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Mathematical model of a multi-loop network of gas pipelines at various modes of current

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Abstract. A method of hydraulic calculation of a multi-loop network of gas pipelines based on Kirchhoff's laws is offered. As completing relations, the formula for the change of pressure on elementary sites of the horizontal gas pipe, received on the basis of Leybenzon's generalized formula of resistance is used.

Keywords: hydraulic calculations; Leybenzon's generalized formula of resistance; Kirchhoff's laws; multi-loop pipeline network.

The wide use of pipeline networks in various branches of a national economy and social sphere is due to their profitability and ecological cleanliness. Application of multi-loop structures at construction of these networks promotes the increase of their reliability and reduction of working costs. The role of mathematical modeling and computing experiments in these processes is extremely important as they replace the natural experiments demanding greater resources in the form of energy, materials and specialists of various branches.

Dependence of objects on many factors, change of the problem statement depending on a stage of application in a life cycle of object (projecting, the analysis of functioning and operating management), nonlinear dependences between the basic parameters of processes, as well as development of numerical methods and the presence of Computer Parks encourage the further development of the given area of research. Including force, power or structural factors and an estimation of their role in work of a hydraulic circuit leads to new steps of development. For example, in our days, systems of central heating and the centralized heat supply work on two kinds of the working agent – water and water vapor. According to this, hydraulic calculation of separately taken linear site of a network should be done within the limits of pressure drop (for heated water) or a square of pressure (for water vapor) depending on the amount of outflow of fluid. Moreover, the value of the resistance factor, expressed usually in the form of a monomial, is established according to a roughness of section of the pipeline and Reynolds's criterion for laminar, transitive and three kinds of turbulent modes of current. For each of these modes the analogue of Kirchhoff's second law should consider different degrees of unknowns. Meanwhile, in an electric circuit of a direct current the potential drops on length of a wire has linear connection with the value of electric current, as it served at the formation of Kirchhoff's second law (the first hypothesis). In this sense, laws of resistance serve as the fifth axiom of Euclidean geometry and it opens new directions for research of simultaneous equations with different degrees of unknowns.

Taking into account the compressibility of ideal gas due to Clapeyron equations or real gas with the super compressibility coefficient of environment the analogue of Kirchhoff's second law shows a linear dependence between difference of squares of pressures and square of the gas outflow at realization of the square law of resistance. Subsequently, the system of the equations becomes complicated even more.

In the given work we will analyze Leybenzon's generalized formula of resistance [1]. The advantages of this formula are the account of all five modes of current, whereas in many other approximated formulas not all modes of current are considered, and its applicability for incompressible and compressible environments in quasi-one dimensional representation. And the account of all modes of current during hydraulic calculations is important, as the pipeline network can work both with small loading, and with an overload.

In opinion of [1, 2] and some other authors, at pipeline transportation of liquids (waters, oil and its derivatives) the most observed modes of current are laminar (where $\mathrm{Re} = wD/\nu < 2200$) and transitive (where $2200 \le \mathrm{Re} \le 4000$), and also a smooth mode of a turbulent flow of a rough surface when the equivalent roughness of the moistened surface of the pipeline has $\mathrm{Re}\, k/D < 10$ condition. For the gas mixture currents, mainly, turbulent modes of current are observed, where Reynolds's criterion has greater values ($\mathrm{Re} > 4000$). These are smooth ($\mathrm{Re}\, k/D < 10$), mixed ($10 \le \mathrm{Re}\, k/D \le 158$) and developed ($\mathrm{Re}\, k/D > 158$) modes of a rough surface turbulent flow. At such classification of modes of current the resistance factor is approximated by this unique formula

$$\lambda = \varsigma \left(k/D\right)^{\theta} \mathrm{Re}^{\varphi} \tag{1}$$

with piecewise constant values of parameters ζ, θ, φ.

As piecewise approximation formulas it is possible to use Stocks's formulas ($\zeta = 64,\ \theta = 0,\ \varphi = -1$) for a laminar current mode, Zaychenko's formulas ($\zeta = 0.0025,\ \theta = 0,\ \varphi = 1/3$) for transitive mode, Blasius's formulas ($\zeta = 0.3164,\ \theta = 0,\ \varphi = -1/4$) for a smooth mode of roughness flow, Leybenzon's formulas ($\zeta = 10^{-0.627},\ \theta = 0.127,\ \varphi = -0.123$) for mixed mode and Shifrinson's formulas ($\zeta = 0.11,\ \theta = 0.25,\ \varphi = 0$) for developed mode.

Parameters of a stationary isothermal mode of gas movement in a linear site of a gas pipeline with the length of *l* and diameter of *D* can be found from the solution of this quasi-one dimensional equation system [1]:

$$\frac{dp}{\rho} + \frac{\lambda w^2}{2}\frac{dx}{D} = 0, \quad M = \rho w F, \quad p = Z \rho R T \tag{2}$$

here, *M* – loss of mass, *T* - absolute temperature, *R* - gas constant, *Z* – super compressibility coefficient of gas and $F = \pi D^2/4$ - the cross-sectional area of gas pipe are considered to be constant.

Let the site be characterized by the mass loss of *M* (in kg/s) and entrance pressure of p_H (in Pa). Having represented Reynolds's criterion $\mathrm{Re} = \frac{wD}{\nu} = \frac{4M}{\rho \pi D \nu} = \frac{4MZRT}{\pi D \nu}\frac{1}{\eth}$ and variables through static pressure, from system (2) we can work out the equation of $\frac{dp^{2+\varphi}}{dx} = -b\,M^{2+\varphi}$, which has a solution of

$$p_K^{2+\varphi} = p_H^{2+\varphi} - bM^{2+\varphi} l\,, \tag{3}$$

where $b = \frac{(2+\varphi)2^{3+2\varphi}\zeta\, k^{\theta}\left(ZRT\right)^{1+\varphi}}{\nu^{\varphi}\pi^{2+\varphi}D^{5+\theta+\varphi}}$.

In practice, calculation of gas outflow is made commercially, i.e. led to standard conditions T_{st}=293.15 *K* and p_{st}=101325 *Pa*. In the formulas resulted above, according to relation of $M = Q\,p_{st}/\left(RT_{st}\right)$, it is possible to get the commercial gas outflow Q. In our mathematical model of a multi-loop gas pipelines we are based on the materials given in [3] those are used for a heat supply system.

Calculation of a multi-loop network of a heat supply, according to [3], is made on the basis of set of the following three groups of the equations:

$$Ax = Q, \quad By = 0, \quad y + H = SXx. \tag{4}$$

Here, column vectors of $x = (x_1,\ x_2, ...,\ x_n)^T$, $y = (y_1,\ y_2, ...,\ y_m)^T$, $Q = (Q_1,\ Q_2, ...,\ Q_m)^T$ and $H = (H_1,\ H_2, ...,\ H_m)^T$ represent outflow and pressure differences in arcs (branches and chords), intensity of selection and pressures on tops. The last from these vectors considers the change of leveling heights of the pipeline axis on a site, and the pressure, produced by a supercharger if the junction represents a pump. Matrices A and B represent a full matrix of incidences and a matrix of loops. In last group of the equations diagonal matrices of resistance and absolute values of outflow amount are used:

$$S = \begin{bmatrix} s_1 & 0 & ... & 0 \\ 0 & s_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & s_n \end{bmatrix}, \quad X = \begin{bmatrix} |x_1| & 0 & ... & 0 \\ 0 & |x_2| & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & |x_n| \end{bmatrix}.$$

In the literature these groups of the equations can be named as representations of Kirchhoff's laws and relations completing them. Solution uniqueness of the equations set (4) for incompressible liquids is provided by removing one equation, corresponding to main junction - to one with known pressure from the first group of the equation, and replacement of last two groups by unique group of $By + BH = BSXx$. Lobachev-Cross, Vyhandu and some other methods are developed for the solution of the system of linear and nonlinear equations like these.

For adaptation of the presented material to compressible gas it is necessary to consider the following features.

First, at building of a column vector $H = (H_1,\ H_2, ...,\ H_m)^T$ gravitational force is not considered. The reason is that, the built of exponential relationship at the solution of [4] that leads to further complication of the equations system. Besides the density of gas in real conditions of transportation is much more less (thousand times) than the density of liquids by virtue of what the necessity of the account of the given factor appears only at greater differences at leveling heights of a gas pipeline axis.

Secondly, according to (3), column vector y expresses not the difference of pressure at the ends of a site, but a difference of $2+\varphi$-th powers (exponents) of pressure at the ends. Accordingly at drawing up of a column vector it is necessary to consider the given fact.

Thirdly, in a diagonal matrix of S it is necessary to take into account b_i, l_i and the scale multiplier, and matrix X should be replaced by

$$X = \begin{bmatrix} \left|x_1^{1+\varphi}\right| & 0 & \dots & 0 \\ 0 & \left|x_2^{1+\varphi}\right| & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \left|x_n^{1+\varphi}\right| \end{bmatrix}.$$

This developed set of the equations represents the mathematical model of a multi-loop network of gas pipelines at various modes of current. The possibility of formation of different modes of current on different parts of a network is not excluded. In such cases, the values of parameters ζ, θ, φ are chosen according to parameters of current on each part.

If the developed mode of a flow of turbulent stream through roughness (i.e. at square law of resistance) is established, for the solution of the equations system it is possible to use known solution methods (e.g. Lobachev-Cross and others) of current distribution problems. In this case, in the equations, concerning the case of an incompressible liquid, changes occur at only pressure differences which are replaced with differences of the squares of pressure.

For the laminar mode of current the offered system of the equations becomes simpler up to a level of linear equations system. The only difference with the case of an electric circuit of a galvanic current will be the additional account of intensities of outflows at the junctions of a gas pipelines network. As computing experiment is down on the track, we have to accept this fact as the proof of reliability of the offered model.

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Аннотация. В рамках законов Кихгофа предложен способ гидравлического расчета многоконтурной сети газопроводов. В качестве замыкающих соотношений использована формула для изменения давления на элементарных участках горизонтального газопровода, полученная на основе обобщенной формулы сопротивления Лейбензона.

Ключевые слова: гидравлический расчет; обобщенная формула сопротивления Лейбензона; законы Кирхгофа; многоконтурная сеть трубопроводов.