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Clustering the Costumers and Assigning Goods to the Clusters through MODM 0/1 Model

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Abstract

In today's competitive market and in service organizations in particular, attending the customer is developing, and institutions invest substantially to recognize behavioral patterns of customers. Since no all customers possess equal value for the organization, clustering the customers, investigating the specific pattern of each cluster, and finally, adopting proper strategies for each cluster can influence the organizational profitability. The development in science and technology, also, has created a wide range of tastes and desires in individuals. As a result, the organization comes across a great number of customers' needs and tastes whose complete responding will be impossible. The researcher aims to take a thorough look at diverse customers through maintenance of comprehensive, relevant, and reasonable criteria for clustering the customers, and to cluster them according to their most similarities in one group (cluster) and most differences in separate groups. Consequently, recognizing the features of customers with different clusters will facilitate, and more appropriate decisions will be made.

Key words

Clustering, Customers, Decision making, MODM

1 INTRODUCTION

Today, data analysis is the most critical tool for beneficial use of diverse resources of data. Precise data with large quantity and low price are produced by different companies and organizations, and they are organized in data warehouses. Data relevant to trade, agriculture, traffic, internet, details of telephone calls, and medical data belong to the examples of such databases. According to the intensity of competition in commercial, scientific, societal, managerial, and political fields, the importance of the factor of speed or data availability for different managerial levels is increasing. Presently, data analysis is regarded as one of the most important technologies in order to effectively and precisely utilize the bulk of data, and this importance is increasing gradually. Through analyzing customer's life cycle, contemporary companies have achieved a growth in customer value. The devices and technologies of data warehouse, data analysis and other customer relationship management techniques fall among methods preparing novel opportunities for trade. As a matter of fact, the product-oriented view is replaced by customer-oriented one. Consequently, compiling the data relevant to the customers, and decision making based on achieved patterns from hidden relationships among data through data analysis tool, one can accomplish the customer-oriented need. In general, the public use of internet and websites as universal informing systems, encounters us with a bulk of data and information. Such explosive growth in stored data has created progressive need to new technologies and automated devices aiding human being intelligently in order to change this bulk of data and information to science; data analysis is regarded as a solution for such problems. Simultaneously, data analysis enjoys some scientific fields such as: databade technology, artificial intelligence, machine learning, neural networks, statistics, pattern recognition, knowledge-based systems, gaining knowledge, information retrieval, and high-speed computing and data visual representation. Clustering is one of the most popular ways of data analysis. Its capability in entering the data space and recognizing its structure has made clustering one of the most ideal mechanisms for work in wide world of data. For the first time, its idea began in 1930s, and now a days, experiencing great developments and mutations, clustering has been introduced in several aspects and functions. Clustering is one of the unsupervised learning branches; it is an automated process during which the samples are divided into groups whose members are similar to each other; such groups are called 'clusters'. As a result, a cluster is a collection containing similar object which differ from objects in other clusters. One can consider numerous criteria for similarity; for instance, one may consider the criterion of distance for clustering and place the closer objects

in one cluster. Such clustering is also called “distance-based clustering”. For example, in figure 1-1, the entering samples on the left are divided into four similar clusters on the right. In this example, each one of the entering samples belongs to (only) one of the clusters, and there is no sample belonging to more than one cluster.

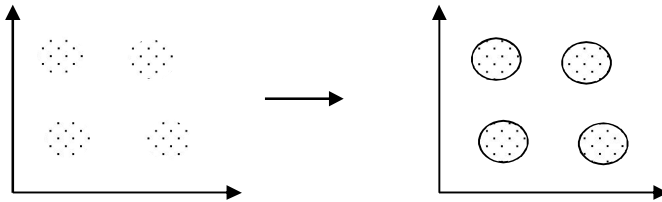


Figure 1-1, the use of distance criterion as dissimilarity among data

Computing the distance between data is important in clustering. The distance, or ‘heterogeneity presenter’, helps us to move within data space and create clusters. Computing the distance between a couples of data, one can find how close they are, and accordingly, put them in one cluster. There are several mathematical functions to compute the distance, such as Euclidean distance, Hamming etc.

In this study, after clustering the customers based on the opinions of experts and by means of the features of each cluster, each one of them is entitled, and then, through an MODM 0 and 1 model, and considering the limitations, the products of the companies are allocated to the clusters.

2 LITERATURE REVIEW

The key methods in data analysis are divided into ‘descriptive’ and ‘predictive’. Some of these methods include: modeling for prediction (such as classifying and regression), subdividing or fractionating (clustering), dependence modeling (visual models or density estimation), summarization, finding the links among fields, concomitance or associative, visualization and modeling, finding changes and deviations in the data and knowledge.

- **Descriptive approaches:** such approaches maintain general features of data. Description aims at finding data patterns interpretable for human being. Clustering is one of descriptive approaches in data analysis.
- **Predictive approaches:** such approaches are being used to forecast future behaviors. Predict means using some variables or fields in database for forecasting future or unknown amounts of interested variables.

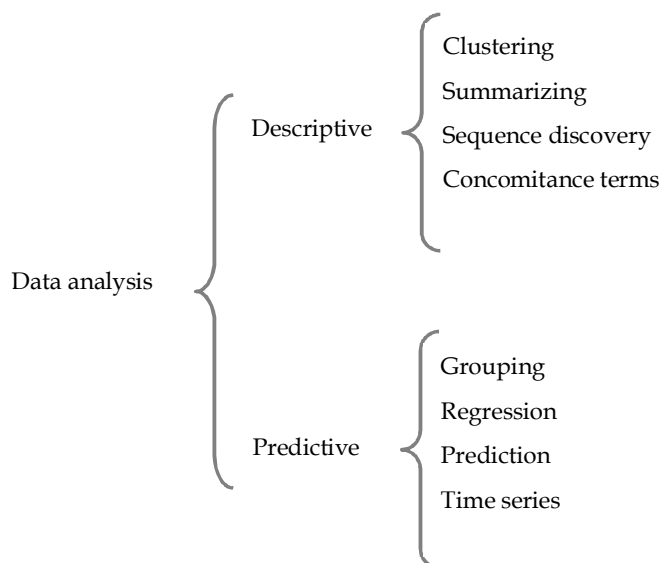


Figure 1-2, data analysis and knowledge discovery, Ghazanfari et. al. 2008

In 18th century, Linaeus and Savages provided a substantive grouping of animals, plants and minerals and diseases. In recent years also Holman had done the same. Today, clustering has wonderfully entered various sciences, and it is usable in different fields (Ghazanfari et. al., 2008)

- Text recognition
- Geographic Information system
- Image processing

- Economic sciences
- Marketing
 - Discovering customer groups to improve marketing program
 - Maintaining customers' purchase pattern
 - Market basket analysis
 - Predicting the amount of customers' purchase
- Insurance: recognizing the customers claiming high costs
- Urban Planning (grouping town houses according to the type, value and location)
- Earthquake studies (grouping observed seismic stations)

3 METHODOLOGY

One of the valid methods in clustering is through K-mean which is performed according to the least distance between each data and the center of cluster (average). This method is very popular due to its ease, and it is performed in different ways.

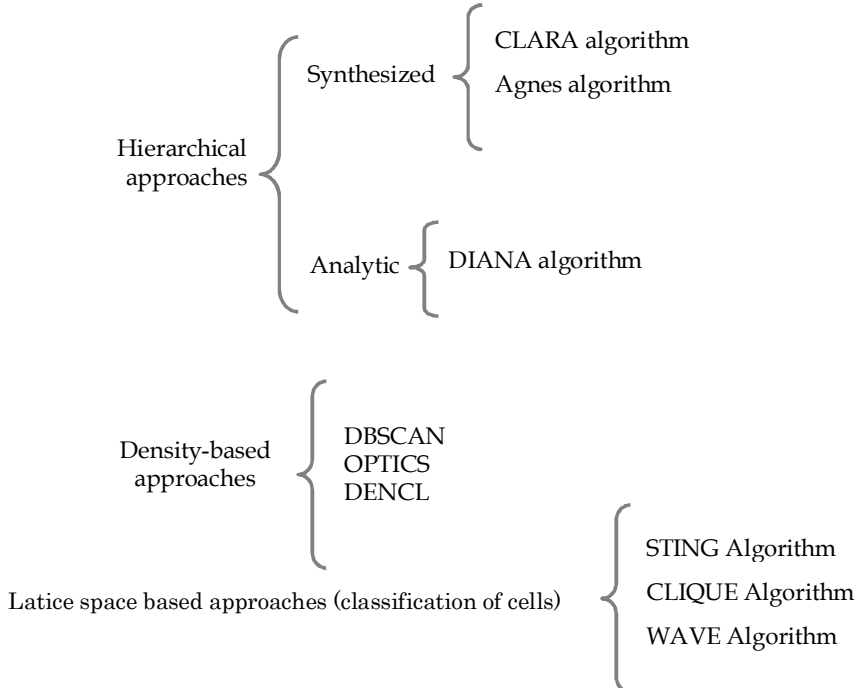
The key stages of K-mean algorithm are as follows:

- Selecting a primary part having K groups, including arbitrary picked samples; computing the average of groups
- Creating a new stage through maintaining each sample around the core of the nearest group.
- Computing the cores of the recent group as the main groups.
- Repeating stages 2 and 3 to reach an optimal value of the performance criterion

Selecting K takes place through several approaches as well

Main approaches of Clustering

- ❖ Secretary approaches (on the basis of the distance between objects) (Ghazanfari et. al., 2008)
 - K-means Algorithm
 - K-modes Algorithm
 - K-medoids Algorithm



Self-organizing approaches or Kohonen's map (illustration of multidimensional data in space)

Allocation Model

The most primary model for allocating is the classic one in which there is a goal (criterion), cost as an instance, and whose variables are presented in 0 and 1 form. In this model, every objects can be assigned to a work, and each work solely applies one individual. Consequently, there are a couple of limitations in this model (a limitation due to allocation of each object to only one work, and limitation of allocating only one object to

one work). The variables of this model also include 0 and 1.

In practice, the decision makers usually consider several and sometimes opposite criteria instead of one criterion. Thus, some models were designed known as multi-criteria decision making or MCDM. Such models are divided into two major categories: multi-objective models and multi-criteria models. The former is used for designing; while the latter is utilized to select the best option, and generally, to rank the options.

A multi-objective question is determined by K objective vector(s) whose mathematical formula is:

$$Z(x) = [Z_1(x), Z_2(x), \dots, Z_k(x)]$$

According to:

$$x \in X$$

Where X is the answer.

$$X = \{x: x \in R^n, g_i(x) \leq \cdot, x_j \geq \cdot \forall i, j\}$$

In mentioned model, R is the set of real numbers, $g_i(x)$ is the i^{th} limitation and x_j is the j^{th} variable. The scale of each objective may differ from that of the others, so one cannot simply add them up (Momeni, 2005).

In general, one cannot optimize the vector of functions simultaneously. To find the optimized result, some information about preferences must be available. Without such information, the objectives will be paradoxical and incomparable, and no optimized result will be achieved, because not all of the feasible results are comparable. In this case, complete ranking of the result will be possible if only value judgments enter decision making process. Many times, a set of non-dominated results are looked for. The set of non-dominant results related to feasible results (X) shown by (S) is defined as:

$$Z_q(x') > Z_q(x) \text{ such that } S = \{x: x \in X\}, x' \in X$$

$$\text{For the values of } q \in \{1, 2, \dots, k\}, Z_k(x') \geq Z_k \text{ for all of } k \neq q$$

Classifying and comparing the multi-objective questions solutions has been reviewed in several references. Kohn and Marquez have introduced three criteria for evaluating solution techniques including: computational efficiency, exchange stipulation, and the amount of information produced for decision making. According to these criteria, they divide multi-objective questions (having one decision maker) into three categories:

- A) Non-dominant set producing method
- B) Methods having the primary list of preferences
- C) Methods with gradual entrance of preferences

Method of weighting the objective functions, method of minimum for each objective, Philip method and Zeleny method fall among the methods in category A. In Philip and Zeleny methods, the multi-objective question does not need to change to single-objective one; they can be applied on objectives vector to find non-dominant results. Both of these methods are solely usable for linear questions. Among methods in category B are ideal planning methods. For the first time, such methods were presented by Dock Stein and Sowarowski, and they are widely used today. Utility function evaluation method, presented by Kinni and Rayfa, are also placed in category B. Another method of this category is exchange of alternative value.

In Compromise programming method, among methods in category C, the ideal result is made, and the result with the least distance to ideal spot is selected. The above mentioned methods were applied for continuous multi-objective questions.

To resolve AHP discrete multi-criteria problems there are also some methods whose mentioning is not required here; only one method is explained used for maintaining the amount of criteria importance, and also ranking the options.

Decision variables, are those which the decision maker aims to maintain their amount. In this model, decision variables have two values (0 & 1), and are defined as:

$$(i=1,2,\dots,n, j=1,2,\dots,m) X_{ij} \text{ Allocating the product } i \text{ to the cluster } j$$

If product i is allocated to cluster j , then $x_{ij}=1$, and if not, then $x_{ij}=0$. The number of decision variables equals the number of products \times number of clusters.

In considering model, the objectives include: Maximizing revenue from deployment and support, maximizing the proportion between the product and the objectived cluster, revenue from training the users.

As a result, we face several objectives which we want maximize them all. If we illustrate the first objective as Z_1 , and the second one as Z_2 , and the k^{th} objective as Z_k , then the objective functions will be as following:

$$MAXZ_1 = \sum_{i=1}^n \sum_{j=1}^m C_{ij1} X_{ij}$$

$$MAX Z_2 = \sum_{i=1}^n \sum_{j=1}^m C_{ij2} X_{ij}$$

$$MAX Z_k = \sum_{i=1}^n \sum_{j=1}^m C_{ijk} X_{ij}$$

Due to non-equality of objectives importance, group analytical hierarchy process is applied to maintain the weight of each objective. For this, some representatives, as samples, are selected and wanted to compare their objective two by two. Then the following geometric mean is used to combine their ideas:

$$a_{ij} = \sqrt[N]{\prod_{k=1}^N a_{ijk}}$$

In which a_{ijk} is numerical value for comparing objective i and objective j by individual k ; N is mean, a comparative bigeminal matrix is emerged and the weight (W) of each objective is computed.

Due to different scales in the coefficients of the objective functions, first the coefficients of the objective becomes normative; this is done by dividing the amounts of objective k by the upper limit of its amount range shown by H_k .

$$Z = \sum_{k=1}^3 W_k * \frac{1}{H_k} * Z_k$$

Then, through multiplying the K^{th} objective weight, achieved from AHP

method in pervious part, by normalized objective function (achieved from multiplying Z_k by $\frac{1}{H_k}$) and their linear addition, the normalized combined objective function (Z) is achieved:

After exchanging the multiple objective to a utility function. Multi-objective linear programming (MOLP), is exchanged to a single-objective linear programming.

4 RESULT

After interviews with senior managers and experts of organization, a need for thorough clustering of the customers in overall organizational level was felt for better recognition of the customers and making decisions according to the needs of each cluster of the customers. From the beginning phase of developing organization support, the customers, based on their geographical location, are divided into three categories:

- ❖ Customers in West Tehran
- ❖ Customers in East Tehran
- ❖ Customers in North Tehran

This has been due to the importance of the geographical locations of the customer for personal visit and servicing. Contemporary to technology developments and the ease of possibility for remote servicing, such clustering is replaced by clustering the customers on the basis of a criterion. Such single criterion belongs to the industrial customers, and it has five categories:

- ❖ Petrochemical, pharmaceutical, oil & gas and cement industries (major industries)
- ❖ Service industries and banks
- ❖ Governments customer
- ❖ Manufacturing industries
- ❖ Construction industry

Since the mentioned single criterion was not sufficient for customer clustering, and there were many similarities among the clusters, and in some cases, inevitably, the customers were moving from one cluster to the other, there was a need for a more thorough division.

Compiling Customers data

In this stage of the study, the data is purified so that the invaluable data is set aside. Therefore, the customers whose information does not exist in the company or it is incomplete, or, due to the security reasons, their personal information will not be disclosed, will be removed from entering the research process. The products and the services with particular aspects that belong to one or more limited customers and cannot be repeated, will be deleted.

Accordingly, from among 2500 customers whose information was updated and available in organization data store, after purgation, 1000 customers having complete information remain. Since presenting the customers' personal information is forbidden for the organization, the information must be set in a way that does not disclose customers' information. Accordingly, the customers' information, after editing and purging, will be

created as an ID for each customer.

Determining criteria

More in this study, determining the criteria needed for clustering is requires; for this, through interviews with experts, the following indicators are achieved:

- The size of database
- Geographic dispersion
- Database volume
- Number of users
- Type of industry
- Work experience with all group customers
- The size of an organization compared to another one in the same group
- The number of purchased systems
- The volume of support contracts amount of customers in 2013
- The percentage of satisfaction per each customer
- The type of customer's support contract
- The duration of received services by each customer from March 21st to June 20th in 2013 (3 months)
- Contract payments procedure (installments/ cash)
- Times of purchase
- Separation of product areas
- If the customers hold a brand

In this stage, to determine proper criteria, the clustering was first performed through the criteria raised by experts, as a test for few numbers of customers. In this test which was performed by K-mean method, and R software, the different Ks were analyzed. At last, among mentioned criteria, through the managers' consult and experts' ideas, the following criteria were chosen for compiling customers' information:

- The size of an organization compared to another one in the same group
- The percentage of satisfaction per each customer
- Work experience with all group customers
- The volume of support contracts amount of customers in 2013
- Geographic dispersion
- The type of customer's support contract
- The duration of received services by each customer from March 21st to June 20th in 2013 (3 months)
- The number of purchased systems

To do this, the information of 2500 customers available in information store was considered. To perform data analysis operations and clustering, the compiled data must be purging. To achieve this objective, we neglect the customers whose information is either incomplete or useless in clustering. As an instance, the information of 5 customers randomly is presented in table 1-4.

Table 1-4, Customers' information

S-QTY	TIME	CON-TYPE	POSITION	AMOUNT	RECENCY	SAT-PERC	SIZE	NO
3	575	Unlimited package	Tehran	86940000	2005	63	large	1
10	1882	morning and evening package	Tehran	156288000	2002	61.9	large	234
8	1013	morning and evening package	Tehran	118560000	2004	70	large	456
3	829	Base	Tehran	4200000	2008	58	small	678
11	3616	Unlimited package	Tehran	228432000	2011	61	large	899

After purgation operation, the data of approximately 1000 customers are prepared for the next stage. The frequency of selected criteria is given in the following graph.

Clustering the customers based on selected criteria and methods

Maybe clustering solely based on variables seem correct, however, it cannot respond the diverse requirements of different industries. The potential of using one partitioning model, such as RFM model that evaluates three variables of novelty, number of transactions, and average balance per customer, is based on the different effects of variables in clustering, and this is achieved through the interference of the experts of different fields in variables weights. As a result, the raw data resulted from recording customer information in a specific period of time, after integrating the weights derived from experts' opinion, will be prepared for clustering.

Normalizing the criteria

For normalizing the information relevant to the criteria, we make use of Fuzzy normalization. In this method, according to the following formula, the information is normalized. If the criterion possesses positive aspect, we use the formula below:

$$x' = (x - x^s) / (x^l - x^s)$$

and if the criterion possesses negative aspect:

$$x' = (x^l - x) / (x^l - x^s)$$

And according to mentioned formulas:

The biggest amount: x^l

The least amount: x^s

Using above equations, the criteria are normalized, and range between 0 and 1. Table 4-2 illustrates a sample of normalized data:

Table 4-2, Decision making Matrix

S-QTY	TIME	CON-TYPE	POSITION	AMOUNT	RECENCY	SAT-PERC	SIZE	NO
0.01	0.07	1	0.08	0.02	0.7	0.55	1	1
0.03	0.24	1	0.08	0.04	1	0.54	1	234
0.03	0.13	1	0.08	0.03	0.08	0.64	0.50	456
0	0.11	0	0.08	0	0.4	0.49	0	678
0.04	0.47	1	0.08	0.06	0.1	0.52	1	899

Measuring the weight of clustering criteria through AHP method

Since not all the criteria have equal values, to create a cluster nearer to reality, weights must be determined for the criteria.

As it was presented in the literature review, such weights are achieved AHP method by dispersing questionnaire among experts.

Table 4-3, Binary Comparison Matrix

Matrix	SIZE	SAT-PERC	RE-CENCY	AMOUNT	POSITION	CON-TYPE	TIME	S-QTY
SIZE	1	0.33	4.00	0.20	5.00	2.00	0.25	0.33
SAT-PERC	3.00	1.00	4.00	2.00	5.00	4.00	2.00	2.00
RECENCY	0.25	0.25	1.00	0.20	1.00	3.00	0.25	0.20
AMOUNT	5.00	0.50	5.00	1.00	7.00	6.00	3.00	2.00
POSITION	0.20	0.20	1.00	0.14	1.00	1.00	0.20	0.20
CON-TYPE	0.50	0.25	0.33	0.17	1.00	1.00	0.20	0.25
TIME	4.00	0.50	4.00	0.33	5.00	5.00	1.00	0.50
S-QTY	3.00	0.50	5.00	0.50	5.00	4.00	2.00	1.00
Sum	16.95	3.53	24.33	4.54	30.00	26.00	8.90	6.48

Next, it turns to calculate the relative weights of criteria. Determining the weights of decision elements compared to each other is performed through a collection of numerical calculations done by data of paired comparison matrix. Next stage, the required calculations for determining the priority of each one of decision elements are made. For this, we add up the numbers in each column of the matrix of paired comparison, and then, we divide the numbers by the sum of the numbers. The new achieved matrix is called normalized comparison matrix. Table 4-4 indicates such matrix:

Table 4-4, normalized comparison matrix

Matrix	SIZE	SAT-PERC	RE-CENCY	AMOUNT	POSITION	CON-TYPE	TIME	S-QTY
SIZE	0.058997	0.09434	0.164384	0.044025	0.166667	0.076923	0.02809	0.051414
SAT-PERC	0.176991	0.283019	0.164384	0.440252	0.166667	0.153846	0.224719	0.308483
RE-CENCY	0.014749	0.070755	0.041096	0.044025	0.033333	0.115385	0.02809	0.030848
AMOUNT	0.294985	0.141509	0.205479	0.220126	0.233333	0.230769	0.337079	0.308483
POSITION	0.011799	0.056604	0.041096	0.031447	0.033333	0.038462	0.022472	0.030848
CON-TYPE	0.029499	0.070755	0.013699	0.036688	0.033333	0.038462	0.022472	0.03856
TIME	0.235988	0.141509	0.164384	0.073375	0.166667	0.192308	0.11236	0.077121
S-QTY	0.176991	0.141509	0.205479	0.110063	0.166667	0.153846	0.224719	0.154242

Calculating the average of the numbers in each line of normalized comparison matrix, the relative weight of the criteria is achieved, and the result of the calculations appears in table 4-5:

Table 4-5, relative weight of criteria

SIZE	SAT-PERC	RE-CENCY	AMOUNT	POSITION	CON-TYPE	TIME	S-QTY
0.0856	0.2398	0.0473	0.2465	0.0333	0.0354	0.1455	0.1667

Preparation of Weighted data for clustering:

Since the different variables in different organizations and depending upon some particular conditions take different values, several weights are related to such criteria. Partitioning technique based on weighting is one of the best ways of partitioning in the world.

First, in this study, the variables are recognized and their amount is compiled for all options. Then, the weights of the variables are calculated through AHP method. After that, through the popular k-mean clustering algorithm, the customers are partitioned.

The weights of the variables are calculated in former stage, and now such weights must be multiplied in relevant variables so that the weighted data are prepared for clustering. An example of weighted data appears in table 4-6:

Table 4-6, weighted data

NO	SIZE	SAT-PERC	RE-CENCY	AMOUNT	POSITION	CON-TYPE	TIME	S-QTY
1	0.222985	0.390048	0.319301	0.367996	0.336938	0.30488	0.213696	0.298075
234	0.274673	0.43728	0.366018	0.394089	0.381224	0.377933	0.249777	0.325512
456	0.229398	0.386319	0.270253	0.396667	0.285841	0.306946	0.235256	0.312031
678	0.119724	0.187798	0.118713	0.245264	0.116363	0.14604	0.136258	0.175521
899	0.318109	0.404485	0.368234	0.369202	0.391306	0.320009	0.253777	0.317105

The next step is to cluster in accordance with new variables. This takes place by clustering technique.

On K-Mean Clustering Method

Unlike classification method, there are no pre-defined groups while to carry out clustering method. In other words, there are no predefined classes in clustering process. Thus, the main concern in the clustering process is about partitioning a given data set into groups (clusters) such that the data points in a cluster are more similar to each other than points in different clusters. On the other hand, classification is a procedure of assigning a data item to a predefined set of categories, whereas clustering is one of the most useful tasks in data

mining process in which there no predefined classes and groups (Han and Kimber, 2001).

In this paper, we have used R Statistical Software to cluster customers. This software uses K-Means, a commonly used algorithm, to cluster customers. In other words, partitioning process begins based on weighted value entries. More specifically, the algorithm begins by initializing a set of c cluster centers. Then, it assigns each object of the dataset to the cluster whose center is the nearest, and re-computes the centers. The process continues until the centers of the clusters stop changing.

The output of partitioning process is to change a set of 1000 customers to a set of 4 clusters of customers. Of course, for different K-values, different results will be achieved. As mentioned before, the optimum K-value will be achieved based on the results obtained from different clustering and meaningful inter-cluster comparison. In any case, the optimum K-value has been assigned 4 in current research.

The R software usually determines the centers of clusters (original clusters) in a randomized basis. Since the centers of original clusters obtained from individual K-Mean clustering process can be different and various, clusters obtained from clustering process are not unique. In K-Mean algorithm, different and various distance criteria can be used to cluster data set based on data type to be going to be clustered. The aim of K-Means clustering is the optimization of an objective function that is described by the equation:

```
Means(x, centers, intermix=10, start=1, algorithm=c("hartigan-wang","loyd","forgy","macquenn"))
```

Where,

x is numerical matrix of data or anything can replace a matrix.

Centers determine the numbers of clusters.

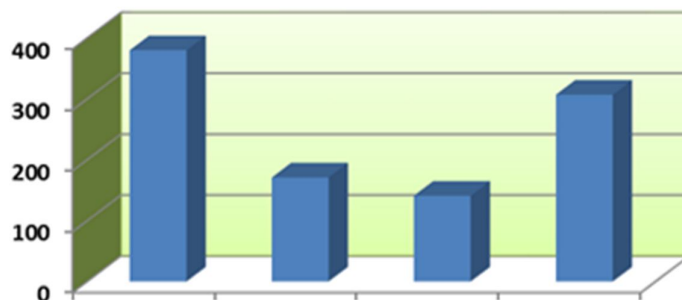
Iter.max is the maximum number of allowable iteration.

N start indicates the numbers of clusters which should be selected in a randomized basis whenever centers are numerical values. The function uses Hurting and Vang algorithm in a default way, otherwise the title of related algorithm should be mentioned. Notepad and Excel applications needed to run R software. To activate the option of data reading from Excel, a package called XL speedwriter should be downloaded and set up. If there are files in Excel format, they can be saved in txt format and then the order read. Table ("fileName.txt") is used.

The result or output of clustering process demonstrates some items as cluster vectors, cluster centers, the number of members in each cluster, and a set of optional information on variances of cluster centers.

The partitioning process, as we mentioned before, and its procedures are shown in figure 4-1.

Figure 4-1 the Results from Clustering Process



Analyzing Client Clusters

As shown in the figure 4-1, a set of 1000 customers is partitioned into a set of 4 clusters of customers such that first, second, third, and fourth clusters (clusters 1, 2, 3, and 4) are composed of 308, 141, 171, and 180 customers respectively. As shown in related tables, all of customers have been analyzed in terms of related values of criteria and then based on these values; the status of each client can be analyzed compared to mean value related to each cluster. In this stage, these four clusters should be named considering the striking similarities between customers found in each cluster and in terms of cluster centers. It would be reasonable to expect customers found in a cluster to show similar behaviors and reactions toward decision-making process. Based on the average parameters used to cluster data set, all clusters can be named in a creative way. To do this, we presented a number of sales managers and sales agents of the organization with the results obtained from clustering process and the average criteria for each cluster and asked them to label each cluster. Table 4-7 has been compiled based on the author's suggestion with a number of elites' comments.

Table 4-7 the Results from Clustering Process

clusters	size	satisfaction	history	price	Place /location	Types of contract	time	System numbers	Cluster labels
cluster 1	1.25	48.50	5.60	32606094.85	0.87	1.44	234.02	2.65	small
cluster 2	2.89	76.22	7.76	267936066.39	1.34	2.69	2021.18	16.13	large/big
cluster 3	2.21	74.73	7.04	90272386.45	1.09	2.13	631.27	5.99	medium/ large
cluster 4	1.40	69.03	6.22	40154995.69	0.92	1.64	288.04	3.04	medium/ small

A Mathematical Modeling of How to Make the Allocation of Products for Client Clusters

Current stage is called allocation process in which we are trying to construct a mathematical model for the allocation of products to client clusters based on pre-determined goals and objectives.

Defining Specific Goals of Allocation Process

In current research, three specific goals have been defined that should be achieved within allocation process. They are as follows:

- guaranteed and secure income and revenue from supporting activities
- the relationship between the size and the amount of a product and a cluster
- revenue from learning process and educational initiatives

The information on guaranteed income and revenue from supporting activities are gained from the research on price ranges for types of customers and customers. These price ranges are fixed based on the size of each client that is a combination of a customer's purchase account, his or her purchase power, and his or her financial flow or status.

On the relationship between the size and the amount of a product and a cluster, a 10-point rating scale (0-10) has been used in the research. The ratio of the numbers of customers having an individual product in a cluster to total customers found in that cluster has been set as a criterion for the relationship between the size and the amount of a product and a cluster.

Revenue from learning process depends on the numbers of users that need learning to use products. First and third parameters are classified as secret documents and they are retrieved from the organization documents. On second parameter, it can be calculated and attained through a method of ratio calculation.

With regard to three specific goals and objectives, it is necessary to design a multi-objective model for the allocation process based on the mathematical concept of binary digit (0, 1). In other words, the variables of decision process are binary in proposed model that may be defined as follows:

For every product i and every cluster j ,

We have,

$i = \{1, 2, \dots, n\}$

And

$j = \{1, 2, \dots, m\}$

If the product i can be allocated to the cluster j , the value of variable equals 1 and otherwise it equals zero.

The product numbers multiplied by cluster numbers is equal to decision process variables.

Table 4-8 the Information on Product Price

	First priority				second priority				third priority			
	Cluster number	Base price	Proportional relationship	Revenue from learning process	Cluster number	Base price	Proportional relationship	Revenue from learning process	Cluster number	Base price	Proportional relationship	Revenue from learning process
Product no.1	2	12800	10	900	3	12000	8	650	1	10500	2	300
Product no.2	2	6300	10	1500	3	6500	10	900	4	6600	9	500
Product no.3	2	7700	10	400	3	7800	7	300	4	7900	5	200
Product no.4	3	1700	10	150	4	1700	7	100	2	1700	7	200
Product no.5	2	14500	10	240	3	14500	6	180	4	14500	4	100

Product no.6	2	5000	10	400	3	5200	6	300	4	5200	4	200
Product no.7	2	7300	10	1000	3	7200	7	700	4	7200	5	400
Product no.8	2	8600	9	450	3	8600	7	300	1	8600	5	100
Product no.9	2	8300	10	1000	3	8300	10	800	4	8300	7	400
Product No.10	2	2080	9	200	3	2000	9	200	4	2100	4	100

The specific objectives of the mentioned model are as follows: maximization of the revenue from establishment and supporting activities, maximization of the relationship between the size and the amount of a product and a cluster, and maximization of learning revenues.

Therefore, we are faced with a multi-objective situation. In other words, there are many objectives that we are trying to maximize all of them. If we assume our desired objectives as z_1, z_2, \dots, z_k , objective functions will be as follows:

$$\begin{aligned} \text{MAX } Z_1 &= \sum_{i=1}^n \sum_{j=1}^m C_{ij1} X_{ij} \\ \text{MAX } Z_2 &= \sum_{i=1}^n \sum_{j=1}^m C_{ij2} X_{ij} \\ \text{MAX } Z_3 &= \sum_{i=1}^n \sum_{j=1}^m C_{ij3} X_{ij} \end{aligned}$$

Here, the coefficients of revenue from establishment and supporting activities C_{ij1} , the relationship C_{ij2} , and learning revenues C_{ij3} are orderly in the 500000-34000000, 1-10, and 0-1800000

Constraints:

In general, there are two major constraints on the above mentioned model. They are as follows:

- It is usable on limited numbers of products per cluster. In other words, the K-mean algorithm is relatively scalable and efficient in processing large data sets.

$$L \leq \sum_j X_{ij} \leq U \quad 3 \dots, n$$

- It is faced with constraints on the value of decision-making variables and it can select them only from a set of binary values (0, 1).

$$X_{ij} = 0 \text{ or } 1, 2, \dots, n \quad 1, \dots, m$$

Based on what mentioned above, objective function is as follows:

$$\text{Max } Z_1 = 12800000 X_{12} + 12000000 X_{31} + 10500000 X_{11} + \dots$$

$$\text{Max } Z_2 = 10 X_{12} + 8 X_{31} + 2 X_{11} + 10 X_{22} + 10 X_{23} + \dots$$

$$\text{Max } Z_3 = 900000 X_{12} + 650000 X_{31} + 300000 X_{11} + \dots$$

And also, model constraints are as follows:

$$\begin{aligned} \sum_{j=1} X_{ij} &\leq 3 \quad i = 1, 2, 3 \dots, n \\ \sum_{j=2} X_{ij} &\leq 20 \quad i = 01, 2, 3 \dots, n \\ \sum_{j=3} X_{ij} &\leq 7 \quad i = 1, 2, 3 \dots, n \\ \sum_{j=4} X_{ij} &\leq 4 \quad i = 1, 2, 3 \dots, n \end{aligned}$$

And 0, 1 constraint:

$$X_{ij} = 0 \text{ or } 1 \quad i = 1, 2, \dots, n \quad j = 1, \dots, m$$

Determining the weight of the criteria

This section is devoted to set the significance of objective functions and to define its related coefficients. To

do this, group-based hierarchical analysis method has been used to set the weight of each objective and define its coefficient. Therefore, a number of agents are sampled and then we ask them to compare objectives in pairs, then following geometric mean value can be used to combine their comments.

$$a_{ij} = \sqrt[N]{\prod_{k=1}^N a_{ijk}}$$

In which, a_{ijk} is the numerical value of compared objectives i & j by individual k ; N refers to the number of the respondents and a_{ij} is the geometric mean. Then, based on the geometric mean value that we have attained, pair wise comparative matrix and the weight of each objective (w) will be calculated. The data on the matrixes of pair wise comparisons and weight coefficients of each objective are shown in table 4-9.

Table 4-9, binary comparisons

objectives	Pair wise comparisons			Objective weights
	Basic price	Proportional relationship	Learning revenues	
Basic price	1.000	0.500	0.244	0.247
Proportional relationship	2.000	1.000	2.500	0.510
Learning revenues	1.000	0.521	1.000	0.244

As shown in the table, the rate of incompatibility is $<.1$ (less than .0821) signifying that obtained weights will be reliable and valid.

Model Constructing Without Using Weighted Average Method

In order to combine three objectives mentioned before, every function should be divided by the upper limit of k th spectrum and be multiplied by its weight:

$$Z = \sum_{k=1}^3 W_k * \frac{1}{H_k} * Z_k$$

Based on the calculations mentioned in pervious section, objective weights are calculated through GAHP method as follows:

$$w_1 = 0.247$$

$$w_2 = 0.510$$

$$w_3 = 0.244$$

Also, the upper limits are calculated as follows:

$$H_1 = 34000$$

$$H_2 = 10$$

$$H_3 = 1800$$

Then,

Considering functions and employing the above formula, a normalized combined function is obtained as follows:

$$= .247 * \frac{1}{34000} * Z_1 + .51 * \frac{1}{10} * Z_2 + .244 * \frac{1}{1800} * Z_3$$

$$= .247 * Z_1 + .51 * Z_2 + .244 * Z_3$$

And according to the Z_1 , Z_2 , Z_3 , functions and using above formula, the normative combined function is achieved:

$$Max Z = .2415 * X_{12} + .1943 * X_{13} + .0729 * X_{11} + \dots + .1259 * X_{314}$$

And also, we have following mathematical model:

$$Max Z = .2415 * X_{12} + .1943 * X_{13} + .0729 * X_{11} + \dots + .1259 * X_{314}$$

$$\sum_{j=1} X_{ij} \leq 3 \quad i = 1, 2, 3, \dots, n$$

$$\sum_{j=2} X_{ij} \leq 20 \quad i = 1, 2, 3, \dots, n$$

$$\sum_{j=3} X_{ij} \leq 7 \quad i = 1, 2, 3, \dots, n$$

$$\sum_{j=4} X_{ij} \leq 4 \quad i = 1, 2, 3, \dots, n$$

$$X_{ij} = 0 \cup 1 \quad i = 1, 2, \dots, n \quad j = 1, \dots, m$$

Now, there is a model with one objective function that can be solved using current software in a simple way. A partial result obtained from allocation process is shown in table 4-10.

Table 4-10 A Partial Result Obtained from Allocation Process

Solution	Cluster 1	Cluster 2	Cluster 3	Cluster 4
P 1		*		
P 2	*	*	*	*
P 3		*		
P 4				
P 5		*		
P 6		*		
P 7		*		
P 8		*		
P 9		*	*	

As shown in the table, product 1 has been allocated to cluster 2 and product 2 allocated to all clusters. Similarly, other products have been allocated. As shown in the table, a large number of products have been allocated to cluster 2 and this is absolutely natural and normal considering that the center of cluster 2 is by far best based on all clustering criteria. On the other hand, some products have not been allocated to any of these clusters because of low proportional relationship and low sales revenues and low revenues from learning process. It is important to mention that the degree of relationship between clients and a cluster has been calculated through customers' previous purchases. Thus, it seems that the degree of relationship between customers and a cluster, among other objectives of allocation process, carries maximum weight being equal to the sum of two other indices.

On the other hand, some of those products which have not been allocated to any of these four clusters are among those products having lower sales price and lower sales volume. One of the reasons of the situation is that customers usually show little inclination to purchase a product over the time and in a long-term period. Thus, considering that a customer is not ready to purchase a product in long-term, its price will possibly be lowered and this cycle of events continually repeats itself.

CONCLUSION

As mentioned before, the customers were first divided and partitioned into four clusters and after that thirty one identified products were allocated to these four clusters in terms of three mentioned criteria. Similarly, following clusters are identified (see table 5-1).

Table 5-1, the results of the analysis

clusters	Size	satisfaction	history	price	Place/ location	Types of contract	time	System numbers	Cluster labels
cluster 1	1.25	48.50	5.60	32606094.85	0.87	1.44	234.02	2.65	small
cluster 2	2.89	76.22	7.76	267936066.39	1.34	2.69	2021.18	16.13	large/ big
cluster 3	2.21	74.73	7.04	90272386.45	1.09	2.13	631.27	5.99	medium/ large
cluster 4	1.40	69.03	6.22	40154995.69	0.92	1.64	288.04	3.04	medium/ small

Also, a partial result obtained from allocation process is shown in table 5-2.

Table 5-2, results of allocation

Solution	Cluster 1	Cluster 2	Cluster 3	Cluster 4
P 1		*		
P 2	*	*	*	*
P 3		*		
P 4				
P 5		*		
P 6		*		
P 7		*		
P 8		*		
P 9		*	*	

As shown in the tables, out of 31 existing products, a set of 20 products has been allocated to cluster 2 and this is completely compatible with the specifications of the cluster, which includes big customers with high purchase power.

On the other hand, out of existing products, allocation process has been made for remaining clusters as follows: 20 products to cluster 2, 7 products to cluster 3, 4 products to cluster 4, and 3 products to cluster 1. As a matter of fact, analyzing the results from the model used in this research reveals that all products been allocated to clusters 4 and 1 have been allocated to clusters 2 and 3. The reason is that clusters composed of large customers or medium/large customers need a large number of products and also they have high purchase power. On product no. 20, it is because of the nature of the product that it has merely been allocated to clusters 1 and 4. It is because of such a specific nature that the product is used to transmit information from different and various locations and because of this, whenever there are a large number of users and many different locations, it is better to employ other product than product no.20. As mentioned above, the degree of relationship between clients and a cluster has been calculated through customers' pervious purchases. Thus, it seems that the degree of relationship between customers and a cluster, among other objectives of allocation process, carries maximum weight being equal to the sum of two other indices. On the other hand, some of those products which have not been allocated to any of these four clusters are among those products having lower sales price and lower sales volume.

Apart from the results on product allocations, customer clustering method can be employed in sensitivity analysis of different sectors of market. In other words, customer clustering method is usually used to measure the sensitivity of market to a specific product in a cluster. For instance, a company may do some effective marketing plan according to the results of customer clustering method. On the other hand, whenever a target product experiences lower sales volume, employing customer clustering method can be effective. On such a case, considering that customers in a similar cluster usually behave similarly, the real cause for lower sales volume of a target product can be determined in terms of market strategies of the organization. A possible solution to the problem of lower sales volume in a specific cluster may be a growing competition among rivals and competitors in production and distribution.

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