Advances in Information Mining, ISSN: 0975–3265, Volume 2, Issue 1, 2010, pp-01-07

Fuzzy multi-objective multi-index transportation problem

Lohgaonkar M.H.[1], Bajaj V.H.[1]*, Jadhav V.A.[2] and Patwari M.B.[1]
[*1]Department of Statistics, Dr. B. A. M. University, Aurangabad, MS, vhbajaj@gmail.com, mhlohgaonkar@gmail.com
[2]Departments of Statistics, Science College, Nanded, MS

Abstract- The aim of this paper is to present a fuzzy multi-objective multi-index transportation problem and develop multi-objective multi-index fuzzy programming model. This model cannot only satisfy more of the actual requirements of the integral system but is also more flexible than conventional transportation problems. Furthermore, it can offer more information to the decision maker (DM) for reference, and then it can raise the quality for decision-making. This paper, we use a special type of linear and non-linear membership functions to solve the multi-objective multi-index transportation problem. It gives an optimal compromise solution.
Keywords- Transportation problem, multi-objective transportation problem, multi-index, linear membership function, non-linear membership function

Introduction

Fuzzy set theory was proposed by L. A. Zadeh and has been found extensive in various fields. Bellman and Zadeh [2] were the first to consider the application of the fuzzy set theory in solving optimization problems in a fuzzy environment, these investigators constraints that both the objective function and the constraints that exist in the model could be represented by corresponding fuzzy set and should be treated in the same manner. The earliest application of it to transportation problems include Prade [11], O'he'igeartaigh [10], Chanas et al. [4]. But these researcher emphases on investigating theory and algorithm. Furthermore, these above investigations are illustrated with simple instance slacking in actual cases of submition. On the other hand, these models are only of single objective and are classical two index transportation problems. In actual transportation problem, the multi-objective functions are generally considered, which includes average delivery time of the commodities, minimum cost, etc. Zimmermann [15] applied the fuzzy set theory to the linear multicriteria decision making problem. It used the linear fuzzy membership function and presented the application of fuzzy linear vector maximum problem. He showed that solutions obtained by fuzzy linear programming always provide efficient solutions and also an optimal compromised solution. Aneja and Nair [1] Showed that the problem model. Multi-index transportation problem are the extension of conventional transportation problems, and are appropriate for solving transportation problems with multiple supply points, multiple demand points as well as problems using diverse modes of transportation demand or delivering different kinds of merchandises. Thus, the forwarded problem would be more complicated than conventional transportation problems. Junginger [9] who proposed a set of logic problems, to solve multi-index transportation problems, has also conducted a detailed investigation regarding the characteristics of multi-index transportation problem model. Rautman et al. [12] used multi-index transportation problem model to solve the shipping scheduling suggested that the employment of such transportation efficiency but also optimize the inegral system.

Mathematical Model

Multi-objective Multi-index Transportation Problem

Let a_{ijl} be multi-dimensional array $1 \le i \le m,\ 1 \le j \le n,\ 1 \le l \le k$ and let $A=(a_{ij}),\ B=(b_{jl}),\ C=(c_{il})$ be multi-matrices then multi-index transportation problem is defined as follows

$$\text{Minimize } Z = \sum_i \sum_j \sum_l a_{ijl} X_{ijl} \quad (1)$$

Subject to

$$\begin{aligned} &\sum_l X_{ijl} = a_{ij} && \forall\ (i,j) \\ &\sum_j X_{ijl} = c_{il} && \forall\ (i,l) \qquad (2) \\ &\sum_i X_{ijl} = b_{jl} && \forall\ (j,l) \\ &X_{ijl} \ge 0 \quad \forall\ (i,j,l) \end{aligned}$$

It is immediate that

$$\sum_i a_{ij} = \sum_l b_{jl}; \quad \sum_j a_{ij} = \sum_l c_{il}; \quad \sum_j b_{jl} = \sum_i c_{il} \quad (3)$$

are three necessary conditions however they are noted to be non sufficient.

Multi-objective double transportation problem as follows

$$\text{Minimize } Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n} k_{ij}^{(1)} x_{ij}^{(1)} + \sum_{i=1}^{m}\sum_{j=1}^{n} k_{ij}^{(2)} x_{ij}^{(2)} \qquad (4)$$

Subject to

$$\sum_{j=1}^{n} x_{ij}^{(1)} = a_{1i} \quad \forall i \qquad (5)$$

$$\sum_{j=1}^{n} x_{ij}^{(2)} = a_{2i} \quad \forall i \qquad (6)$$

$$\sum_{i=1}^{m} x_{ij}^{(1)} = b_{1j} \quad \forall j \qquad (7)$$

$$\sum_{i=1}^{m} x_{ij}^{(2)} = b_{2j} \quad \forall j \qquad (8)$$

$$x_{ij}^{(1)} + x_{ij}^{(2)} = c_{ij} \quad \forall i,j \qquad (9)$$

$$x_{ij}^{(1)}, x_{ij}^{(2)} \geq 0 \quad \forall i,j \qquad (10)$$

It may be easily seen that for existence of solution following set of conditions are necessary.

$$\sum_{j=1}^{n} c_{ij} = a_{1i} + a_{2i} \quad \forall i \qquad (11)$$

$$\sum_{i=1}^{m} c_{ij} = b_{1j} + a_{2j} \quad \forall j \qquad (12)$$

$$\sum_{i=1}^{m} a_{1i} = \sum_{j=1}^{n} b_{1j} \qquad (13)$$

$$\sum_{i=1}^{m} a_{2i} = \sum_{j=1}^{n} b_{2j} \qquad (14)$$

$$\sum c_{ij} \leq \text{Min}(a_{1i} + b_{1j}) + \text{Min}(a_{2i} + b_{2j}) \quad \forall (i,j) \qquad (15)$$

It may be easily seen that DTP is composed of two transportation tables and one C matrix as given below.

$$C_1 = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} \cdots & k_{1n}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} \cdots & k_{2n}^{(1)} \\ \cdots & & \\ k_{m1}^{(1)} & k_{m2}^{(1)} \cdots & k_{mn}^{(1)} \end{bmatrix} \begin{matrix} a_{11} \\ a_{12} \\ \cdots \\ a_{1m} \end{matrix} \qquad (16)$$
$$b_{11} \quad b_{12} \quad \cdots \quad b_{1n}$$

$$C_2 = \begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} \cdots & k_{1n}^{(2)} \\ k_{21}^{(2)} & k_{22}^{(2)} \cdots & k_{2n}^{(2)} \\ \cdots & & \\ k_{m1}^{(2)} & k_{m2}^{(2)} \cdots & k_{mn}^{(2)} \end{bmatrix} \begin{matrix} a_{21} \\ a_{22} \\ \cdots \\ a_{2m} \end{matrix} \quad \text{and } C = (c_{ij})_{m \times n} \qquad (17)$$
$$b_{11} \quad b_{12} \quad \cdots \quad b_{1n}$$

$$T_1 = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} \cdots & k_{1n}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} \cdots & k_{2n}^{(1)} \\ \cdots & & \\ k_{m1}^{(1)} & k_{m2}^{(1)} \cdots & k_{mn}^{(1)} \end{bmatrix} \begin{matrix} a_{11} \\ a_{12} \\ \cdots \\ a_{1m} \end{matrix} \qquad (18)$$
$$b_{11} \quad b_{12} \quad \cdots \quad b_{1n}$$

$$T_2 = \begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} \cdots & k_{1n}^{(2)} \\ k_{21}^{(2)} & k_{22}^{(2)} \cdots & k_{2n}^{(2)} \\ \cdots & & \\ k_{m1}^{(2)} & k_{m2}^{(2)} \cdots & k_{mn}^{(2)} \end{bmatrix} \begin{matrix} a_{21} \\ a_{22} \\ \cdots \\ a_{2m} \end{matrix} \quad \text{and } C = (c_{ij})_{m \times n} \qquad (19)$$
$$b_{21} \quad b_{22} \quad \cdots \quad b_{2n}$$

Fuzzy Algorithm to solve multi-objective multi-index transportation problem

Step 1:

Solve the multi-objective multi-index transportation problem as a single objective transportation problem P times by taking one of the objectives at a time

Step 2 :

From the results of step 1, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find pay-off matrix as follows

$$\begin{array}{c|cccc} & Z_1(X) & Z_2(X) & \ldots & Z_p(X) \\ \hline X^{(1)} & Z_{11} & Z_{12} & \cdots & Z_{1p} \\ X^{(2)} & Z_{21} & Z_{22} & \cdots & Z_{2p} \\ . & \cdots & \cdots & \cdots & \cdots \\ . & Z_{p1} & Z_{p2} & & Z_{pp} \\ X^{(P)} & & & & \end{array}$$

Where, $X^{(1)}, X^{(2)}, \ldots, X^{(p)}$ are the isolated optimal solutions of the P different transportation problems for P different objective functions

$Z_{ij} = Z_j(X^i)$ $(i=1,2,\ldots,p \ \ \& \ j=1,2,\ldots,p)$ be the i-th row and j-th column element of the pay-off matrix.

Step 3:

From step 2, we find for each objective the worst (U_p) and the best (L_p) values corresponding to the set of solutions, where,

$$U_p = \max(Z_{1p}, Z_{2p}, \ldots, Z_{pp}) \text{ and}$$

$$L_p = Z_{pp} \qquad p=1,2,\ldots,P$$

An initial fuzzy model of the problem (4)-(10) can be stated as

Find X_{ij} $\quad i=1,2,\ldots,m \quad j=1,2,\ldots,n,$

so as to satisfy $Z_p \tilde{<} L_p \quad p=1,2,\ldots,P$

subject to (4)-(10)

Step 4: Case (i)

Define membership function for the p-th objective function as follows:

$$\mu_p(X) = \begin{cases} 1 & \text{if } Z_p(X) \leq L_p \\ \dfrac{U_p - Z_p(X)}{U_p - L_p} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } Z_p \geq U_p \end{cases} \qquad (20)$$

Step 5:

Find an equivalent crisp model by using a linear membership function for the initial fuzzy model

$$\text{Maximize } \lambda$$

$$\lambda \leq \frac{U_p - Z_p(X)}{U_p - L_p} \qquad (21)$$

subject to (5)-(10)

Step 6: Solve the crisp model by an appropriate mathematical programming algorithm.

$$\text{Maximize } \lambda$$

$$\text{Subject to} \qquad (22)$$

$$C_{ij}^{p} X_{ij} + \lambda (U_p - L_p) \le U_p \qquad p=1,2,...,P$$

$$\text{subject to (5)-(10)}$$

Now, by using hyperbolic membership function for the P-th objective function

$$\mu^{H} Z_p(x) = \begin{cases} 1 & \text{if } Z_p \le L_p \\ \dfrac{1}{2} \dfrac{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} - e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}}{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} + e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}} + \dfrac{1}{2} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } Z_p \ge U_p \end{cases}$$

(23)

Where, $\alpha_p = \dfrac{3}{U_p - L_p} = \dfrac{6}{U_p - L_p}$

Crisp model for the fuzzy model can be formulated as:

Maximize λ subject to

$$\lambda \le \frac{1}{2} \frac{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} - e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}}{e^{\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p} + e^{-\{\frac{(U_p+L_p)}{2} - Z_p(x)\}\alpha_p}} + \frac{1}{2} \qquad (24)$$

$$\text{subject to } (5)-(10) \quad \& \quad \lambda \ge 0$$

Solve the crisp model as

$$\text{Maximize } X_{mn+1}$$

$$\text{subject to} \qquad (25)$$

$$\alpha_p Z_p(x) + X_{mn+1} \le \alpha_p (U_p + L_p)/2, \qquad p = 1,2,\text{----}P$$

$$\text{subject to (5)-(10) and } X_{mn+1} \ge 0$$

Where, $X_{mn+1} = \tanh^{-1}(2\lambda - 1)$

Now, by using exponential membership function for the p th objective function and is defined as

$$\mu^{E} Z_p(x) = \begin{cases} 1, & \text{if } Z_p \le L_p \\ \dfrac{e^{-S\Psi_p(X)} - e^{-S}}{1-e^{-S}}, & \text{if } L_p < Z_p < U_p \\ 0, & \text{if } Z_p \ge U_p \end{cases} \qquad (26)$$

Where, $\Psi_p(X) = \dfrac{Z_p - L_P}{U_p - L_p} \qquad p=1,2,...,P$

S is a non zero parameter, prescribed by the decision maker

Numerical Examples

Example 1

$$C_1 = \left[\begin{array}{ccc|c} 4 & 3 & 5 & 9 \\ 8 & 6 & 2 & 14 \\ 7 & 4 & 1 & 6 \\ 9 & 10 & 12 & 7 \\ \hline 14 & 12 & 10 & \end{array}\right] \quad C_2 = \left[\begin{array}{ccc|c} 8 & 6 & 3 & 6 \\ 5 & 4 & 1 & 7 \\ 9 & 2 & 6 & 5 \\ 4 & 9 & 3 & 6 \\ \hline 5 & 8 & 11 & \end{array}\right] \quad C = \begin{bmatrix} 5 & 7 & 3 \\ 8 & 4 & 9 \\ 4 & 1 & 6 \\ 2 & 8 & 3 \end{bmatrix} \qquad (27)$$

Example 2

$$T_1 = \left[\begin{array}{ccc|c} 5 & 6 & 7 & 9 \\ 4 & 5 & 2 & 14 \\ 1 & 3 & 4 & 6 \\ 4 & 2 & 3 & 7 \\ \hline 14 & 12 & 10 & \end{array}\right] \quad T_2 = \left[\begin{array}{ccc|c} 10 & 9 & 9 & 6 \\ 7 & 9 & 2 & 7 \\ 8 & 7 & 9 & 5 \\ 8 & 4 & 5 & 6 \\ \hline 5 & 8 & 11 & \end{array}\right] \quad C = \begin{bmatrix} 5 & 7 & 3 \\ 8 & 4 & 9 \\ 4 & 1 & 6 \\ 2 & 8 & 3 \end{bmatrix} \qquad (28)$$

Example 1 is simplified as

$$\begin{aligned} \text{Minimize } Z_1 = {} & 4X_{11}^{(1)} + 3X_{12}^{(1)} + 5X_{13}^{(1)} + 8X_{21}^{(1)} + 6X_{22}^{(1)} + 2X_{23}^{(1)} + 7X_{31}^{(1)} + 4X_{32}^{(1)} + \\ & X_{33}^{(1)} + 9X_{41}^{(1)} + 10X_{42}^{(1)} + 12X_{43}^{(1)} + 8X_{11}^{(2)} + 6X_{12}^{(2)} + 3X_{13}^{(2)} + 5X_{21}^{(2)} + \\ & 4X_{22}^{(2)} + X_{23}^{(2)} + 9X_{31}^{(2)} + 2X_{32}^{(2)} + 6X_{33}^{(2)} + 4X_{41}^{(2)} + 9X_{42}^{(2)} + 3X_{43}^{(2)} \end{aligned} \qquad (29)$$

Subject to

$$X_{11}^{(1)} + X_{12}^{(1)} + X_{13}^{(1)} = 9$$
$$X_{21}^{(1)} + X_{22}^{(1)} + X_{23}^{(1)} = 14$$
$$X_{31}^{(1)} + X_{32}^{(1)} + X_{33}^{(1)} = 6$$
$$X_{41}^{(1)} + X_{42}^{(1)} + X_{43}^{(1)} = 7$$
$$X_{11}^{(2)} + X_{12}^{(2)} + X_{13}^{(2)} = 6$$
$$X_{21}^{(2)} + X_{22}^{(2)} + X_{23}^{(2)} = 7$$
$$X_{31}^{(2)} + X_{32}^{(2)} + X_{33}^{(2)} = 5$$
$$X_{41}^{(2)} + X_{42}^{(2)} + X_{43}^{(2)} = 6$$
$$X_{11}^{(1)} + X_{21}^{(1)} + X_{31}^{(1)} + X_{41}^{(1)} = 14$$
$$X_{12}^{(1)} + X_{22}^{(1)} + X_{32}^{(1)} + X_{42}^{(1)} = 12$$
$$X_{13}^{(1)} + X_{23}^{(1)} + X_{33}^{(1)} + X_{43}^{(1)} = 10$$
$$X_{11}^{(2)} + X_{21}^{(2)} + X_{31}^{(2)} + X_{41}^{(2)} = 5$$
$$X_{12}^{(2)} + X_{22}^{(2)} + X_{32}^{(2)} + X_{42}^{(2)} = 8$$
$$X_{13}^{(2)} + X_{23}^{(2)} + X_{33}^{(2)} + X_{43}^{(2)} = 11$$
$$X_{11}^{(1)} + X_{11}^{(2)} = 5$$
$$X_{12}^{(1)} + X_{12}^{(2)} = 7$$
$$X_{13}^{(1)} + X_{13}^{(2)} = 3$$
$$X_{21}^{(1)} + X_{21}^{(2)} = 8$$
$$X_{22}^{(1)} + X_{22}^{(2)} = 4$$
$$X_{23}^{(1)} + X_{23}^{(2)} = 9$$
$$X_{31}^{(1)} + X_{31}^{(2)} = 4$$
$$X_{32}^{(1)} + X_{32}^{(2)} = 1$$
$$X_{33}^{(1)} + X_{33}^{(2)} = 6 \qquad (30)$$
$$X_{41}^{(1)} + X_{41}^{(2)} = 2$$
$$X_{42}^{(1)} + X_{42}^{(2)} = 8$$
$$X_{43}^{(1)} + X_{43}^{(2)} = 3$$

Example 2 is simplified as

$$\text{Minimize } Z_2 = 5X_{11}^{(1)}+6X_{12}^{(1)}+7X_{13}^{(1)}+4X_{21}^{(1)}+5X_{22}^{(1)}+2X_{23}^{(1)}+1X_{31}^{(1)}+3X_{32}^{(1)}+ \quad (31)$$
$$4X_{33}^{(1)}+4X_{41}^{(1)}+2X_{42}^{(1)}+3X_{43}^{(1)}+10X_{11}^{(2)}+9X_{12}^{(2)}+9X_{13}^{(2)}+7X_{21}^{(2)}+$$
$$9X_{22}^{(2)}+2X_{23}^{(2)}+8X_{31}^{(2)}+7X_{32}^{(2)}+9X_{33}^{(2)}+8X_{41}^{(2)}+4X_{42}^{(2)}+5X_{43}^{(2)}$$

Subject to

$$X_{11}^{(1)}+X_{12}^{(1)}+X_{13}^{(1)}=9$$
$$X_{21}^{(1)}+X_{22}^{(1)}+X_{23}^{(1)}=14$$
$$X_{31}^{(1)}+X_{32}^{(1)}+X_{33}^{(1)}=6$$
$$X_{41}^{(1)}+X_{42}^{(1)}+X_{43}^{(1)}=7$$
$$X_{11}^{(2)}+X_{12}^{(2)}+X_{13}^{(2)}=6$$
$$X_{21}^{(2)}+X_{22}^{(2)}+X_{23}^{(2)}=7$$
$$X_{31}^{(2)}+X_{32}^{(2)}+X_{33}^{(2)}=5$$
$$X_{41}^{(2)}+X_{42}^{(2)}+X_{43}^{(2)}=6$$
$$X_{11}^{(1)}+X_{21}^{(1)}+X_{31}^{(1)}+X_{41}^{(1)}=14$$
$$X_{12}^{(1)}+X_{22}^{(1)}+X_{32}^{(1)}+X_{42}^{(1)}=12$$
$$X_{13}^{(1)}+X_{23}^{(1)}+X_{33}^{(1)}+X_{43}^{(1)}=10$$
$$X_{11}^{(2)}+X_{21}^{(2)}+X_{31}^{(2)}+X_{41}^{(2)}=5$$
$$X_{12}^{(2)}+X_{22}^{(2)}+X_{32}^{(2)}+X_{42}^{(2)}=8$$
$$X_{13}^{(2)}+X_{23}^{(2)}+X_{33}^{(2)}+X_{43}^{(2)}=11$$
$$X_{11}^{(1)}+X_{11}^{(2)}=5$$
$$X_{12}^{(1)}+X_{12}^{(2)}=7$$
$$X_{13}^{(1)}+X_{13}^{(2)}=3$$
$$X_{21}^{(1)}+X_{21}^{(2)}=8$$
$$X_{22}^{(1)}+X_{22}^{(2)}=4$$
$$X_{23}^{(1)}+X_{23}^{(2)}=9$$
$$X_{31}^{(1)}+X_{31}^{(2)}=4$$
$$X_{32}^{(1)}+X_{32}^{(2)}=1$$
$$X_{33}^{(1)}+X_{33}^{(2)}=6$$
$$X_{41}^{(1)}+X_{41}^{(2)}=2 \quad (32)$$
$$X_{42}^{(1)}+X_{42}^{(2)}=8$$
$$X_{43}^{(1)}+X_{43}^{(2)}=3$$

For objective Z_1, we find the optimal solution as

$$X^{(1)}=\left\{\begin{array}{l} X_{11}^{(1)}=5;\ X_{12}^{(1)}=4;X_{21}^{(1)}=8,X_{22}^{(1)}=2, \\ X_{23}^{(1)}=4;\ X_{33}^{(1)}=6;\ X_{41}^{(1)}=1;X_{42}^{(1)}=6; \\ X_{12}^{(2)}=3;\ X_{13}^{(2)}=3;X_{22}^{(2)}=2;\ X_{23}^{(2)}=5, \\ X_{31}^{(2)}=4;\ X_{32}^{(2)}=1;\ X_{41}^{(2)}=1;X_{42}^{(2)}=2; \\ X_{43}^{(2)}=3 \end{array}\right\}$$

$$Z_1=300$$

For objective Z_2, we find the optimal solution as

$$X^{(2)}=\left\{\begin{array}{l} X_{11}^{(1)}=4;\ X_{12}^{(1)}=5;X_{21}^{(1)}=8,X_{22}^{(1)}=4, \\ X_{23}^{(1)}=2;\ X_{31}^{(1)}=1;X_{33}^{(1)}=5;\ X_{41}^{(1)}=1; \\ X_{42}^{(1)}=3;X_{43}^{(1)}=3;X_{11}^{(2)}=1;\ X_{12}^{(2)}=2; \\ X_{13}^{(2)}=3;X_{23}^{(2)}=7;X_{31}^{(2)}=3;\ X_{32}^{(2)}=1; \\ X_{33}^{(2)}=1;\ X_{41}^{(2)}=1;X_{42}^{(2)}=5; \end{array}\right\}$$

$$Z_1=283$$

Now for $X^{(1)}$ we can find out Z_2,

$$Z_2(X^{(1)})=291$$

Now for $X^{(2)}$ we can find out Z_1

$$Z_1(X^{(2)})=330$$

Pay-off matrix is

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \end{array}\begin{array}{c} Z_1 \quad\quad Z_2 \\ \begin{bmatrix} 300 & 291 \\ 330 & 283 \end{bmatrix}\end{array}$$

From this matrix $U_1=330,\quad U_2=291,\quad L_1=300,\quad L_2=283$

$$\text{Find}\left\{X_{ij},\ i=1,2,3,\ j=1,2,3\right\},\ \text{so as to satisfy}\quad Z_1\le 300 \text{ and } Z_2\le 283,$$

Define membership function for the objective functions $Z_1(X)$ and $Z_2(X)$ respectively

$$\mu_1(X)=\begin{cases} 1, & \text{if} \quad Z_1(X) \le 300 \\ \dfrac{330-Z_1(X)}{330-300}, & \text{if} \quad 300 < Z_1(X) < 330 \\ 0, & \text{if} \quad Z_1(X) \ge 330 \end{cases}$$

;

$$\mu_2(X)=\begin{cases} 1, & \text{if} \quad Z_2(X) \le 283 \\ \dfrac{291-Z_2(X)}{291-283}, & \text{if} \quad 283 < Z_2(X) < 291 \\ 0, & \text{if} \quad Z_2(X) \ge 291 \end{cases}$$

Find an equivalent crisp model

$\text{Maximize } \lambda$, $\lambda+Z_1(X) \le 330$ and

$5\lambda+Z_2(X) \le 291$

Solve the crisp model by using an appropriate mathematical algorithm.

$$4X_{11}^{(1)}+3X_{12}^{(1)}+5X_{13}^{(1)}+8X_{21}^{(1)}+6X_{22}^{(1)}+2X_{23}^{(1)}+7X_{31}^{(1)}+4X_{32}^{(1)}+$$
$$X_{33}^{(1)}+9X_{41}^{(1)}+10X_{42}^{(1)}+12X_{43}^{(1)}+8X_{11}^{(2)}+6X_{12}^{(2)}+3X_{13}^{(2)}+5X_{21}^{(2)}+$$
$$4X_{22}^{(2)}+X_{23}^{(2)}+9X_{31}^{(2)}+2X_{32}^{(2)}+6X_{33}^{(2)}+4X_{41}^{(2)}+9X_{42}^{(2)}+3X_{43}^{(2)}+30\lambda \le 330$$

$$5X_{11}^{(1)}+6X_{12}^{(1)}+7X_{13}^{(1)}+4X_{21}^{(1)}+5X_{22}^{(1)}+2X_{23}^{(1)}+1X_{31}^{(1)}+3X_{32}^{(1)}+$$
$$4X_{33}^{(1)}+4X_{41}^{(1)}+2X_{42}^{(1)}+3X_{43}^{(1)}+10X_{11}^{(2)}+9X_{12}^{(2)}+9X_{13}^{(2)}+7X_{21}^{(2)}+$$
$$9X_{22}^{(2)}+2X_{23}^{(2)}+8X_{31}^{(2)}+7X_{32}^{(2)}+9X_{33}^{(2)}+8X_{41}^{(2)}+4X_{42}^{(2)}+5X_{43}^{(2)}+8\lambda \le 291$$

Subject to (30)

The optimal compromise solution of the problem is represented as

$$\boxed{\lambda=0.6521}$$

$$X^{(*)}=\begin{Bmatrix} X_{11}^{(1)}=5\,;\ X_{12}^{(1)}=2.2608\,;X_{13}^{(1)}=1.7391\,;\ X_{21}^{(1)}=8; \\ X_{22}^{(1)}=3.7391;X_{23}^{(1)}=2.2608;\ X_{33}^{(1)}=6\,;\ X_{41}^{(1)}=1\,; \\ X_{42}^{(1)}=6;\ X_{12}^{(2)}=4.7391;X_{13}^{(2)}=1.2608;X_{23}^{(2)}=6.7391; \\ X_{31}^{(2)}=4;\ X_{32}^{(2)}=1\,;\ X_{41}^{(2)}=1\,;X_{42}^{(2)}=2;X_{43}^{(2)}=3 \\ \\ Z_1^{*}=309.3902 \quad \text{and} \quad Z_2^{*}=283.4329 \end{Bmatrix}$$

If we use hyperbolic membership function with

$$\alpha_1=\frac{6}{U_1-L_1}=\frac{6}{330-300}=\frac{6}{30}\;;\quad \alpha_2=\frac{6}{U_2-L_2}=\frac{6}{291-283}=\frac{6}{8}$$

$$\frac{U_1+L_1}{2}=\frac{630}{2}=315\;;\quad \frac{U_2+L_2}{2}=\frac{574}{2}=287$$

Then we get the membership functions $\mu_1^H(Z_1)$ and $\mu_2^H(Z_2)$ for the objectives Z_1 & Z_2 respectively, are defined as follows:

$$\mu^H Z_1(x)=\begin{cases} 1, & \text{if } Z_1(X) \le 300 \\ \dfrac{1}{2}\tanh\left\{315-Z_1(X)\right\}\dfrac{6}{30}+\dfrac{1}{2} & \text{if } 300 < Z_1(X) < 330 \\ 0, & \text{if } Z_1(X) \ge 330 \end{cases}$$

$$\mu^H Z_2(x)=\begin{cases} 1, & \text{if } Z_2(X) \le 283 \\ \dfrac{1}{2}\tanh\left\{287-Z_2(X)\right\}\dfrac{6}{8} & \text{if } 283 < Z_2(X) < 291 \\ 0, & \text{if } Z_2(X) \ge 291 \end{cases}$$

We get an equivalent crisp model

$\text{Maximize } X_{mn+1}$

Subject to

$$\alpha_1 Z_1(X)+X_{10} \le \frac{\alpha_1}{2}(U_1+L_1)$$

$$\frac{6}{30}(4X_{11}^{(1)}+3X_{12}^{(1)}+5X_{13}^{(1)}+8X_{21}^{(1)}+6X_{22}^{(1)}+2X_{23}^{(1)}+7X_{31}^{(1)}+4X_{32}^{(1)}+$$
$$X_{33}^{(1)}+9X_{41}^{(1)}+10X_{42}^{(1)}+12X_{43}^{(1)}+8X_{11}^{(2)}+6X_{12}^{(2)}+3X_{13}^{(2)}+5X_{21}^{(2)}+$$
$$4X_{22}^{(2)}+X_{23}^{(2)}+9X_{31}^{(2)}+2X_{32}^{(2)}+6X_{33}^{(2)}+4X_{41}^{(2)}+9X_{42}^{(2)}+3X_{43}^{(2)})+X_{mn+1} \le \frac{6}{30}\,315$$

$$24X_{11}^{(1)}+18X_{12}^{(1)}+30X_{13}^{(1)}+48X_{21}^{(1)}+36X_{22}^{(1)}+12X_{23}^{(1)}+42X_{31}^{(1)}+24X_{32}^{(1)}+$$
$$6X_{33}^{(1)}+54X_{41}^{(1)}+60X_{42}^{(1)}+72X_{43}^{(1)}+48X_{11}^{(2)}+36X_{12}^{(2)}+18X_{13}^{(2)}+30X_{21}^{(2)}+$$
$$24X_{22}^{(2)}+6X_{23}^{(2)}+54X_{31}^{(2)}+12X_{32}^{(2)}+36X_{33}^{(2)}+24X_{41}^{(2)}+54X_{42}^{(2)}+18X_{43}^{(2)}+30X_{mn+1} \le 1890$$

And

$$\alpha_2 Z_2(X)+X \le \frac{\alpha_2}{2}(U_2+L_2)$$

$$\frac{6}{8}(5X_{11}^{(1)}+6X_{12}^{(1)}+7X_{13}^{(1)}+4X_{21}^{(1)}+5X_{22}^{(1)}+2X_{23}^{(1)}+1X_{31}^{(1)}+3X_{32}^{(1)}+$$
$$4X_{33}^{(1)}+4X_{41}^{(1)}+2X_{42}^{(1)}+3X_{43}^{(1)}+10X_{11}^{(2)}+9X_{12}^{(2)}+9X_{13}^{(2)}+7X_{21}^{(2)}+$$
$$9X_{22}^{(2)}+2X_{23}^{(2)}+8X_{31}^{(2)}+7X_{32}^{(2)}+9X_{33}^{(2)}+8X_{41}^{(2)}+4X_{42}^{(2)}+5X_{43}^{(2)})+X_{mn+1} \le \frac{6}{8}291$$

$$30X_{11}^{(1)}+36X_{12}^{(1)}+42X_{13}^{(1)}+24X_{21}^{(1)}+30X_{22}^{(1)}+12X_{23}^{(1)}+6X_{31}^{(1)}+18X_{32}^{(1)}+$$
$$24X_{33}^{(1)}+24X_{41}^{(1)}+12X_{42}^{(1)}+18X_{43}^{(1)}+60X_{11}^{(2)}+54X_{12}^{(2)}+54X_{13}^{(2)}+42X_{21}^{(2)}+$$
$$54X_{22}^{(2)}+12X_{23}^{(2)}+48X_{31}^{(2)}+42X_{32}^{(2)}+54X_{33}^{(2)}+48X_{41}^{(2)}+24X_{42}^{(2)}+30X_{43}^{(2)})+8X_{mn+1} \le 1746$$

Subject to (30)

The problem was solved by using the linear interactive and discrete optimization (LINDO) software, the optimal compromise solution is

$$X_{mn+1}=1.9608$$

$$X^{(*)}=\begin{Bmatrix} X_{11}^{(1)}=5;\ X_{12}^{(1)}=3.1304;\ X_{21}^{(1)}=8;X_{22}^{(1)}=2.8695; \\ X_{23}^{(1)}=3.1304;\ X_{33}^{(1)}=6;\ X_{41}^{(1)}=1;X_{42}^{(1)}=6; \\ X_{12}^{(2)}=3.8695;X_{13}^{(2)}=2.1304;X_{22}^{(2)}=1.1304; \\ X_{23}^{(2)}=5.8695;X_{31}^{(2)}=4;\ X_{32}^{(2)}=1;\ X_{41}^{(2)}=1;X_{42}^{(2)}=2;X_{43}^{(2)}=3 \\ Z_1^*=300.8683 \quad \text{and} \quad Z_2^*=282.3024 \\ \lambda=0.9804 \end{Bmatrix}$$

$$\mu^E_{Z_1}(x)=\begin{cases} 1, & \text{if } Z_1 \le 300 \\ \dfrac{e^{-1\Psi_1(X)}-e^{-1}}{1-e^{-S}}, & \text{if } 300 < Z_1 < 330 \\ 0, & \text{if } Z_1 \ge 330 \end{cases}$$

;

$$\mu^E_{Z_2}(x)=\begin{cases} 1, & \text{if } Z_2 \le 283 \\ \dfrac{e^{-1\Psi_2(X)}-e^{-1}}{1-e^{-S}}, & \text{if } 283 < Z_2 < 291 \\ 0, & \text{if } Z_2 \ge 291 \end{cases}$$

Then an equivalent crisp model for fuzzy model can be formulated as

Maximize λ subject to

$$\lambda \le \frac{e^{-s\psi_p(x)}-e^{-s}}{1-e^{-s}}, \qquad p = 1,2,\text{-----}P \quad \text{and}$$

subject to (7)-(9)

$$\Psi_1(X)=\frac{Z_1-L_1}{U_1-L_1}=\frac{Z_1-300}{330-300}=\frac{Z_1-300}{30} \qquad \text{and}$$

$$\Psi_2(X)=\frac{Z_2-L_2}{U_2-L_2}=\frac{Z_2-283}{291-283}=\frac{Z_2-283}{8}$$

$$\Psi_2(X)=(-4X_{11}^{(1)}-3X_{12}^{(1)}-5X_{13}^{(1)}-8X_{21}^{(1)}-6X_{22}^{(1)}-2X_{23}^{(1)}-7X_{31}^{(1)}-4X_{32}^{(1)}- X_{33}^{(1)}-9X_{41}^{(1)}-10X_{42}^{(1)}-12X_{43}^{(1)}-8X_{11}^{(2)}-6X_{12}^{(2)}-3X_{13}^{(2)}-5X_{21}^{(2)}- 4X_{22}^{(2)}-X_{23}^{(2)}-9X_{31}^{(2)}-2X_{32}^{(2)}-6X_{33}^{(2)}+4X_{41}^{(2)}-9X_{42}^{(2)}-3X_{43}^{(2)}+300)/30$$

$$\Psi_2(X)=(-5X_{11}^{(1)}-6X_{12}^{(1)}-7X_{13}^{(1)}-4X_{21}^{(1)}-5X_{22}^{(1)}-2X_{23}^{(1)}-1X_{31}^{(1)}-3X_{32}^{(1)}- 4X_{33}^{(1)}-4X_{41}^{(1)}-2X_{42}^{(1)}-3X_{43}^{(1)}-10X_{11}^{(2)}-9X_{12}^{(2)}-9X_{13}^{(2)}-7X_{21}^{(2)}- 9X_{22}^{(2)}-2X_{23}^{(2)}-8X_{31}^{(2)}-7X_{32}^{(2)}-9X_{33}^{(2)}-8X_{41}^{(2)}-4X_{42}^{(2)}-5X_{43}^{(2)}+283)/8$$

Then the problem is $\lambda \le \dfrac{e^{\psi_1(x)}-e^{-1}}{1-e^{-1}}$, and

$$\lambda \le \frac{e^{\psi_2(x)}-e^{-1}}{1-e^{-1}},$$

And subject to (30)

Then the problem can be simplified as

Maximize λ

Subject to

$$e^{-S\Psi_p(X)}-(1-e^{-S})\lambda \ge e^{-S} \qquad p=1,2,...,P$$

$$(3.2)-(3.4) \quad \forall \ i,j \quad \& \quad \lambda \ge 0$$

$$\Rightarrow \text{Maximize } \lambda$$

$$e^{-\Psi_1(X)}-(1-e^{-1})\lambda \ge e^{-1} \Rightarrow e^{-\Psi_1(X)}-(1-0.368)\lambda \ge 0.368 \Rightarrow e^{-\Psi_1(X)}-(0.6321)\lambda \ge 0.368$$

$$e^{-\Psi_2(X)}-(1-e^{-1})\lambda \ge e^{-1} \Rightarrow e^{-\Psi_2(X)}-(0.6321\lambda \ge 0.368$$

The problem is solved by the general interactive optimization (LINGO) software

$$\lambda=0.7084$$

$$X^{(*)}=\begin{Bmatrix} X_{11}^{(1)}=5;\ X_{12}^{(1)}=2.3703;\ X_{13}^{(1)}=1.6296;X_{21}^{(1)}=8;X_{22}^{(1)}=4; \\ X_{23}^{(1)}=2;\ X_{33}^{(1)}=6;\ X_{41}^{(1)}=1;X_{42}^{(1)}=5.6296;X_{43}^{(1)}=0.3703 \\ X_{12}^{(2)}=4.6296;X_{13}^{(2)}=1.3703;X_{31}^{(2)}=4;\ X_{32}^{(2)}=1;\ X_{41}^{(2)}=1; \\ X_{42}^{(2)}=2.3703;X_{43}^{(2)}=2.6296 \\ Z_1^*=306.1085 \quad \text{and} \quad Z_2^*=270.6274 \end{Bmatrix}$$

Conclusion

In this paper multi-objective multi-index transportation problem is defined and problem is solved by using fuzzy programming technique (Linear, Hyperbolic and Exponential membership function). The multi-index transportation problem can represent different modes of origins and destination or it may represent a set of intermediate warehouse. If we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution of hyperbolic membership function changes significantly if we compare with the solution obtained by the linear membership function but the optimal compromise solution of exponential membership function does not change significantly if we compare with the solution obtained by the linear membership function.

References

[1] Aneja V.P. and Nair K.P.K. (1979) *Management Science,* 25, 73-78.

[2] Bellman R. E. and Zadeh L. A. (1970) *Management science,* 17, 141-164.

[3] Bit A. K., Biswal M.P. and Alam S. S. (1993) *Industrial Engineering Journal XXII*, No. 6, 8-12.

[4] Chanas S., Kolodzejczyk W. and Machaj A. (1984) *Fuzzy set and systems*, 13, 211-221.

[5] Gwo-Hshiung Tzeng, Dusan Teodorovic and Ming-Jiu Hwang (1996) *European Journal of Operations Research*, 95, 62-72.

[6] Haley K. B. (1963) *Operations Research* 10, 448-463.
[7] Haley K. B. (1963) *Operations Research* 11, 369-379.
[8] Haley K. B. (1965) *Operations Research* 16, 471-474.
[9] Junginger W. (1993) *European Journal of Operational Research* 66, 353-371.
[10] Oheigeartaigh M. (1982) *fuzzy sets and systems*, 8 , 235-243.
[11] Prade H. (1980) Fuzzy sets. Theory and applications to policy analysis and information Systems. Plenum Press, new work, 155-170.
[12] Rautman C.A. Reid R.A. and Ryder E.E. (1993) *Operations Research* 41, 459-469.
[13] Verma Rakesh, Biswal M.P. and Biswas A. (1997) *Fuzzy sets and systems* 91, 37-43.
[14] Waiel F. and Abd El- Wahed (2001) *fuzzy sets and systems,* 117, 26-33.
[15] Zimmermann H. J. (1978) *fuzzy set and system* 1**,** 45-55.