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PRE-SERVICE PHYSICS TEACHERS' CONCEPT MAPS REPRESENTING RELATIONAL STRUCTURE OF PHYSICS CONCEPTS

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Abstract. *In this study, a parsimonious set of quantities is developed to characterise and evaluate the quality of concept maps. It is shown here how such a set can be based on measurements of the connectivity, clustering and cohesion of concept maps. A structural model of the data is introduced so that concept maps can be evaluated and classified by using only two quantities called here richness of content (D) and quality of structure (q). The model is used in monitoring changes in the concept maps (N=33) consisting of two sets of maps (initial and final ones), made before and after teaching. Such an approach gives access to monitor changes of the concept maps during a teaching sequence.*

Key words: *knowledge organisation, learning process, physics concept maps.*

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Introduction

Concept maps and other similar graphical representations, where concepts are linked to each other, are widely used in the teaching and learning of physics (Ingeç, 2008; van Zele, Lenaerts &Wieme, 2004; Yin, Vanides, Ruiz-Primo, Ayala & Shavelson, 2005), chemistry (Nicoll, Francisco & Nakhleh, 2001) and biology (Kinchin, De-Leij & Hay, 2005; Kinchin, Hay & Adams, 2000), as well as in different levels of education from the elementary to advanced levels. Concept maps can also been used as tools of assessment and evaluation and as tools of monitoring development during learning (Ruiz-Primo & Shavelson, 1996; van Zele et al., 2004; Yin et al., 2005).

The structure of concept maps indicating of the quality of students' knowledge has been the focus of many studies. Attention is then paid to how concepts are interconnected locally or to what the "semantic fields" are that are provided by these connections. Consequently, counting of the links can be used to quantitative classification and the scoring of concept maps (McClure, Sonak & Suen, 1999; Nicoll et al., 2001; Novak & Gowin, 1984; Ruiz-Primo & Shavelson, 1996). Recently it has been suggested that both good understanding and the high quality of students' knowledge is reflected as structures, which are tightly connected sets of several nodes and web-like (Derbentseva, Safayeni & Cañas, 2007; Ingeç, 2008; Liu, 2004; Kinchin et al., 2000, Kinchin et al., 2005; Safayeni, Derbentseva & Cañas, 2005; Vanides, Yin, Tomita & Ruiz-Primo, 2005; van Zele et al., 2004; Yin et al., 2005). Such qualitative methods for analysing the concept maps have thus revealed that global topological features which are chain-, spoke- or web-like carry important information about the quality of knowledge represented in the maps (Kinchin et al., 2000; Kinchin et al., 2005; van Zele et al., 2004).

In this study, a new quantitative method is introduced to reveal the important structural feature of connectedness in concept

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maps and on this basis, monitor changes in the pre-service physics teachers' concept maps when maps before teaching and after teaching are compared. The key structural features identified here are related to the local interconnectedness in form of the clustering and to the global interconnectedness in form of the cohesion. Clustering (denoted by C) means a property, where concepts form small and local interconnected cluster. The cohesion (denoted by Ω) is closely related to the overall interconnectedness in large scale and navigation due to longer cycles.

The objective is to develop suitable quantities for the classification of the concept maps, so that the classification is based on the richness of content and the quality of structural features. This approach is based on the normalised structural variables C and Ω introduced here. A structural model of the data, which allows us to predict the expected deterministic increase of the clustering and cohesion, is introduced. The analysis of the variations in the sample allows then detection of individual cases which are better than expected. These quantities can be used in classifying and monitoring the changes in student's concept maps, which represent the students' ideas of the relations between physics concepts. This eventually enables the description of the structural quality (denoted by q) of the concept maps in greatly condensed form by using only one variable termed the structural quality and makes possible to monitor the students' development on organising the knowledge by using the maps (and thus supposedly also conceptual development) during the teaching.

Methodology of Research

The concept maps discussed here were constructed by following design principles where links represent either modelling procedures or procedures of quantitative experiments. The most essential difference between the sample of maps discussed here and the traditional concept maps is that, in our case, the links are not propositions but based on a more detailed description, for example, how a certain experiment or model is used to motivate the introduction of a new concept**.** A justification was required for each link, so that students were asked to write down a short description. In what follows, it is important to remember that in order to establish a link students must be able to justify the link so that it is ontologically correct (correct entities are used), epistemologically acceptable (acceptable attempt is made to justify the reasoning and logic), and methodologically plausible (experiment can be made, model involves correct mathematics). This means that every link represents knowledge, which in principle is usable for purposes of teaching, i.e. "usable knowledge". Therefore, all links can be taken as "correct" ones. Other aspects of the design principles are discussed in more detail elsewhere (Koponen & Pehkonen, 2010; Nousiainen & Koponen, 2010).

The concept maps constructed on basis of the above mentioned design principles have been applied in physics teacher education in a teacher preparation courses for third year pre-service physics teachers during two academic years 2009 and 2010. Students were asked as a course task to draw concept maps from given subject areas of physics. The goal of the course is to form a holistic overview of physics by producing concept maps and writing down descriptions about the linking procedures. Within the course "new physics" is not really taught but the focus is on the organising of concepts and binding together the pieces of knowledge students already had. Students worked on their own representation but collaboration between students were encouraged during process. The feedback from the students on use of the concept maps has been appreciative, many students noting the advantages of being able to visualise complex conceptual connections by using the concept maps.

Sample of Research

The sample of concept maps (N=33) to be analysed here were produced by third-year university students during physics teacher education courses in academic years 2009 and 2010. The courses were similar, of 7 weeks duration both. All the maps were drawn from the point of view as to how teacher students conceived the concept and laws of electromagnetism as they were introduced in the first-year university course, and how they thought the concepts and laws are connected through the aforementioned procedures. The number of the concepts were limited to n=34 most central concepts, while students were free to introduce as many links as they found necessary. An example of the concept map with n=34 nodes is shown in Figure 1. This concept map is an example of the most extensive and best ones. In these kinds of concept maps the most well connected concepts are typically linked to 3-5 other concepts. Some connecting procedures are given in Table 1 with numbering in reference to Figure 1. It should be noted that the numbering of links denotes the sequence of steps in the construction of the concept map.

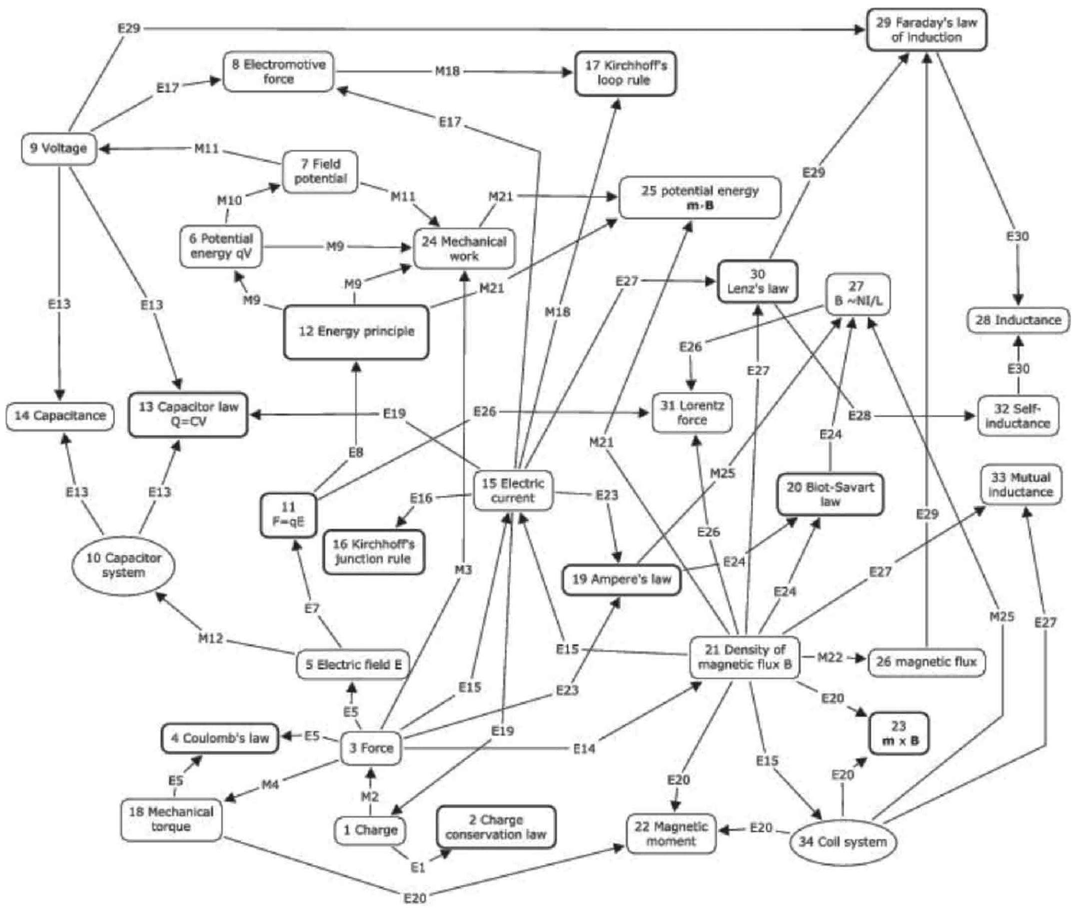

Figure 1: An example of a concept map: concepts are illustrated as boxes, laws and principles boxes with thick borders and conceptual models of a real system are shown as ellipses. Linking procedures are either models (M) or experiments (E).

The original concept maps drawn by students each have a personal graphical layout (i.e. a way to draw the map), which often makes it difficult to see important topological features or which may misguide the attention away from the relational structure to mere appearance. The visual appearance of maps was made comparable by using systematic re-drawings of the maps so that the same rules for ordering the nodes are used in all cases. In graph theory this is called embedding of the graph (i.e. here the map). Here graph-embedding method called the spring-embedding has been used, which is obtained when each link is assumed to behave like a spring and which minimises the total energy of the spring system (for details see Pemmaraju & Skiena, 2006). Examples of the maps in the spring-embedded form are shown in Figures 2 and 3 for maps with n=34 nodes, respectively.

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Table 1. Some examples of modelling and experimental procedures from the concept map illustrated in Figure 1.

Modelling Procedures M (only part)	M11: Definition of electrostatic voltage as a potential difference. M21: Magnetic potential energy identified when turning an electrical circuit in magnetic field. Interpretation through analogy to a particle's potential energy in electric field.
Experimental procedures E (only part)	E5: Coulomb's experiment and quantification electrostatic force. E13: Capacitance and capacitor law defined using electrostatic generator E23: Ampere's experiment and quantification of current and force between current carrying wires.

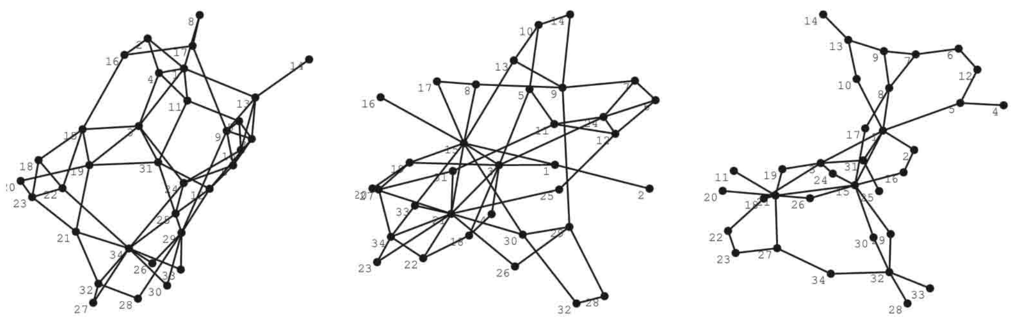

Figure 2: Three examples of spring embedded concept maps with different level of clustering show that somewhat similar structure can be found in each case.

The structural features of the maps – clustering-like interconnectedness locally, cyclical interconnectedness in global scale and the overall web-like character – displayed in Figure 2 are those essentially brought forward by qualitative analysis of concept maps (c.f. Kinchin et al., 2000; Kinchin et al., 2005; van Zele et al., 2004). All these qualitative notions based on visual inspection will now be operationalised and made quantitative.

The Method of Analysis

The structure of the concept maps can be analysed as network structures the analysis of which takes place in three steps:

1. The qualitative features of interest are transformed in a form amenable to quantitative analysis. This part is carried out by using the graph-theoretical approach.
2. A quantitative analysis of measured properties is carried out. The purpose of quantitative analysis is to reveal those measurable properties and their systematic dependencies which allow us to classify the maps on the basis of the richness of their content and the quality of their structure.
3. On the basis of the classification a quantitative measure for the structural quality is introduced.

The qualitative structural features of primary interest are clustering (local connectedness) and cohesion (global connectedness). Clustering means here a property by which concepts tend to form an interconnected local cluster. Cohesion means a connections provided by larger clusters, which provide paths to navigate in structure and which hold the structure together. These features can be operationalised by defining the following variables for each node *k* locally (Kolaczyk, 2009):

1. The degree D_k is the number of links connected to a given node *k*. The degree D_k contains the incoming and outgoing links. Therefore, it directly measures the connectivity of the node.
2. The clustering coefficient C_k measures the local interconnectedness of concepts through as the ratio of triangles to the all triply connected neighbours around a given concept. C_k obtains values between 0 and 1, where 1 corresponds to the maximal number of triangles obtained by connecting all neighbours. The clustering is a kind of "family tie" measure.
3. The cohesion Ω_k is defined so that it will be related to importance of longer cycles. The main interest is in longer cycles consisting of combinations based on triangles, because they provide cross-links between nodes, i.e. short-cuts. These are essentially odd cycles, i.e. 3-cycles, 5-cycles etc. Therefore, cohesion is defined as the ratio of the number of odd-cycles to all cycles. The cohesion then directly relates to the overall connectedness of the network and to interconnectedness in large scale and navigation in providing cohesion.

The values of the variables *D*, *C* and Ω on a given concept map (network) can be calculated when the variables a_{ij} (adjacency matrix **a**) are known. Their detailed mathematical definitions are given in Appendix A.

The local analysis of the concept maps produces a great amount of detailed data, which need to be reduced for the practical purposes of classification and comparison of different concept maps. Of the average degree of nodes, called degree *D*, is then the basic variable related to the connectivity and the number of the links (richness of content), *C* to clustering (local interconnectedness) and Ω to cohesion (local interconnectedness). Because the properties of being hierarchical and clustered are important qualitative features, it is clear that in the structurally good concept maps all the quantities *C* and Ω need to have large values.

In order to analyse the systematic dependencies and the variations around the mean the variables *C* and Ω on *D* are regressed, but it should be noted that the dependence (regression) is not linear. The regression means that the connectivity or the richness of content *D* is selected as an independent (or explanatory) variable, while the set $\{C;\Omega\}$ consists of the dependent (or response) variables. This selection is motivated on the basis of a theoretical understanding of the expected dependencies and in practice the selection is the first step in the structural modelling of the data (see e.g. Mulaik, 2009). Such a structural modelling aims to condense the variables in a form of multivariate distribution (probability density) P($C;\Omega$) which tells the frequency (probability) to obtaining a given set of values $\{C;\Omega\}$. This probability density function can then be used as the basis for defining the structural validity so that the values larger/smaller than the average indicate structurally better/poorer than average maps. In practice, direct measure for that is given by the cumulative distribution function (CDF) $\Phi(C,\Omega) = \text{Prob}\left(\text{P}; C' < C; \Omega' < \Omega\right)$ of probability density function P in defining the structural quality in the form

$$q = \Phi(C,\Omega) - \Phi(0,0)$$

The quality *q* then plays a direct role in interpreting how much the map with the given values of *C* and Ω deviate from the expected average map. Values $q < 0$ indicate structurally poorer than average maps while $q > 0$ indicates structurally better maps. Of course "structurally good" is then simply a statement that clustering and cohesion are all above average values, while "structurally poor" means that all values of structural variables are below average values. Similarly, "structural quality" refers only to these properties. The structural quality is completely independent of *D* and therefore *q* and *d* represent truly independent dimensions; structural qualities and richness of content.

Results of Research

The sample of the concept maps studied here consists of two sets of maps, but in both cases the

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design principles are the same. The first set is the maps made by students before group discussions, where maps are compared and evaluated. This set is called *the initial maps*. The second set is done after the group discussions, with the purpose of improving the maps. This set is called *the final maps*. Some examples of the initial and final maps in the spring-embedded form are shown in Figure 3.

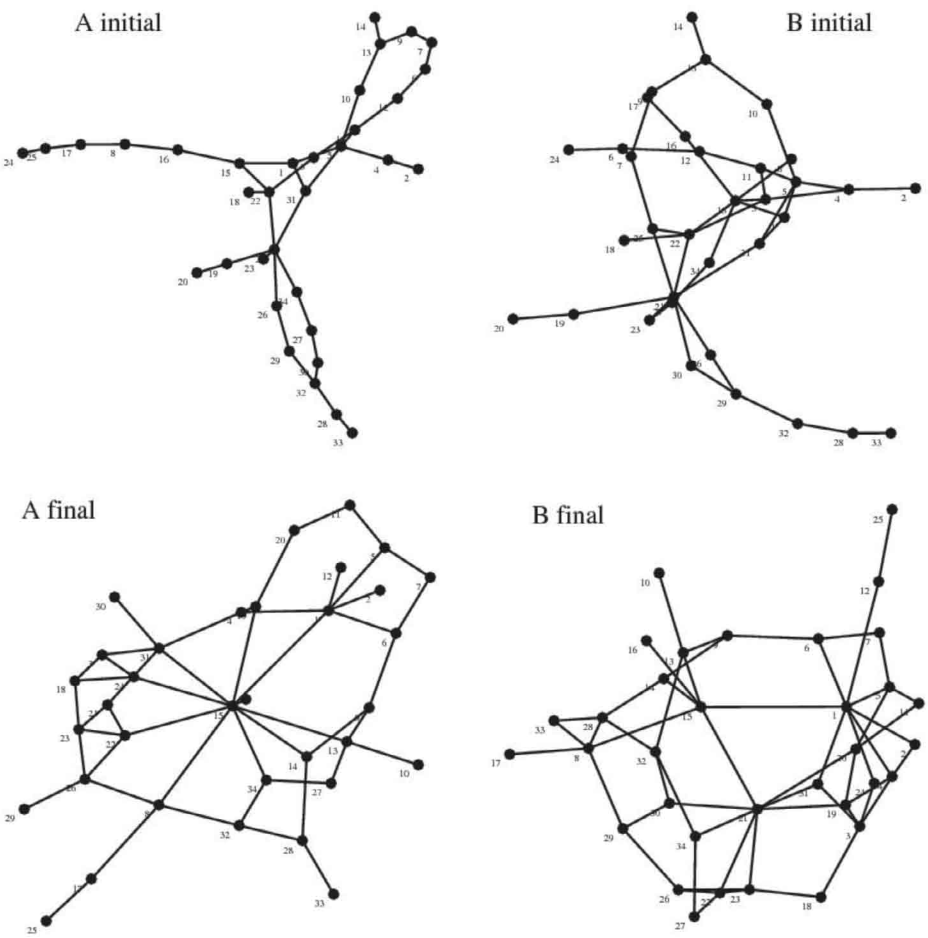

Figure 3: Concept maps *n*=34 drawn in the spring-embedded form, initial maps have *D*=2.23 *Q*=-0.13 (map A) and *D*=2.88 *Q*=-0.31 (map B) and final maps *D*=2.65 *Q*=-0.06 (map A) and *D*=3.29 *Q*=0.33 (map B).

The measured values of the variables *C* and *Ω* as regressed on the values of *D* are shown in Figure 4 for the sample consisting of N=33 concept maps. Analysis of the distribution shows that the distribution of *C* and *Ω* can be represented by multi-normal distribution with standard deviations σ_{C}, =0.038 and $\sigma_{\Omega,}$ =0.048 respectively (see Appendix B). The correlation coefficients between variables are given by *η*=0.82. The goodness of fit is tested for the marginal normal distributions by using the standard χ^2-test (Ruskeepää, 2004), yields p-values 0.94 indicating very high goodness of fit.

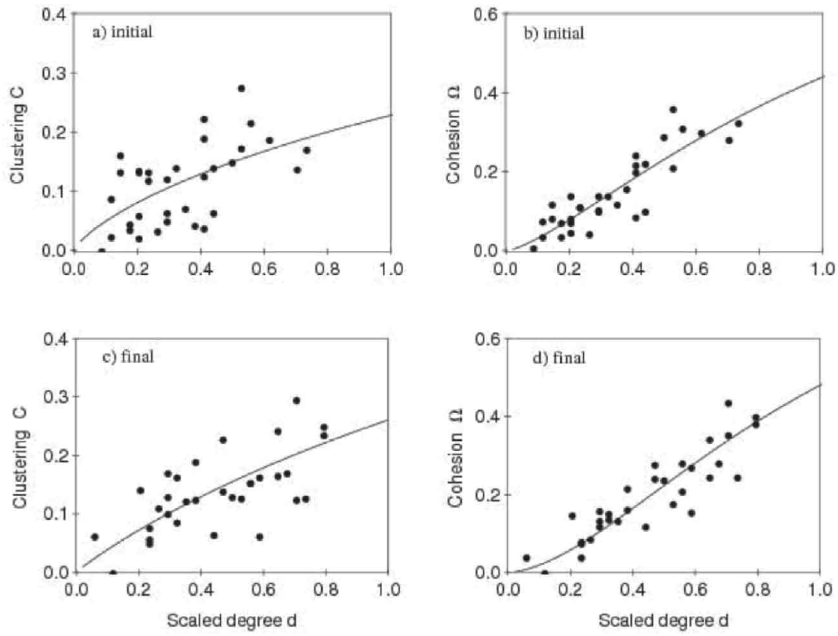

Figure 4: The clustering *C* and cohesion *Ω* as they depend on scaled degree *d*. Fits are given by Eq. (1) with parameters κ_C=3.77 and α_C=0.75 for the clustering *C*, and κ_Ω=1.29 and α_Ω=1.40 for the cohesion *Ω*.

Clustering. The results in Figure 4 for *C* show that also clustering increases with increasing *D* in initial and final maps alike. This reflects the fact that, if concepts A and B and B and C are related, it is very probable that also concept B and C are related (the transitivity condition), therefore giving rise to triangular patterns. The high clustering can be thus taken as a sign of tight interconnectedness in close neighbourhood of given concept.

Cohesion. From the results in Figure 4 can be seen that the cohesion *Ω* of the concept maps increases rather rapidly with the increasing scaled number *d* of connections per node. Cohesion measures the networks' stability to stay in one piece; that means, the larger the cohesion is, the larger is the probability that the network cannot be divided into two subsets of nodes where nodes in the given subset are not connected.

The structural quality *q* of the concept map, based on CDF of the multivariate distribution P and defined in Eq. (4), is shown in Figure 5. The quality of initial concept maps is shown by stars while the quality of the final maps is given by bullets. It should be noted that average structural quality corresponds case *q*=0. The concept maps with the poorer than average quality have negative values of *q* while those which have better than average quality have positive values. From the result it is seen that *q* indeed classifies the different maps so that there is a continuum of values but *q* does not depend on *d*.

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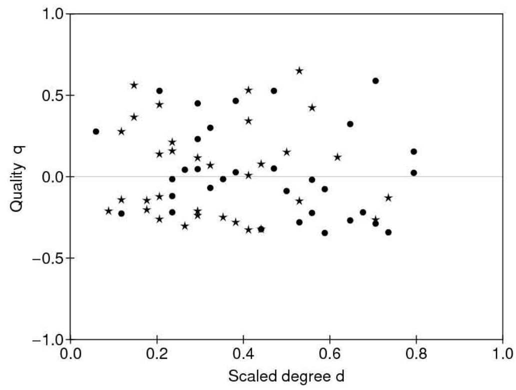

Figure 5: The structural quality *q* of the concept maps for maps with scaled degree *d*. The initial concept maps are marked by stars the final maps by bullets.

The connectivity *d* and the quality *q* represent very different qualitative properties of the maps. Taking into account that all links are procedural connections, based either on experimental or modelling procedures, *d* simply represents the richness of the procedural knowledge that students have at their command. Therefore, *d* and *q* represent independent dimensions of properties; richness of content and quality of structure. Having now the two quantities *d* and *q* which can be used to characterise the structural quality of the concept maps, they can be used to monitor the change by using these quantities. The differences can be formed $\Delta q = q_{final} - q_{initial}$ and $\Delta d = (d_{final} - d_{initial})/d_{final}$, where Δd is scaled to give the relative change. These changes from the initial to final maps are shown in Figure 6.

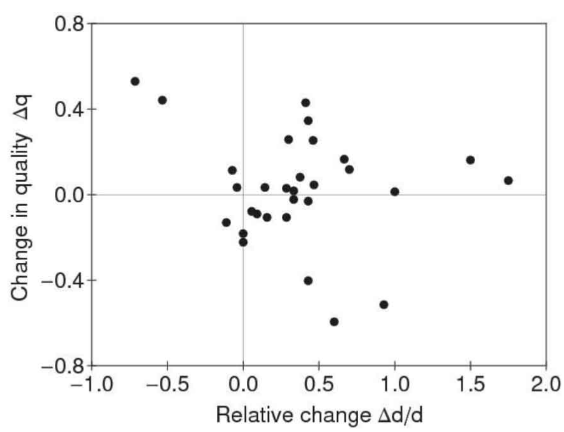

Figure 6: The change in quality in the concept maps with relative change of scaled degree *d*.

The figure 6 describes the changes in the structure of concept maps obtained during the learning (i.e. learning outcomes) and show that there is notable development of structure, and thus expectedly also in students' conceptual understanding. The relative change in the richness of content (in x-axis) means that students have managed to integrate new content in their concept maps. For example relative change of 0.5 means 50% of new information as final map is compared to the initial one. The most important result is that most of the students are located right upper quadrant in Figure 6 which tells us these students really have made progress. Of these students some have achieved substantial changes in quality of the structure but only small relative changes in the richness of content. There is a small group of students, which have reached substantial increase in richness of content but small changes in quality of structure. The results seem to indicate that it is not easy to perform well in both aspect (increasing the content and improving the structure) and that the most productive learning apparently takes place in the region where large changes of structure take place but changes in richness of content are moderate.

If there are large changes in richness of content, change in structure remains moderate or the structural quality can even become poorer as in the lower right quadrant. It might be that students have drawn initial map in too optimistic way (i.e. drawn too much links, maybe based on loose association) and realised afterwards that they are not able to justify all links and thus have deleted some, with the result of deteriorating the structure. In some cases, located in upper left quadrant students have apparently improved the structure simply by "pruning", by reducing the richness of content in favour of better connected structures. Finally, the lower left quadrant represents cases where all has gone amiss - the content is impoverished and structure deteriorated.

Discussion

The most important finding of this study is that structural variables describing the topology of the concept maps can be used as basis to define unambiguously the quality of the structure. The structural validity of the analysis concerns the question of whether or not the variables provide information on the structural aspect. In regard to the clustering and cohesion, this question is answered nearly trivially by noting that: 1) the variables are constructed so that they operationalise the qualitative structural features of interest, 2) the definition of variables is mathematically exact (see Table A1 in Appendix A), and 3) the analysis itself is based only on these well-defined topological measurements of the concept maps.

The content validity is to a large degree resolved by the notion that students need to justify each link and explain its (procedural) content in written report coming with the map. Although the depth and thoroughness of these explanations vary, they are always acceptable (i.e. correct models and experiments) from the point of view of physics content knowledge. Therefore, the number of links is directly related to the richness of correct content (connectivity), while the good structure is related to the ability to organise this content.

The remaining question is the reliability of the analysis, concerning the statistical significance of the averaged values of variables (i.e. local variables averaged over the whole map). It has been shown in this study that the suitably scaled data can be described quite reliably by means of a multivariate normal distribution, which suggests that the residuals form a set of data, which is distributed independently, identically and normally, that is, it represents the so-called heteroscedastic data set. The possibility to have a data model, where the structural model describes the expected mean and the probability density P the deviations from the mean, forms the basis on which to attribute one quantity q to each concept map so that it describes the quality of the map in terms of the probability measure of the expected deviation.

The research carried out here has concentrated on finding structural model to describe concept maps and to develop unambiguous measures to monitor the structural change. However, from point of view of learning and teaching the main focus is the development of method to monitor conceptual change and development in learning during a teaching sequence. The mutual changes in relatedness of concepts is directly reflected in the structure of concept maps, which suggests that such changes can be used in monitoring conceptual development, as has been discussed for example by di Sessa and Sherin (1998) and Liu (2004). The development in richness of content and quality of structure are both important aspects when evaluating students' concept maps. Therefore, the richness of content and the quality of structure as introduced here open up the possibility to monitor changes in conceptual structures and make inferences of the students' development during a teaching sequence. The results show that in the most cases there is positive development where both richness of the content and quality of structure are improved, but only in one or another dimension. Progression in both dimensions (quality of structure q or richness of content d) is not easily reached and it might well be too complicated task for the students. The moderate changes either in quality of structure q or richness of content d probably means that teaching and learning happens near students' *the zone of proximal development* (ZPD) as described by Vygostky (1978). According to Vygotsky's theory, teaching and learning is productive and

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meaningful if its goals are set so that they are a bit more demanding than the goals easily achieved by the students in their starting position. In that case teaching promotes development of actions, which are soon developing, i.e. actions which are in the ZPD. Thus, in co-operation and supervision (guided learning) students are able to solve more complex tasks than alone. Apparently in this case, improving either structure or content is within the ZPD whereas, improving them both overshoots ZPD.

Conclusions

The structural variables describing the topology of the concept maps can be used as basis to define the quality of the structure. A quantitative method of analysis of concept maps is introduced here, which provides tools for analysing relevant complex features with quantitative accuracy and for assessing the quality of the concept maps on this by using valid and reliable measurements. It is shown that such an approach gives access to monitoring change of the concept maps during the teaching, and that the changes brought forward can be meaningfully seen as conceptual development or change. These notions encourage thinking that the methods developed here provide fruitful starting point for monitoring learning outcomes and for their assessment and evaluation.

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Appendix A

In order to make the detailed analysis of the concept maps amenable first the variables are defined. In a map (network) of *n* nodes, the variables a_{ij} form a *nxn* dimensional so called adjacency matrix $\boldsymbol{a}$. The independent variables are binary variables a_{ij}, which indicate the connections between nodes *i* and *j* so that if nodes are connected, then a_{ij}=1 and if there is no connection, then a_{ij}=0. The dependent variables are the number of links of each node (connectivity) and the clustering and cohesion as defined in Table A1

Table A1. Mathematical definitions of variables for measuring the topology of the networks. Note that in the definition of *Ω* the numerator is the number of odd cycles and denominator the number of all cycles S_k going through node *k*. The subscripts *k* refer to the *k*th node. The adjacency matrix is given by a and it has elements a_{ij}.

Variable		Definition
Connectivity	Dk	$\sum_i (a_{ik} + a_{ki})$
Clustering	Ck	$\sum_{i>j} a_{kj} a_{ij} a_{ki} / \sum_{i>j} a_{ik} a_{kj}$
Cohesion	Ω	$\dfrac{\left(\sum_i (a^{2i+1})_{kk} / (2i+1)!\right)}{\left(\sum_i (a^i)_{kk} / i!\right)}$

The classification of the maps, definition of the quality of the maps and ultimately the monitoring of the changes in the quality will be done, it what follows, by using the values of *C* and *Ω* averaged over the all nodes.

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Appendix B

In order to make the detailed analysis of the concept maps amenable first the variables are defined. The systematic dependence of the averages <X> of variables *X*={*C*; *Ω*} on *D* in the studied sample of maps is modelled by the structural equations of form

$$\langle X \rangle = \left(1 + \kappa_X \, d^{-\alpha_X}\right)^{-1}, \quad d = D / D_0 - 1$$

In the Eq. (1) parameters κ_x and α_x define the nonlinear dependence on suitably scaled parameter $d=D/D_0-1$, with D_0 as a scale parameters. Note that the model is valid only for $D > D_0$, where value $D_0 \approx 2.0$ is related to the threshold when maps begin to have an appreciable number of 3-cycles. All parameters are defined empirically, by using least squares fit to the pool of data. The justification of the structural model is given *a posteriori* by inspection of the residuals of the data. In the best case the structural model helps to represent the data so that the residuals *x* = *X* - <*X*> are normally, identically and independently (NIID) distributed. In this case the so-called heteroscedasticity condition of the residuals is fulfilled, which is needed in order to carry out unbiased and reliable statistical analysis (Johnson & Wichern, 2007). When the NIID condition is fulfilled the normal distribution can be used to describe the data, with average values given by the structural model and with variances obtained from the heteroscedastic distribution of residuals. In this case the distribution of variables *X*={*C*; *Ω*} is then given by the multivariate normal distribution.

$$\mathrm{P}[\boldsymbol{X};\Xi] = Z^{-1}\exp[-\frac{1}{2}\boldsymbol{X}^T \Xi^{-1}\,\boldsymbol{X}]$$

where ***X*** is a vector of variables and Z is the normalisation factor. The covariance matrix **Ξ** is given by

$$\Xi = \begin{pmatrix} \sigma_C^2 & \eta\sigma_C\sigma_\Omega \\ \eta\sigma_C\sigma_\Omega & \sigma_\Omega^2 \end{pmatrix}$$

where standard deviations of *C* and *Ω* are σC, and σ_{Ω}, respectively. The correlation coefficients between variables are given by *η*. When the values of the variables are distributed according to Eq. (2)-(3) it is possible to tell directly what the probability P(*C*;*Ω*) is for obtaining the set of given values {*C*;*Ω*}.

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