PAMUKKALE ÜNİVERSİTESİ MÜHENDİSLİK FAKÜLTESİ
PAMUKKALE UNIVERSITY ENGINEERING COLLEGE
MÜHENDİSLİK BİLİMLERİ DERGİSİ
JOURNAL OF ENGINEERING SCIENCES
YIL : 1999
CİLT : 5
SAYI : 2-3
SAYFA : 1017-1023

ELASTO-PLASTIC STRESS ANALYSIS OF THERMOPLASTIC MATRIX COMPOSITE LAMINATED PLATES UNDER IN-PLANE LOADING

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Geliş Tarihi : 06.03.1999

ABSTRACT

Thermoplastic matrix reinforced with metal fiber, composite laminated plates were manufactured by using moulds. The symmetric and antisymmetric laminated plates were loaded by in-plane forces. An elastic-plastic numerical solution has been carried out by finite element technique (FEM) for some load steps. Residual stresses and expansion of plastic zone have been illustrated in tables and figures.

Key Words : Elasto-plastic stress, Thermoplastic matrix, Composite laminated plate

TERMOPLASTİK MATRİSLİ TABAKALI KOMPOZİT PLAKTA ELASTİK-PLASTİK GERİLME ANALİZİ

ÖZET

Metal lif takviyeli termoplastik matrisli tabakalı kompozit plaklar sıcak pres metoduyla imal edilmiştir. Simetrik ve anti-simetrik tabakalı kompozit plaklar kendi düzlemine paralel yüklere maruzdur. Elastik-plastik sayısal çözüm bazı yük adımları için sonlu elemanlar yöntemiyle (SEM) gerçekleştirilmiştir. Plaklardaki artık gerilmeler ve plastik bölgeler tablolar halinde verilmiş ve şekillerle gösterilmiştir.

Anahtar Kelimeler : Elastik-plastik gerilme, Termoplastik matris, Tabakalı kompozit plak

1. INTRODUCTION

Composite materials offer great potential and flexibility in structural design because of the anisotropy of material properties, unique ply-by-ply constructions, and novel fabrication methods (Alexander and Tzeng, 1997). For applications in advanced composite systems made up of polymeric-matrix materials and high strength and stiffness reinforcements such as carbon fibers, thermosetting polymers have been utilized almost exclusively for several decades. However, several years ago, the introduction and development of new thermoplastic, e. g., Torlon, PEEK and Ryton, followed by numerous other candidates, as matrix materials in structural composites have given another dimension to the advanced application of reinforced plastic materials (Chen et al., 1993.). This new generation of engineering materials, thermoplastic polymer matrix continuous fiber composites, offer the potential of significant improvement and advantage in many aspects, compared with conventional reinforced thermosets (Hoggatt, 1973, 1975; Hartness, 1980, 1982, 1984; Rigby, 1982; Hergenrother et all., 1984; Muzzy and Kays, 1984).

A number of papers have appeared on stress analysis of fibrous composite structures. Stress concentration factors for different structures have been investigated (Jong, 1981; Karakuzu et all., 1990; Okur et all., 1993;). Anisotropic strength of composites has been studied (Azzi and Tsai, 1965; Tsai and Wu, 1971; Chou et all., 1973). Two-dimensional finite element analyses (plane stress, plane strain) of different isotropic structures have been obtained (Karakuzu and Sayman, 1991; Toparlı and Aksoy, 1991a). The elasto-plastic analyses of composite materials and structures have been made (Bahei-El-Din et all., 1981; Avcı and Akdemir, 1990; Karakuzu and Sayman, 1991).

In this study, Low Density Polyethylene (LDPE-F2.12) matrix metal fiber composite laminated plates with a hole, have been subjected to in-plane loading. The residual stresses and expansion of the plastic zone were determined for some load steps. Residual stresses in composite materials are important because they can lead to premature or increased failure (Jeronimidis and Parkyn). Prediction and measurement of residual stresses are therefore important in the production, design and performance of composite component (Karakuzu et all., 1997).

2. MATHEMATICAL FORMULATION

The metal matrix laminated plate of constant thickness is composed of orthotropic layers bonded symmetrically or antisymmetrically about the middle surface.

The solution of laminated plate elements includes transverse shear deformations. Therefore, the stress-strain relation for an orthotropic layer in any orientation angle in the plane of the layer is given as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad (1)$$

Where the transformed reduced stiffnesses, Q_{ij} are given in terms of the orientation angle and the engineering constants of the material.

In this investigation, transverse shear deformation theory is used. The theory assumes that the particles of the plate, originally on a line that is normal to the undeformed middle surface, remain on a straight line during deformation, but this line not necessarily normal to the deformed middle surface. Therefore, the displacement components of a point of coordinates x, y, z for small deformations are,

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\psi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\psi_y(x,y) \\ w(x,y,z) &= w(x,y) \end{aligned} \qquad (2)$$

Where u_0, v_0 and w are the displacements of a point on the middle surface, and ψ_x, ψ_y are the rotation angles of normal to the y and x-axes, respectively. By using the strain-displacement relations, bending strains are found to vary linearly through the plate thickness, whereas shear strains are assumed to be constant throughout the thickness as,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ -\dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} - \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix} \qquad (3)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} + \psi_x \end{vmatrix}$$

To obtain the element equilibrium equations, the total potential Π is written with p equal to the transverse loading per unit area and in plane forces $\overline{N_n^b}, \overline{N_s^b}$,

$$\Pi = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\int_A (\sigma_n \varepsilon_n + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}) dA \right] dz + \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\int_A (\tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}) dA \right] dz$$
$$- \int_A wp dA - \int_{\partial R} \left(\overline{N}_n^b u_n^0 + \overline{N}_s^b u_s^0 \right) ds \qquad (4)$$

where $dA = dxdy$ and in-plane forces are applied on the boundary ∂R.

The resultant forces N_x, N_y, N_{xy}, and moments M_x, M_y and M_{xy} and shearing forces Q_x and Q_y per unit length of the cross section of the laminated plate are given as,

$$\begin{vmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{vmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (1, z) dz$$

$$\begin{vmatrix} Q_x \\ Q_y \end{vmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \qquad (5)$$

For equilibrium, the potential energy Π must be stationary. It is obtained so that $\delta\Pi = 0$ which may be regarded as the principle of virtual displacement for the plate element (Bathe, 1982).

3. FINITE ELEMENT MODEL

In order to find the residual stresses and the yield points of the laminates, a nine-node finite element was employed. The symmetric or antisymmetric laminated plates are composed of four or two layers. The laminated plates are divided into 8 imaginary parts to obtain more accurate results in the solution.

The stiffness matrix of the plate is obtained by using the minimum potential energy principle. Bending and shear stiffness matrices are,

$$|K_b| = \int_A |B_b|^T |D_b| |B_b| dA$$
$$|K_s| = \int_A |B_s|^T |D_s| |B_s| dA \tag{6}$$

where,

$$|D_b| = \begin{vmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{vmatrix}$$
$$|D_s| = \begin{vmatrix} k_1^2 A_{44} & 0 \\ 0 & k_2^2 A_{55} \end{vmatrix} \tag{7}$$
$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz \quad (i, j = 1, 2, 6)$$

$$(A_{44}, A_{55}) = \int_{-h/2}^{h/2} (Q_{44}, Q_{55}) dz$$

D_b and D_s are the bending and shear parts of the material matrix, respectively. A_{45} is negligible in comparison with A_{44} and A_{55}. k_1 and k_2 represent the shear correction factors for rectangular cross sections and are given as (Lin and Kuo, 1989) $k_1^2 = k_2^2 = 5 / 6$.

In this solution the external forces are applied transversely and are increased incrementally. Because the calculated stresses do not coincide with the true stresses in a nonlinear stress analysis, the unbalanced nodal forces and the equivalent nodal forces must be calculated for each iteration. The equivalent nodal forces that correspond to the element stresses can be expressed as,

$$\{R\}_{\text{equivalent}} = \int_{vol} |B|^T |\sigma| dA = \int_{vol} |B_b|^T |\sigma_b| dA + \int_{vol} |B_s|^T |\sigma_s| dA \tag{8}$$

Hence, the unbalanced nodal forces can be determined as follows,

$$\{R\}_{\text{unbalanced}} = \{R\}_{\text{applied}} - \{R\}_{\text{equivalent}} 1 \tag{9}$$

The unbalanced nodal forces are used to obtain increments and satisfy the convergence tolerance in a nonlinear analysis throughout the complete history per step of load application (Karakuzu et all., 1997). The stress-strain relation in plastic region is given as,

$$\sigma = \sigma_0 + K\varepsilon_p^n \tag{10}$$

In the plastic solution Tsai-Hill theory was used as a yield criterion.

4. PRODUCTION OF LAMINATED PLATES

Fiber-reinforced plastics are associated with products in which a polymeric matrix is combined with reinforcing fibers. Products of polymeric composite materials are numerous and steadily growing. In this study, low density polyethylene (LDPE-F2.12) was used as a thermoplastic matrix and galvanized steel wire as fiber materials. The mould is prepared and pressed as shown in Figure 1.

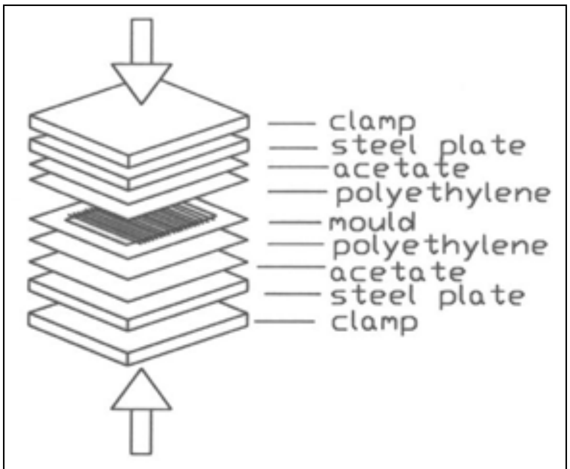


Figure 1. Production of the plate

The mould temperature is increased to 160^0C in five minutes without a pressure. The hot mould is waited five minutes under 2.5 MPa. In the cooling process the temperature has been decreased to 30 ^{0}C under 15 MPa pressure in three minutes. Thus the composite combination is obtained. The thickness of a layer is manufactured as 2.5 mm.

5. EXPERIMENTAL CHARACTERIZATION

The major focus of the experiments was to characterize the elasto-plastic behavior of metal reinforced thermoplastic matrix laminated plates and to evaluate the results with finite element solution.

The layer is loaded in principal material directions by Instron tensile machine; thus the yield points in principal material directions and shear are found as given in Table 1. Mechanical properties of the layer are obtained experimentally by using strain gages.

Table 1. Mechanical Properties and Yield Points of a Layer

Mechanical Properties		Yield Strengths and Parameters		
E_1	4300 (MPa)	Axial Strength	X	21.01 (MPa)
E_2	957 (MPa)	Transverse Strength	Y	5.22 (MPa)
G_{12}	241 (MPa)	Shear Strength	S	5.85 (MPa)
ν_{12}	0.4	Hardening Parameter	K	47.183 (MPa)
		Strain -Hardening Parameter	n	0.713

6. NUMERICAL RESULTS AND DISCUSSION

The laminated plate with a hole (Figure 2) is assumed to be under uniformly distributed loads at the opposite edges of the plate.

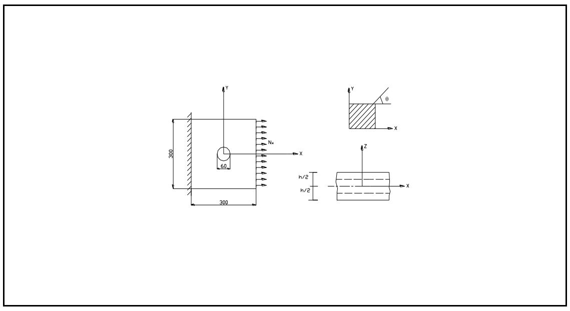


Figure 2. In-plane loading of the plate

They are composed of four orthotropic or generally orthotropic layers bounded symmetrically or antisymmetrically. The plates are simply supported. Loading is gradually increased up to the plastic zone that is not allowed to be large. In the iterative solution the overall stiffness matrix of the laminated plate is the same at each loading step. The incline load (N_x) is increased 0.01 N/mm per step.

One quarter of the plate is enough to find the expansion of the plastic zone and the residual stresses in the cross-ply symmetric laminated plate ($(0^0, 90^0)_2$) without a hole. Residual stress components are given in Table 2.

Table 2. Residual Stress Components in the Symmetric Cross-ply, ($(0, 90)_2$), Laminated Square Plate Without a Hole for 200 Load Steps

Orientation Angle	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{xz} (MPa)	τ_{yz} (MPa)
0^0	-0.282	-0.143	0.000	0.000	0.000
90^0	0.282	0.143	0.000	0.000	0.000

The layer of 0^0 orientation angle yields earlier than the layer of 90^0 orientation angle. Because E_1/E_2 is greater than X/Y, therefore the layer of 0^0 orientation angle takes higher stress component than that in the layer of 90^0 orientation angle. The layer of 0^0 orientation angle has permanent deformation, therefore it applies a tensile force to the layer of 90^0 orientation angle. For static equilibrium, the layer of 90^0 orientation angle applies a compressive force to the layer of 0^0 orientation angle. Therefore, the residual stress components in the layers of 0^0 and 90^0 orientation angles are, σ_x= -0.282 MPa, σ_y= -0.143 MPa and σ_x= 0.282 MPa, σ_y= -0.143 MPa, respectively.

The yield points in laminated plates with a hole are given in Table 3. As seen from this table, the yield points in symmetric laminated plates are greater than those in antisymmetric laminated plates. The yield point in the symmetric angle-ply, ($(30^0,-30^0)_2$), is maximum (N_x = 55.40 N/mm).The expansion of the plastic zones for the layers or orientation angles 0^0 and 90^0 in symmetric laminated square plate with a hole under in-plane loading is illustrated in Figure 3.

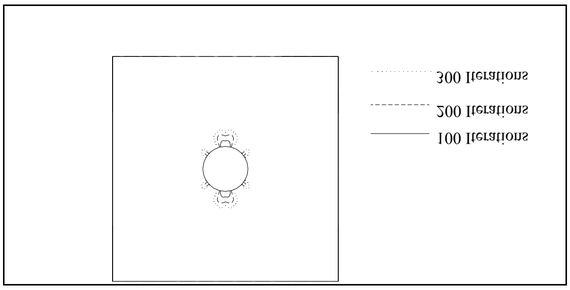


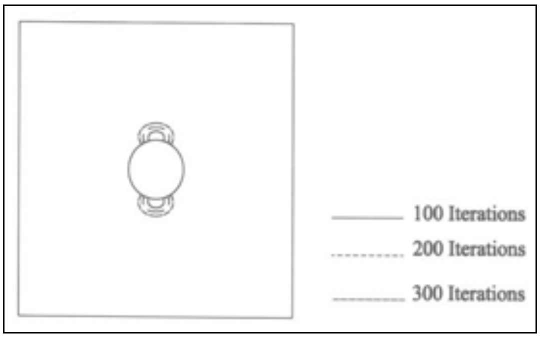
Figure 3. Expansion of the plastic zone in symmetric cross-ply laminated plate a) 0^0 b) 90^0

It is seen from this figure that the expansion of plastic zone for of 0^0 and 90^0 orientated layers is similar and the plastic zone starts at different points around the hole. The difference between 0^0 and 90^0 orientated layers is very small. But the expansion of plastic zone for the symmetric and antisymmetric cross-ply, ($(0^0,90^0)_2$), laminated plates is different.

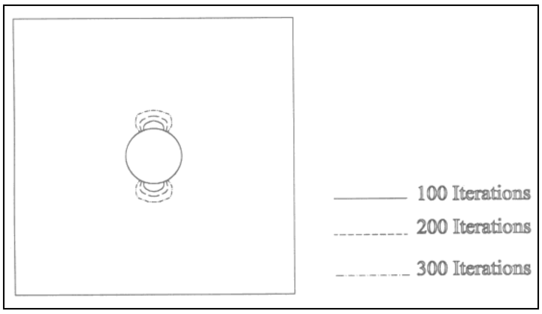
Table 3. Yield Points in Laminated Plates With a Hole

	$((0^o, 90^o)_2)$	$((30^o, -30^o)_2)$	$((45^o, -45^o)_2)$	$((60^o, -60^o)_2)$
Symmetric N_x (N/mm)	42.30	55.40	45.70	27.50
Antisymmetric N_x (N/mm)	41.70	54.90	45.30	27.00

The expansions of plastic zones for the antisymmetric plates are shown in Figure 4. The plastic zone in the orientation angle 90^0 is slightly larger than that of 0^0 orientation angle. In the layer of 90^0 orientation angle, for 300 load steps, the residual stress component σ_x along A-B is given in Figure 5.



(a)



(b)

Figure 4. Expansion of the plastic zone in antisymmetric cross-ply laminated plate a) 0^0 b) 90^0

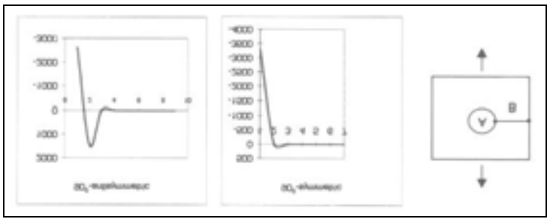


Figure 5. Residual stresses along the A-B line a) A-B line b) Antisymmetric plate b) Symmetric plate

The effect of the orientation angle on the expansion of plastic zone is presented in Figure 6, for $((30^0,-30^0)_2)$ symmetric and antisymmetric angle-ply laminated plates with simply supported edges. When the external force N_x reaches 55.40 N/mm, the yielding occurs in the symmetric layer, and when it is further increased incrementally, the plastic zone expands around the hole. The expansions of the plastic zones are nearly the same in both symmetric and antisymmetric plates.

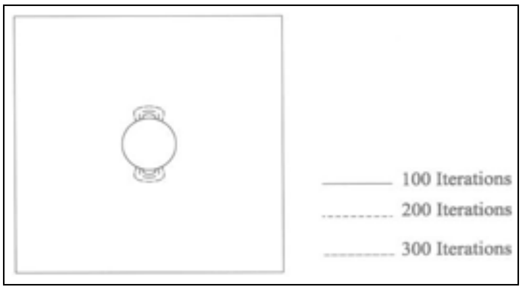


Figure 6. Expansion of the plastic zone in symmetric and antisymmetric $((30^0.-30^0))$ laminated plates

The effect of orientation angles on the expansion of the plastic zone are shown in Figure 7, for $((45^0,-45^0)_2)$ symmetric and antisymmetric angle -ply laminated plates with simply supported edges. When we increase the external force gradually, the plastic zone expands around the hole. It is nearly the same for both 45^0 and -45^0 orientated layers.

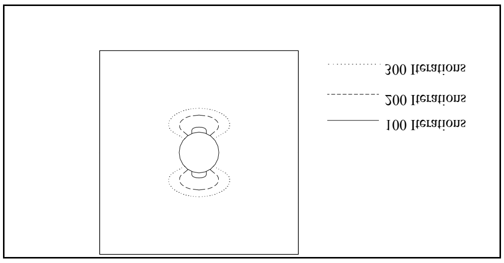


Figure 7. Expansion of the plastic zone in symmetric and antisymmetric $((45^0.-45^0))$ laminated plates

The expansion of the plastic zone in $((60^0,-60^0)_2)$ symmetric and antisymmetric angle -ply laminated plates is shown in Figure 8. Plastic zones for the layers of orientations 60^0 and -60^0 are nearly the same at each case.

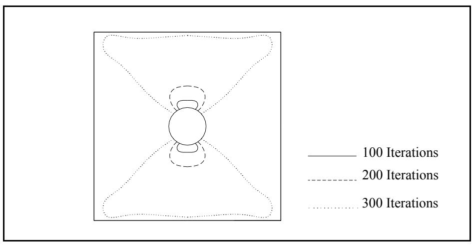


Figure 8. Expansion of the plastic zone in symmetric and antisymmetric $((45^0.-45^0))$ laminated plates

The in-plane force (N_x) is given at Table 4, for the case when the laminated plates without a hole reach yielding.

Table 4. Yield Points in Laminated Plates Without A Hole

	$((0^o, 90^o)_2)$	$((30^o,-30^o)_2)$	$((45^o,-45^o)_2)$	$((60^o,-60^o)_2)$
Symmetric N_x (N/mm)	127.70	139.20	96.00	57.70
Antisymmetric N_x (N/mm)	108.00	139.20	96.00	57.70

It is seen that the in-plane loads at the yield points for the symmetric and antisymmetric angle-ply laminates are the same, but for the cross-ply laminates there is a different situation. The yield points of the symmetric cross-ply laminates are higher than the antisymmetric laminates. If the orientation angle is increased in angle-ply laminated plates, the yield points become smaller. The yield point is maximum in the angle-ply, $((30,-30)_2)$, laminated plate in comparison with the others.

7. CONCLUSIONS

Elasto-plastic stress analysis has been carried out by using the first order shear deformation theory in thermoplastic matrix-metal fiber laminated plates. The expansion of plastic zone and residual stresses are obtained in symmetric and antisymmetric cross-ply and angle-ply composite laminated plates.

1. The yield point in the symmetric cross-ply, $((0^0,90^0)_2)$, laminated plate without a hole is higher than that in antisymmetric cross-ply laminated plate. The yield point in the angle-ply, $((30^0,-30^0)_2)$, laminated plate is maximum in comparison with the others.
2. The yield point is the same in the symmetric and antisymmetric angle-ply laminated plates for the same orientation angles. The yield point is maximum in the symmetric angle-ply, $((30^0,-30^0)_2)$, laminated plate with a hole.
3. The yield point in the symmetric laminated plates is higher than those in the antisymmetric laminated plates.
4. The expansion of the plastic zone is different in the symmetric and antisymmetric cross-ply, $((0^0, 90^0)_2)$, laminated plates with a hole.

If the orientation angle is increased the plastic zone becomes larger in the angle-ply laminated plates.

The residual stress components in the symmetric cross-ply, $((0^0, 90^0)_2)$, laminated plate with a hole are greater than those in antisymmetric cross-ply laminated plate.

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