possible project structures: (1) there is a (successive) relation between task $\mathbf{A}$ and task $\mathbf{B}$, therefore task $\mathbf{A}$ and task $\mathbf{B}$ must be realized in a sequence; or (2) task $\mathbf{A}$ and task $\mathbf{B}$ are independent from each other, therefore task $\mathbf{A}$ and task $\mathbf{B}$ can be realized in parallel (see Table 3).

Table 3. Possible project structures.


When applying Numerical DSM, researchers (Browning \& Eppinger, 2002; Tang, Zhu, Tang, Xu \& He, 2009) suggested that uncertain relations between the two tasks should be classified into high, medium and low dependencies. Empty cells mean independency. Within the SNPM method there can be values between 0 and 1 instead of the? mark to express the uncertainty of relations where both value 0 or empty cell means independency, the value 1 represents the certain (successive) relation between the two tasks, and if there is a number between 0 and 1 in a cell it refers to the uncertain (successive) relation between the two tasks.

There is a special case where the value of? marks are between 0 and 1 at both relations in the matrix (see in Table 4). It can mean, that these tasks are coupled, so it can represent a circle. However, it can represent those cases as well, where the order of these tasks can be sequential, so task A can follow task $\mathbf{B}$, or task $\mathbf{B}$ can follow task $\mathbf{A}$, or they can be independent from each other, being carried out in parallel. The following table represents these possible realizations. The value 0.5 represents very special situation. In this case the probability of each solution is the same.

Table 4. Possible realization ways of the uncertain relations.


It can happen with certain kinds of projects that the iterative relation is notallowed, in these cases the task realisations happen according to the other possibilities. In the method introduced here the relation strength is used, which can mean the probability or the importance of the relations. The value of the relation strength between two tasks can be a real value from 0 to 1. Value 0 means the certain independency between the two tasks and value 1 means the certain dependency between the two tasks. Other values (between 0 and 1 ) of the relation strength between two tasks are called uncertain relation.
(1) Uncertainty of relation between tasks can be handled as probability, so the value of an uncertain (successive) relation can mean probability: (1.a) if there is prior information about realization for the given tasks from already completed projects, the (weighted) relative frequencies of occurred relations between tasks will be the (objective) probability of dependencies; (1.b) if there is more than one possible technological plan or project experts' plan in different kinds of DSM matrices, the (weighted) relative frequencies of occurred relations between tasks should be considered as the (subjective) probability of dependencies (see Figure 1). Different kinds of project plans from several project experts can also be reconciled by using the AHP (Analytical Hierarchy Process) method (Chen \& Lin, 2002).
(2) The other interpretation of uncertain relations is the relative importance level of the relations between the two tasks. In this case the importance level of the relation between task $\mathbf{A}$ and task $\mathbf{B}$ shows how important to realize task $\mathbf{B}$ followed by task $\mathbf{A}$.

Since the probability of relation and the importance of relation are distinguished from each other we nominated them with different symbols: $p_{(\mathrm{A}, \mathrm{B})}$ means the probability of dependency (probability of succession) between task $\mathbf{A}$ and task $\mathbf{B}$. This way 1-p $p_{(\mathrm{A}, \mathrm{B})}$ means the probability of independency (probability of non-succession) between task $\mathbf{A}$ and task $\mathbf{B}$. If $p_{(\mathbf{A}, \mathbf{B})}=1-$ $p_{(\mathbf{A}, \mathbf{B})}=0.5$ shows indifferent relations between the two tasks, than the probability of sequential and parallel realization of the tasks is equal (see Table 3 if "?" $=0.5$ ). Likewise the probability the importance of the dependency (succession) and the importance of the independency (nonsuccession) can be defined similarly: $\rho_{(\mathrm{A}, \mathrm{B})}$ nominates the relative importance of dependency between task $\mathbf{A}$ and task $\mathbf{B}$, and $1-\rho_{(A, B)}$ nominates the relative importance of independency between task $\mathbf{A}$ and task $\mathbf{B}$. This way either the sequential or the parallel realization of the tasks can be preferred.

## Research Focus

For understanding the aim of this paper two definitions have to be declared, what project scenario and project structure mean. Project scenarios define project plans build up from different tasks. It can represent the different alternatives like the GERT method or the eEPC diagram. Project structures represent different structures, possible realization orders of the same tasks. They can be depicted by simple logic plans, by network plans (like e.g. CPM or

MPM), furthermore by process management methods (like e.g. eEPC).
The aim of this research is to support the logic planning phase of projects with the help of newly developed matrix-based methods. The basic matrix (introduced in the next section) has been assumed to include the realization priority/occurrence probability of the tasks and the uncertain/possible precedence relations between the two tasks. Based on the values in the matrix, all possible project scenarios and all possible project structures belonging to a given project scenario can be determined and ranked. With the help of the new method it is possible to select the permitted solutions within the time, resource and budget constraints from all the possible project scenarios/ project structures and the optimal solution can be determined according to one or more objective function(s).

## Handling the Uncertainty of Task Realizations

The uncertain task realizations can be handled by the enhanced variation of SNPM called the PEM (Project Expert Matrix) as well (Kiss \& Kosztyán, 2009a, b). Similarly to the uncertain relations the uncertain realizations of the tasks can mean (1) the probability of task realizations (where the probabilities can be determined similarly to the formerly introduced way) or (2) the (relative) importance of task realizations. The uncertainty of the task realizations can be notated into the diagonal of the PEM matrix (see Table 5), where the certain task realization is denoted by 1 or " X " while uncertain task realisation is represented by? mark. As we mentioned before, it can happen at some projects, that project tasks exceeds the time and resource limits, consequently some tasks have to be postponed for later projects or have to be omitted. This is the reason why is it important to prioritise the task realisations.

Table 5. Generating all possible project networks from PEM.


The duration, demands of cost and resources of the tasks can be represented within the matrix diagonal of the enhancing Project Expert Matrix. Apart from the diagonal the lags can be represented next to the strength (probability/importance) of relations in the off-diagonal cells (see Table 6).

Table 6. Extended Project Expert Matrix (ePEM).

| ePEM | Task A |  | Task B |  |
| :--- | :--- | :--- | :--- | :--- |
| A | Uncertainty of <br> realization | Duration | Uncertainty of <br> the Relation | Lag |
|  | Cost Demands | Resource <br> Demands | Uncertainty of <br> realization | Duration |
|  | Uncertainty of <br> the Relation | Lag | Cost Demands | Resource <br> Demands |

As introduced previously, if the project structures of former similar projects are known, then the prior experience of the possible project realizations (logic plans) can be taken into account within the PEM method, allowing the probability of task realizations to be estimated. However, instead of probability, the relative priority/importance of task realizations can be used considering the logic plans of project experts. Despite the fact that PEM matrices can handle the probability as well as the relative priority/relative importance of task realizations, it is expedient to take a difference between probability and importance. Probability of realization of task A noted as $p_{\mathrm{A}}$ and relative importance/relative priority of realization of task $\mathbf{A}$ noted as $\rho_{\mathrm{A}}$.

If a relative importance (probability) of task realization is $\rho_{\mathrm{A}}\left(p_{\mathrm{A}}\right)$, it is assumed that the relative importance (probability) of task non-realization is $1-\rho_{\mathrm{A}}\left(1-p_{\mathrm{A}}\right)$. (Because of the ranking of tasks we prefer the use of the relative importance instead of probability in this paper.) This way the relative importance of tasks, which must be realized, are 1 , and relative importance of non-realization of these tasks are 0 . Similarly as previously observed the relative importance of the realization of a task, which will not be realized, is 0 , and the relative importance of nonrealization of a task is 1 . The value of task realizations, which should be realized, is higher than 0.5 . The value 0.5 means indifferent task realization, where the task is either realized or not.

If the probability/importance of task realizations and the probability/importance of relations between tasks are estimated, the probability/importance of project scenarios, and the probability/importance of project structures of a given project scenario can be specified according to the data of the matrix. The importance of a project scenario can be calculated as an average of the importance of those tasks which need to be realized, and the complementer of the importance of those tasks which are not to be realized. If probabilities are used to express the uncertainties of task realizations the probability of a project scenario can be defined as a geometric average of probability of task realizations, which tasks are to be realized and one minus probability of task realizations, representing those tasks which are not to be realized.

It is also possible in a similar way to determine the probability/importance of a project scenario, the probability/importance of the project structures of a given project scenario can be calculated as an average of uncertain relations between tasks, which relations are to be realized and one minus uncertainty of relations, which relations are not to be realized.

There is a need to differentiate between importance and probability. The decision can be based upon the type of the project and the source of the information. Probability can be used in these cases by using estimates based upon the experience of prior similar projects and/or the opinions of experts. This applies to maintenance projects (Kosztyán, Hegedűs \& Kiss, 2010), as well. However, in case of IT and innovation projects it is practical to use the importance instead of the probability (Kiss \& Kosztyán, 2009a, b). There are also differences in the calculation. The average importance (probability) value is calculated taking the average of the summary (product) of the relation strengths which are realised and the relation strengths minus from one which are not realised within the project structure. The arithmetic average is used for the importance, and the geometric average is applied for the probability. In this example we used
the arithmetic average for the calculation of the average importance values.
Figure 1 shows how to consider former project plans and its DSM matrices. In this very simple case the values of the PEM matrix elements will be the relative frequency of the occurrences of tasks and relations. The values of the Project Expert Matrix can be determined using prior experience of similar projects as it can be seen on the left side of Figure 1, and it is possible to get all deterministic solutions based on the values of the PEM matrix with the mediation of other matrix-based methods. The solutions, the so called project structures can be represented by different methods, e.g. Precedence Diagramming Method (PDM), Critical Path Method (CPM) or extended Event-driven Process Chain (eEPC) (see the right side of Figure 1). It could be an interesting question, how the values of the PEM matrix can be determined. This will be the topic of our next paper.


Figure 1: Previous project plans and their DSM matrices; PEM matrix contains the relative frequencies of the occurrences of the tasks and their relations; Generated possible project networks (Kosztyán \& Kiss, 2009a).

Figure 2 summarizes how the Project Expert Matrix can be applied for reusing prior project plans to calculate the importance of both the task realizations and the precedence relations. Based on the values of the PEM matrix, project managers can choose what (which tasks) and how (in what kind of order) they want to implement during the project. To determine all possible project scenarios it is a combinatorial problem, the computation time can be decreased extensively by using genetic algorithms. This PEM method needs lots of calculations; however, as a part of an expert system it can take the work of project planners and managers much easier. It can be a universal method to solve the occurred problems during planning and scheduling of special projects as well.


Figure 2: How to apply matrix-based project planning methods.

Following from the first probable project scenario should be determined from the PEM matrix. In this step we need to specify WHAT to do in the project. Using the probability/ importance of project scenarios the probable project scenarios could be ranked. There could be uncertain relations, which can be represented by an SNPM or a NDSM matrix. In the second step the probable project structures can be determined and can be ranked by using probability/ importance value of project structures belonging to a given project scenario. In this phase we can arrange tasks and can answer the question: HOW to do this project. If the PEM matrix is ordered by the uncertainty of task realizations, it helps project managers to find which task realizations are the most important and which tasks could be omitted from this project.

This paper focuses on the matrix-based methods, but uncertainty of task realizations and uncertain relations can be represented by an AoN logic network, known as the Project Expert Graph (PEG). First, through a naive algorithm it can be shown how to determine the most important project scenario and the most important project structure for this project scenario taking into account the project constraints (time, cost and resource constraints). Since this problem is a combinatorial one, we also show a genetic algorithm to solve this problem for large matrices.

Table 7. Initial PEM matrix, PEM matrix where tasks descent ordered by importance of task realizations and PEG network.

| 今 | A | B | C | D | 今 | B | A | C | D |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0.8 | 1 | 0.8 | 0.2 | B | 1 |  | 0.6 | 0.4 |  |
| B |  | 1 | 0.6 | 0.4 | A | 1 | 0.8 | 0.8 | 0.2 |  |
| C |  |  | 0.5 |  | C |  |  | 0.5 |  |  |
| D |  |  | 0.1 | 0 | D |  |  | 0.1 | 0 |  |
| Initial PEM matrix |  |  |  |  | PEM matrix where tasks descent ordered by importance of task realizations |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

In the first step all possible project structures are determined based on the PEM matrix and the PEG network. These project scenarios can be represented in the SNPM or NDSM matrices as well (as you can see on the left side of Table 8). In the second step the possible realizations of the chosen tasks are specified based on each SNPM or NDSM matrices. These possible task orders are the project structures (see on the right side of the Table 8). Both project scenarios and project structures can be ranked according to their values, calculation way of these values was written previously. The number of possible project scenarios/project structures is $2^{\mathrm{k}}$, where k means the number of uncertain tasks/relations in the matrix.

Table 8. Steps of the method, determining all possible solutions.


In the phase of determining and ranking the probable project scenarios we can ask WHAT to do in the project (which tasks will be realized). In the next phase we can answer HOW to do these tasks (in which order the tasks will be realized) in this project.

In the $1^{\text {st }}$ step, SNPM matrices of probable project scenarios were determined based on the diagonal cells of the PEM. There are two uncertain tasks in the example; it means $2^{2}=4$ possible project scenarios as it can be seen on the left side of Table 8.

In the $2^{\text {nd }}$ step, the probable project structures can be determined according to the SNPM matrix of a feasible project scenario, if similarly to the uncertainties of task realizations the uncertainties of relations between tasks are taken into account. In this step deterministic project networks and their DSM matrices can be determined. For example the first project scenario has $2^{2}=4$ project structures, because there are 2 uncertain relations between the tasks of the project scenario (see on the right side of Table 8).

## Empirical Test

For determining feasible project scenarios and feasible project structures a program using genetic algorithms can be used in order to decrease the need of computation resources. Genetic algorithms can be used for NP complete or NP hard problems (Hartmann, 1998) and also can be used where an optimal solution should be determined from the large set of probable/ feasible solutions. When using genetic algorithms for project scheduling, the initial population will be the set of probable project scenarios and project structures of a project scenario. Evolution operators (selection, mutation, recombination, etc.) are executed on the entities of the population. Every entity (project structures of a project scenario, which can be represented by a DSM matrix) has a fitness value. To increase the effectiveness and decrease the computation time, the fitness value is the combination of probability/importance of project scenario, the resource and time constraints. If the project scenario or the project structure is infeasible, the fitness value is 0 . The effectiveness of genetic algorithms can be improved if we use distributive architectures (multiple CPUs or computers). For handling the numerous probable solutions and computations, we used a promising distributive technology called Compute Unified Device Architecture (CUDA, 2007), which distributes computation tasks amongst the Graphical Process Units (GPUs).

However, genetic algorithms are not guaranteed to find the optimal solution even with the help of appropriate fitness function and evolution operators, but they were proven to be able to give a good approximation of the optimal solution in almost every case. In order to compare the solutions of various genetic algorithms, all possible project scenarios and all possible project structures are ranked and the most important feasible project structures are selected for the test projects. After the genetic algorithm framework finds and ranks the possible project scenarios and structures, then the interface program exports the optimal project structure to Microsoft Project.

## Results of Research

To compare the different kinds of optimisation algorithms, different sizes of random PEM matrices were generated, where the number of uncertain tasks and the number of uncertain relations can be defined as a percentage of all tasks and possible relations. Duration of the tasks, project budget and 3 different kinds of resource demands of the tasks were generated randomly. In the comparison test, the time constraint was specified. This time constraint was half of the project duration, with all uncertain relations specified to 1 . In this case all uncertain relations between tasks were considered as a certain dependency. The resource constraints were the half of the maximal resource demands, when all uncertain relations are considered as a certain independency and the tasks scheduled for the earliest start time. The percentage of uncertain relations between two tasks was $50 \%$.

Table 9. The comparison of the run time of the algorithms.

| Size of PEM <br> matrices | Number of all possible <br> solutions | Runtime of full <br> evaluating algorithm | Runtime of genetic <br> algorithm |
| :---: | :---: | :---: | :---: |
| $10 \times 10$ | $2^{23}=8,388,608$ | $1,215 \mathrm{~ms}$ | 42 ms |
| $20 \times 20$ | $2^{95}$ | 2.1 hours | 580 ms |
| $50 \times 50$ | $2^{613}$ | 12.2 hours | 83 sec |

As it can be seen from the table of the simulation's results, genetic algorithms can give a near optimal solution based on the values of the Project Expert Matrix within a short runtime, in case of the relative high number of project tasks as well.

## Case Study

In this section our PEM method is applied to a practical example in the case of an ERP system implementation project. To begin with the tasks are illustrated in the Work Breakdown Structure (WBS) of the project (see figure 3). Then those tasks and their dependencies are represented by the PEM matrix (see Figure 4).


Figure 3: WBS of the ERP implementation project.

The WBS contains 43 tasks and 6 milestones. Only some of the tasks can be handled as uncertain tasks, however, the order of tasks can be different. The values of the tasks are determined according to the importance of each task; the relations between tasks are specified according to the logical sequence of the project tasks. The tasks within task groups 4 and 5 (Gap analysis and Implementation of modules) can be executed in different ways, that is why these task groups are represented as overall tasks (4.1-9 and 5.1-9) in the matrix (see Figure 4) and their tasks are isolated and represented in individual sub PEM matrices. The relations between
these tasks are represented mostly with 0.5 in the sub PEMs.


Figure 4: PEM matrix of the ERP implementation project.
There are two task groups represented in the previous matrix as the overall tasks (4.19 and 5.1-9) which can be realised in different order, so the tasks of task group 4 and 5 are separated into individual matrices containing tasks 4.1-4.9 and tasks 5.1-5.9. In both task groups there are $9 *(9-1)=72$ possible relations between the tasks. Let us first take a look at table 9 which sub PEM matrix include the tasks from 4.1 to 4.9 and their relations. The sequence of these tasks is optional that is why the relation between tasks are 0.5 , consequently there are indifferent relations between the tasks in all cases. It means that each task pair can be completed in serial and parallel as well.

Table 10. The PEM matrix of the task group 4.

| 4.1 |  |  | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.1 | 1 | 0.5 | 0.5 | 0.5 |  |  |  |  |  |  |
| 4.2 |  | 1 | 0.5 | 0.5 | 0.5 |  |  |  |  |  |
| 4.3 |  |  | 1 | 0.5 | 0.5 | 0.5 |  |  |  |  |
| 4.4 |  |  |  | 1 | 0.5 | 0.5 | 0.5 |  |  |  |
| 4.5 |  |  |  |  | 1 | 0.5 | 0.5 | 0.5 |  |  |
| 4.6 |  |  |  |  |  | 1 | 0.5 | 0.5 | 0.5 |  |
| 4.7 |  |  |  |  |  |  | 1 | 0.5 | 0.5 |  |
| 4.8 |  |  |  |  |  |  |  | 1 | 0.5 |  |
| 4.9 |  |  |  |  |  |  |  |  | 1 |  |


| Duration <br> (day) | Resource <br> (head) |
| :---: | :---: |
| 2 | 1 |
| 2 | 1 |
| 3 | 1 |
| 3 | 1 |
| 3 | 1 |
| 2 | 1 |
| 4 | 1 |
| 3 | 1 |
| 2 | 1 |

There are two extreme cases, when all tasks are accomplished in a sequence and when they are all executed in parallel (see Figure 5 and 6). In the course of the realisation of the project is much better if tasks are independent from each other and they can be carried out
parallel. Thus there are no dependencies in the matrix. The problem occurs whether there are time and resource constraints of the project.


24 days
Figure 5: Serial realization of tasks.


Figure 6: Parallel realization of tasks.
The other sub PEM matrix contains the tasks from 5.1 to 5.9 of the task group 5 and their relations. To show the difference between indifferent values and other (not 0.5 ) uncertain values between 0 and 1, there are some other relation strengths in this sub PEM matrix (see Table 11).

Table 11. The PEM matrix of the task group 5.

| $5.1-$ <br> 9 | 5.1 | 5.2 | 5.3 | 5.4 | 5.5 | 5.6 | 5.7 | 5.8 | 5.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.1 | 1 | 0.6 | 0.5 | 0.5 |  |  |  |  |  |
| 5.2 |  | 1 | 0.5 | 0.5 | 0.5 |  |  |  |  |
| 5.3 |  |  | 1 | 0.5 | 0.5 | 0.5 |  |  |  |
| 5.4 |  |  |  | 1 | 0.7 | 0.5 | 0.5 |  |  |
| 5.5 |  |  |  |  | 1 | 0.6 | 0.5 | 0.5 |  |
| 5.6 |  |  |  |  |  | 1 | 0.8 | 0.5 | 0.5 |
| 5.7 |  |  |  |  |  |  | 1 | 0.5 | 0.5 |
| 5.8 |  |  |  |  |  |  |  | 1 | 0.5 |
| 5.9 |  |  |  |  |  |  |  |  | 1 |


| Duration <br> (day) | Resource <br> (head) |
| :---: | :---: |
| 2 | 1 |
| 3 | 1 |
| 2 | 1 |
| 2 | 1 |
| 3 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 1 |
| 3 | 1 |

3 different cases are shown in the following as a comparison. The first one is when all tasks are performed in a sequence, this is the optimisation for the minimal resource demand
(see Figure 7). The second case is when the minimal lead time is the objective function, so tasks are executed parallel (see Figure 8). The third one is the most important now, because it depends on the non-indifferent uncertain values. If the relation strength is higher than 0.5 , tasks are completed in serial, whilst the other tasks can be in parallel, it can be called the mixed realisation of tasks (see Figure 9). In this case the maximal importance (or maximal probability) is the objective function.


Figure 7. Task realization in a sequence.


Figure 8: Parallel task real- Figure 9: Task realization with maximal imization.
portance value.
According to the figures of realisation structures it can be seen that in this case the most favourable solution considering the precedent relations between the tasks, when there are no dependencies between tasks. This parallel case gives the highest flexibility to complete the tasks.

After the overview of the simulations' results and the case study, we will now evaluate
the results of the real practical exercise. Table 12 summarizes the lead time, resource demand and average importance value of each case to both sub PEM matrix. The program using genetic algorithms (GA) was run on the sub PEM matrices. The program gave solutions within 1 ms . Using this program is more practical when multiple objective functions need to be taken into consideration. GA gave a near optimal solution within 91 and 95 ms taking into account the average importance value and the lead time of the project, and within 112 ms in case of three objective functions (the minimal resource demand is the third objective function next to the average importance value and the lead time). The objective function of the program (GA) was the combination of the minimal lead time and the maximal importance value on the same proportion. After determining the best solution the program calculated the resource need of the project.

## Table 12. The results of the case study.

|  | Task group 4 |  |  | Task group 5 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Realisation ways | Serial | Paral- <br> le | GA | Serial | Parallel | Mixed | GA |
| Average importance <br> value | 0.5 | 0.5 | 0.5 | 0.5097 | 0.4903 | 0.5097 | 0.5050 |
| Lead time (day) | 24 | 4 | 7 | 24 | 4 | 10 | 7 |
| Resource demand <br> (person) | 1 | 9 | 4 | 1 | 9 | 5 | 4 |

The table demonstrates that the program using genetic algorithms can give a better solution taking multiple objective functions into account instead of the extreme (serial or parallel) cases.

## Discussion

During this study the newly developed Project Expert Matrix method is introduced for planning and scheduling such projects, which can hardly be planned by using traditional methods. The mentioned methods cannot solve the problem of handling the circles during the project, they cannot contribute to logic planning of projects by restructuring the project tasks based on the uncertain relations and they cannot categorize the tasks according to their probabilities or importance. We noted in the Introduction that there were different methods for project planning and process management; however, our proposed matrix-based methods can be used for the planning and the organizing of projects and the processes as well.

## Conclusion

The PEM is a matrix-based method, which can be applied for planning, scheduling and even restructuring the logic plans of the projects. The values in the Project Expert Matrix can represent the probabilities of the task realisations (in diagonal cells) and the probabilities of the precedence relations (in off-diagonal cells). All possible project scenarios (building from different tasks) can be defined and ranked using the probability values of the task realizations. All possible project structures (different realization ways) can be determined within each project scenario. The suggested method is able to aid in the selection of the optimal solution according to a given objective function within the given restriction(s) from the ranked possible project scenarios and the belonging possible project structures.

Using the PEM matrix, the prior plans of similar projects can be taken into consideration
and all possible project scenarios can be determined. The project constraints of feasible project structures can be ranked and optimal solution(s) determined. With the help of a program using genetic algorithms the effectiveness of computation can be increased extensively and if there are more than 20 uncertain relations the computation time can be decreased significantly. This program is designed for providing a good solution within a relative short time to solve this multi objective optimization problem.

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