Applications of fuzzy multiple attribute decision making method solving by interval numbers

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Abstract- This paper is an applied approach to grey relation analysis to select representative criteria among a large set of available choices. The method of grey related analysis to solve Fuzzy Multiple Attribute Decision Making (FMADM) problem, using interval fuzzy numbers is considered. The method standardizes inputs through norms of interval number vectors. Interval valued indices are used to apply multiplicative operations over interval numbers instead of that In this paper, the method of grey related analysis use the idea of minimizing a distance function. However, grey related analysis reflects a form of fuzzification of inputs, and uses different calculations, to include different calculation of norms. The method is demonstrated on a practical problem that selection of materials related to the wind turbine blades for decision maker estimates of alternative performance on different scales.
Keywords- Multi-criteria decision making, Grey theory, Fuzzy membership function, Ranking

1. Introduction

In the last two decades years there has been a great deal published concerning decision theory and multiple attribute decision making. This research activity has spanned decision science, system engineering, management science, operations research, and many practical fields of application. Contemporary decision making is conducted in a highly dynamic environment, involving complex tradeoffs and high levels of uncertainty. Practical decision problems involve uncertainty with respect to all elements of the basic decision making model i.e., relative attribute weights by decision maker, index values of how well available attributes are expected to perform on each of these attributes. The uncertainty and fuzziness inherent in decision making makes the use of precise numbers problematic in multiple attribute models. Decision makers are usually more comfortable providing intervals for specific model input parameters. Interval input in multiple attribute decision making has been a very active field of research. Methods applying intervals have included, use of interval numbers as the basis for ranking alternatives, error analysis with interval numbers, use of linear programming and object programming with feasible regions bounded by interval numbers use of interval number ideal alternatives to rank alternatives by their nearness to the ideal; Sengupta and Pal [11], Yoon [13]. In this paper, the method of grey related analysis as a means to reflect uncertainty in multiple attribute models through interval numbers are presented. Grey system theory was developed by Deng [4] based upon the concept that information is sometimes incomplete or unknown. The intent is the same as with factor analysis, cluster analysis, and discriminant analysis, except that those methods often don't work well when sample size is small and sample distribution is unknown, Wang, Ilo, Feng and Fang [12]. With grey related analysis, interval numbers are standardized through norms, which allow transformation of index values through product operations. The method is simple, practical, and demands less precise information than other methods. Grey related analysis and TOPSIS Hwang and Yoon [7], Lai, Liu and Hwang [8], Yoon and Hwang [14] both use the idea of minimizing a distance function. However, grey related analysis reflects a form of fuzzification of inputs, and uses different calculations, to include different calculation of norms. Feng [6] applied grey relation analysis to select representative criteria among a large set of available choices, and then used TOPSIS for outranking. AHP was presented Saaty [10], as a way to take subjective human inputs in a hierarchy and convert these to a value function. This method has proven extremely popular. Saaty used the eigenvector approach to reconcile inconsistent subjective inputs. Lootsma [9] proposed a different scaling method in his REMBRANDT system. Many researchers their interval method using linear programming over the constrained space of weights and values as a means to incorporate uncertainty in decision maker inputs to AHP hierarchies. Fuzzy AHP was proposed as another way to reflect uncertainty in subjective inputs to AHP in the same group context by Buckley [1-3]. This paper presents the method of grey related analysis to solve FMADM problem, using interval fuzzy numbers. The method standardizes inputs through norms of interval number vectors. Interval valued indices are used to apply multiplicative operations over interval numbers. The method is demonstrated on a practical problem that selection of materials related to the wind turbine blades for decision maker estimates of alternative performance on different scales.

2. The Method of Grey Related Analysis

Grey related analysis has been used in a number of applications. We shall use the concept of the norm of an interval number column vector, the distance between intervals, product operations, and number-product operations of interval numbers.

Let

$$a=[a^{-},a^{+}]=\{x \mid a^{-}\le x\le a^{+}, a^{-}\le a^{+}, a^{-}, a^{+}\in R\}$$

We call $a=[a^{-},a^{+}]$ an interval number.

If $0\le x\le a^{+}$, we call interval number $a=[a^{-},a^{+}]$ a positive interval number.

Let $X=([a_1^{-},a_1^{+}],[a_2^{-},a_2^{+}],...,[a_n^{-},a_n^{+}])^{T}$ be an n-dimension interval number column vector.

Definition 1: If $X=([a_1^{-},a_1^{+}],[a_2^{-},a_2^{+}],...,[a_n^{-},a_n^{+}])^{T}$ is an arbitrary interval number column vector, *the* norm of X is defined here as,

$$\|X\|=\max\left(\max\left(\left|a_1^{-}\right|,\left|a_1^{+}\right|\right),\max\left(\left|a_2^{-}\right|,\left|a_2^{+}\right|\right),...,\max\left(\left|a_n^{-}\right|,\left|a_n^{+}\right|\right)\right)^{T}$$

Definition 2: If $a=[a^{-},a^{+}]$ and $b=[b^{-},b^{+}]$ are two arbitrary interval numbers, the distance from $a=[a^{-},a^{+}]$ to $b=[b^{-},b^{+}]$.

$$|a-b|=\max\left(\left|a^{-}-b^{-}\right|,\left|a^{+}-b^{+}\right|\right)$$

Definition 3: If k is an arbitrary positive red number, and $a=[a^{-},a^{+}]$ is an arbitrary interval number, then $k\cdot[a^{-},a^{+}]=[ka^{-},ka^{+}]$ will be called the number product between k and $a=[a^{-},a^{+}]$.

Definition 4: I f $a=[a^{-},a^{+}]$ is an arbitrary interval number, and $b=[b^{-},b^{+}]$ are arbitrary interval numbers, we shall define the interval number product $[a^{-},a^{+}]\,.\,[b^{-},b^{+}]$ as follows,

(1) when $b^{+}>0$

$$[a^{-},a^{+}]\cdot[b^{-},b^{+}]=[a^{-}b^{-},a^{+}b^{+}].$$

(2) When $b^{+}<0$

$$[a^{-},a^{+}]\cdot[b^{-},b^{+}]=[a^{+}b^{-},a^{-}b^{+}].$$

I f $b^{+}=0$, the interval reverts to a point, and thus we would return to the basic crisp model.

3. Proposed Methodology

Suppose that multiple attribute decision making problem with interval numbers has m feasible alternatives $X_1+X_2+...+X_m$, *n* indices, the weight value w_j of index G_j is uncertain, but we know that $w_j\in[c_j,d_j]$. Here,

$$0\le c_j\le d_j\le 1,\ j=1,2,...,n,$$

$w_1+w_2+...+w_n=1$, the index value of j^{th} index G_j of feasible alternative X_i is an interval number $[a^{-},a^{+}]$, $i=1,2,...,m$, $j=1,2,...,n$.

When $c_j=d_j$, $j=1,2,...,n$, the multiple attribute decision making problem with interval numbers is an interval valued multiple attribute decision making problem with crisp weights. When a_{ij}^{-},a_{ij}^{+}, $i=1,2...,m$, $j=1,2,...,n$, the alternative scores over criteria are crisp. The principle and steps of this method are given below:

Step 1: Construct decision matrix A with index number of interval numbers If the index value of j^{th} index G_j of feasible alternative X_i is an interval number $[a^{-},a^{+}]$, $i=1,2,...,m$, $j=1,2,...,n$, decision matrix A with index number of interval numbers is defined as follows:

$$A=\begin{bmatrix}[a_{11}^{-},a_{11}^{+}] & [a_{12}^{-},a_{12}^{+}] & \cdots & [a_{1n}^{-},a_{1n}^{+}]\\ [a_{21}^{-},a_{21}^{+}] & [a_{22}^{-},a_{22}^{+}] & \cdots & [a_{2n}^{-},a_{2n}^{+}]\\ \cdots & \cdots & \cdots & \cdots\\ [a_{m1}^{-},a_{m1}^{+}] & [a_{m2}^{-},a_{m2}^{+}] & \cdots & [a_{mn}^{-},a_{mn}^{+}]\end{bmatrix}$$

Step 2: Transform contrary index into positive index.

The index is called a positive index if a greater index value is better. The index is called a contrary index if a smaller index value is better. We may transform contrary index into positive index if j^{th} index G_j is contrary index

$$[b_{ij}^{-},b_{ij}^{+}]=[-a_{ij}^{+},-a_{ij}^{-}]\quad i=1,2,...,m$$

Without loss of generality, in the following, we supposed that all the indeces are "positive indeces".

Step3: Standardize decision matrix A with index number of interval numbers to gain standardizing decision matrix $R=[r_{ij}^{-},r_{ij}^{+}]$.

If we mark the column vectors of decision matrix A with interval-valued indeces with $A_1,A_2,...,A_n$, the element of standardizing decision matrix $R=[r_{ij}^{-},r_{ij}^{+}]$ is defined as the following:

$$[r_{ij}^{-},r_{ij}^{+}]=\left[\frac{a_{ij}^{-}}{\|A_j\|}\ \ \frac{a_{ij}^{+}}{\|A_j\|}\right],\ i=1,2,...,m,\ j=1,2,...,n.$$

Note that TOPSIS uses the root mean square to evaluate distance. Grey related analysis uses a different norm, based on minimization of maximum distance.

Step 4: Calculate interval number weighted matrix $C=\left(\left[c_{ij}^{-},c_{ij}^{+}\right]\right)_{m\times n}$.

The formula for calculation of the interval number weighted matrix $C=\left(\left[c_{ij}^{-},c_{ij}^{+}\right]\right)_{m\times n}$ is,

$$[c_{ij}^{-},c_{ij}^{+}]=[c_jd_j]\cdot[r_{ij}^{-},r_{ij}^{+}],\ i=1,2,...,m,\ j=1,2,...,n.$$

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Step 5: Determine reference number sequence.

The vector for the reference number sequence is determined as the set of optimal weighted interval values associated with each of the n attributes.

$$U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], ..., [u_0^-(n), u_0^+(n)])$$

is called a reference number sequence if

$$u_0^-(j) = \max_{1 \le i \le m} c_{ij}^- (1) u_0^+(j) = \max_{1 \le i \le m} c_{ij}^+, \quad j = 1, 2, ..., n$$

Step 6: Calculate the connection between the sequence composed of weight interval number standardizing index value of every alternative and reference sequence.

The connection coefficient $\xi_i(k)$, between the sequence composed of weight interval number standardizing index value of every alternative $U_0 = ([c_{i1}^-, c_{i1}^+], [c_{i2}^-, c_{i2}^+], ..., [c_{in}^-, c_{in}^+])$ and reference number sequence

$$U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], ..., [u_0^-(n), u_0^+(n)])$$

is calculated. The formula of $\xi_i(k)$ is,

$$\xi_i(k) = \frac{\min\limits_i \min\limits_k \left\| \left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right] \right\| + \rho \max\limits_i \max\limits_k \left\| \left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right] \right\|}{\left\| \left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right] \right\| + \rho \max\limits_i \max\limits_k \left\| \left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right] \right\|}$$

The resolving coefficient $\rho \in (0, +\infty)$ is used. The smaller ρ, the greater it's resolving power. Usually, $\rho \in [0, 1]$. The value of ρ reflects the degree to which the minimum scores are emphasized relative to the maximum scores. A value of 1.0 would give equal weighting. After calculating $\xi_i(k)$, the connection between i^{th} alternative and reference number sequence will be calculated according to the following formula

$$r_i = \frac{1}{n} \cdot \sum_{k=1}^{n} \xi_i(k), \qquad i = 1, 2, \ldots, m.$$

Step 7: Determine optimal alternative

The feasible alternative X_t is optimal by grey related analysis if $r_t = \max_{1 \le i \le m} r_i$.

4. Numerical Example

We shall analyze the following example with the method of grey related analysis to multiple attribute decision making problem with interval numbers. Assume a multiple attribute decision making problem for selection of materials related to the wind turbine blades and are tabulated as the interval number decision matrix A contains decision maker estimates of alternative performances on different scales as in table -1. The weights w_l, w_2, w_3, w_4, w_5 of attributes G_1, G_2, G_3, G_4, G_5 are uncertain, but the experts can specify the following weight ranges:

$w_l \in$ [0.10,0.10], $w_2 \in$ [0.20,0.20], $w_3 \in$ [0.20,0.20], $w_4 \in$ [0.30, 0.30], $w_5 \in$ [0.40, 0.40]. Without loss of generality, we suppose that all the index values are positive.

(1) Standardize the interval number decision matrix A. Let A_1, A_2, A_3, A_4, A_5 denote the close interval column vector of index interval number decision matrix A, respectively, then $||A_1||$= 355, $|| A_2||$ = 4100, $|| A_3||$= 9,
$|| A_4||$ = 20, $|| A_5||$ = 580.

Standardizing the interval number decision matrix converts the initial divergent measures to a common 0-1 scale. Here, we obtain matrix R as follows.

R=

$$\begin{bmatrix} [0.0704, 0.0986] & [0.0439, 0.0487] & [0.6666, 1.0000] & [0.5000, 1.0000] & [0.9138, 1.0000] \\ [0.0141, 0.0423] & [0.0195, 0.0244] & [0.1333, 0.4666] & [0.5000, 0.7500] & [0.6552, 0.7241] \\ [0.1887, 0.2197] & [0.8293, 0.8781] & [0.1111, 0.4444] & [0.0500, 0.2500] & [0.5689, 0.6552] \\ [0.9718, 1.0000] & [0.9512, 1.0000] & [0.0555, 0.3333] & [0.0500, 0.1000] & [0.8103, 0.9138] \\ [0.3239, 0.3521] & [0.8536, 0.9024] & [0.0555, 0.2222] & [0.4500, 0.6000] & [0.3793, 0.4827] \end{bmatrix}$$

(2) Calculate the interval number weighted decision matrix C by multiplying the weight intervals by matrix R.

C=

$$\begin{bmatrix} [0.0070, 0.0098] & [0.0088, 0.0097] & [0.1333, 0.2000] & [0.1500, 0.3000] & [0.3650, 0.4000] \\ [0.0014, 0.0042] & [0.0039, 0.0048] & [0.0266, 0.0933] & [0.1500, 0.2250] & [0.2621, 0.2896] \\ [0.0188, 0.0219] & [0.1658, 0.1756] & [0.0222, 0.0888] & [0.0150, 0.0750] & [0.2276, 0.2621] \\ [0.0972, 0.1000] & [0.1902, 0.2000] & [0.0111, 0.0666] & [0.0150, 0.0300] & [0.3241, 0.3655] \\ [0.0324, 0.0352] & [0.1707, 0.1805] & [0.0111, 0.0444] & [0.1350, 0.1800] & [0.1517, 0.1931] \end{bmatrix}$$

(3) Determine the reference number sequence Uo.

U_o = ([0.972,0.1000], [0.1902,0.2000], [0.1333,0.2000], [0.1500,0.2250], [0.3655,0.40])

(4) Calculate the connection between the sequences composed of weighted interval number standardizing index value of every alternative and reference number sequence (table -2).

Let $\Delta_i(k) = \left\| \left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right] \right\|$. The connection coefficient $\xi_i(k)$ (a distance function) is then calculated by the formula as follows:

$$\xi_i(k) = \frac{\min\limits_i \min\limits_k \Delta_i(k) + \rho \max\limits_i \max\limits_k \Delta_i(k)}{\Delta_i(k) + \rho \max\limits_i \max\limits_k \Delta_i(k)}$$

In the example, ρ = 0.5. When $\xi_i(k)$ is determined $\min\limits_i \min\limits_k \Delta_i(k)$ and $\max\limits_i \max\limits_k \Delta_i(k)$ will be calculated as follows:

From the above chart (Table 2), we know that $\min\limits_i \min\limits_k \Delta_i(k) = 0$, $\max\limits_i \max\limits_k \Delta_i(k) = 0.4207$.

This is used in the connection coefficient formula to identify distances (larger values means greater distance).

$$\xi_1 = (0.5383, 0.3614, 1.0000, 1.0000, 1.0000)$$

$$\xi_2 = (0.5233, 0.3554, 0.4918, 0.7372, 0.4959)$$

$$\xi_3 = (0.5734, 0.8117, 0.4853, 0.3688, 0.4327)$$

$$\xi_4 = (1.0000, 1.0000, 0.4514, 0.3418, 0.7348)$$

$$\xi_5 = (0.6187, 0.8436, 0.4309, 0.6091, 0.3333)$$

By these results, we know that the connection between every alternative and reference number sequence is, respectively $r_1 = 0.7799$, $r_2 = 0.5187$,
$r_3 = 0.5344$, $r_4 = 0.7056$, $r_5 = 0.5671$.
Ranking feasible alternatives from largest to smallest r_i the rank order of feasible alternatives is
X_1, X_4, X_5, X_3, X_2.

5. Conclusion

The method of grey related analysis to multiple attribute decision making problems with interval numbers given in this paper concerns interval fuzzy input parameters. It applies the traditional method of grey related analysis. The method reflects decision maker or group uncertainty concerning multiple criteria decision input parameters. The method presented here is simple, practical, and requires less rigid input from decision makers. Weight inputs are entered as fuzzy interval numbers. Alternative performance scores also can be entered as general interval fuzzy numbers. Both weight and performance scores are standardized, and composite utility value ranges obtained. The grey related analysis is very simple and less tedious and solved in less time duration. In the final selection of material for the wind turbine blade by using this method in the above order of the ideal solutions, best alternative found is Steel. But Alloy Steel has lower compressive strength, poor machineability, poor environmental stability and poor temperature strength. Therefore, second best alternative i.e. Carbon fiber material A_4 is selected as the connection co-efficient indicate that the Carbon is better than the other four alternatives.

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Table1. Step 2: Transform "contrary index" into positive index.

Properties Materials	Stiffness (GPA)	Tensile strength (Mpa)	Density (g/cm3)	Elongation at break (~)	Max temp
Steel	[25,35]	[180,200]	[6, 9]	[10, 20]	[530, 580]
Aluminium	[5,15]	[80,100]	[1.2,4.2]	[10, 15]	[380, 420]
Glass-E	[67,78]	[3400,3600]	[1, 4]	[1, 5]	[330, 380]
Carbon	[345,355]	[3900,4100]	[0.5, 3]	[1, 2]	[470, 530]
Aramid	[115,125]	[3500,3700]	[0.5,2]	[9, 12]	[220, 280]

Table 2 Standardizing Index Value

	G1	G2	G3	G4	G5	$\min_k \Delta_i(k)$	$\max_k \Delta_i(k)$
$\Delta_1(k)$	0.1804	0.3717	0	0	0	0	0.3717
$\Delta_2(k)$	0.1926	0.3815	0.2174	0.075	0.2138	0.075	0.3815
$\Delta_3(k)$	0.1565	0.0488	0.2231	0.36	0.2758	0	0.36
$\Delta_4(k)$	0	0	0.2556	0.405	0.0759	0	0.405
$\Delta_5(k)$	0.1296	0.039	0.2778	0.135	0.4207	0	0.4207
$\min_i \min_k \Delta_i(k)$						0	
$\max_i \max_k \Delta_i(k)$							0.4207