# CONTRIBUTION TO THE DEFINITION OF THE MOST IMPORTANT PARAMETERS FOR TIRE MODELS

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## INTRODUCTION

Reliable prediction of vehicle motion using different simulation models is almost impossible without precisely defined forces at the contact of tire and road. Throughout the history, a large number of different models that describe the available adhesion between tire and road, and define the tire forces and moments has been developed. A useful brief overview of the different developed models is given in [3], [6]. The developed models, based on its approach, can be divided into several categories.

On one side, there are the experimental models, usually given in the form of tables or interpolation formulas (Pacejka „Magic formula“, Burckhardt). Given that they are developed on the basis of experimental results, these models show very good match with measured data. From the standpoint of time and computer resources they are suitable for use in vehicle dynamic simulation models. However, these models do not provide insight into the physical phenomena involved in the process of generating forces at the contact of tire and road. In addition, these models are valid only for specific tire in specific exploitation conditions. Changing any parameter causes a change in the final values of forces, that is changes the set of coefficients in the mathematical formulas.

On the other side, there are models based on physical assumptions and phenomena involved in the process of generating forces at the contact of tire and road. For a detailed analysis of the tire behavior complex models are used based on finite element methods. Due to the complexity and time needed for the calculation these models are not used within vehicle dynamics simulation models. For this purpose, more convenient are simple physical models (Brush, Dugoff, etc) that are easy on the budget, while allowing the physical understanding of tire behavior. These models show good match with experimental data, provided good definition of the values of influential parameters figured in equations (tire stiffness, friction coefficients, etc.), that change with a change of tire slip, vertical load, surface condition, etc.

The aim of this study is to, based on analysis of experimental data sets, define the dependence of tire stiffness and adhesion coefficient as a function of the vertical load and the longitudinal and lateral tire slip.

## EXPERIMENTAL DATA AND TIRE MODELING

The experimental data presented in [1] show the value of pure longitudinal force at the contact of tire and road during braking (Fig. 1), and pure lateral forces during cornering (Fig. 2). Results are given for dry asphalt surface.

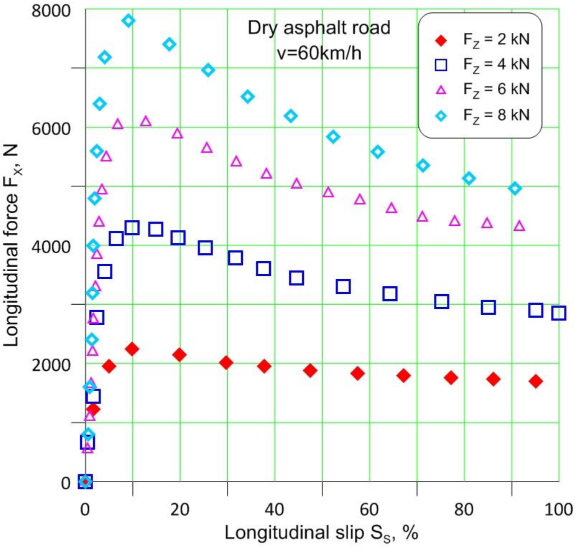

***Figure 1** Experimental data for pure longitudinal force (pure braking), for different vertical loads [1]*

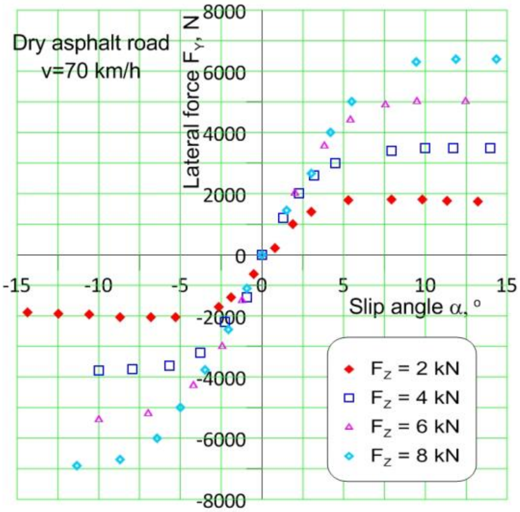

***Figure 2** Experimental data for pure lateral force (pure cornering,) for different vertical loads [1]*

By different models of adhesion, it is tried as accurately as possible to describe the available adhesion between tire and ground, i.e. to define the tire forces. Two simple physical models that have found extensive application in vehicle dynamics models are Dugoff model, and the Brush model.

Dugoff model [2], [3], [6], allows calculation of the tire longitudinal force $F_x$ and tire lateral force $F_y$, assuming a uniform pressure distribution at the tire-road contact area.

$$F_x = C_s \frac{S_s}{1-S_s} f(\lambda) \tag{1}$$

$$F_y = C_\alpha \frac{\tan\alpha}{1-S_s} f(\lambda) \tag{2}$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & \lambda < 1 \\ 1, & \lambda \geq 1 \end{cases} \tag{3}$$

$$\lambda = \frac{\mu F_z (1-\sigma_x)}{2\sqrt{(C_s S_s)^2 + (C_\alpha \tan\alpha)^2}} \tag{4}$$

where $C_S$ and $C_\alpha$ are tire stiffness in longitudinal and lateral direction, $S_S$ tire longitudinal slip, $\alpha$ tire slip angle, $\mu$ tire friction coefficient, $F_z$ tire vertical load.

Longitudinal slip is calculated by the formula:

$$S_s = \frac{v_x - r_d \omega_w}{v_x} \tag{5}$$

where $v_x$ is longitudinal wheel center velocity, $\omega_w$ wheel angular velocity, $r_d$ wheel rolling radius.

More realistic, parabolic distribution of tire-road contact pressure is taken into account by the Brush model with parabolic distribution of contact pressure [4]. A Pacejka's interpretation of the Brush model with a parabolic distribution of contact pressure [3], [5] is shown below.

Longitudinal force $F_x$,

$$0 < S_s < S_{sc}$$

$$F_x = 3\mu_x F_z \theta_x S_s \left[ 1 - |\theta_x S_s| + \frac{1}{3}(\theta_x S_s)^2 \right] \tag{6}$$

$$S_s < S_{sc}$$

$$F_x = \mu_x F_z \tag{7}$$

where $S_{Sc}$ is critical value of longitudinal slip at which complete tire began to slide, $\theta$ parameter which depends of tire stiffness, vertical load, and friction coefficient.

Value of tire slip $S_S$ and $S_{Sc}$, as well as parameter $\theta$ are calculated by the equations:

$$S_s = \frac{V_x - r_d \omega_w}{V_x}, \quad S_{sc} = \frac{1}{\theta_x} = \frac{3\mu F_z C_s}{C_s}, \quad \theta_x = \frac{C_s}{3\mu F_z} \tag{8}$$

Lateral force $F_y$,

$$0 < S_\alpha < S_{\alpha c}$$

$$F_y = 3\mu_y F_z \theta_y s_\alpha \left[ 1 - \left| \theta_y S_\alpha \right| + \frac{1}{3} \left( \theta_y S_\alpha \right)^2 \right] \tag{9}$$

$$S_\alpha < S_{\alpha c}$$

$$F_y = \mu_y F_z \tag{10}$$

where is:

$$S_\alpha = \tan\alpha, \quad S_{\alpha c} = \frac{1}{\theta_y} = \frac{3\mu F_z C_\alpha}{C_\alpha}, \quad \theta_y = \frac{C_\alpha}{3\mu F_z} \tag{11}$$

One can see from the above models, the value of the available adhesion between tire and road, or tire forces are given in dependence of the longitudinal and lateral tire stiffness, friction coefficient between tire and road, and the tire vertical load.

Bellow, a brief analysis of the influence of aforementioned parameters on adhesion will be made, and the dependence of tire stiffness and adhesion coefficient as a function of the vertical load and the longitudinal and lateral tire slip will be defined.

## ANALYZE OF INFLUENTIAL PARAMETERS

As mentioned above, as parameters influential to the process of generating forces at the contact of tire and road, in simplified physical models appear tire longitudinal and lateral stiffness, and friction coefficient between tire and road.

### *Tire stiffness*

Tire longitudinal stiffness is defined as the slope of the curve of the longitudinal force at zero slip:

$$C_s = \left. \frac{dF_x}{dS} \right|_{S=0} \tag{12}$$

For experimental data presented in [1], tire longitudinal stiffness for different values of vertical loads can be calculated (Table 1).

***Table 1*** *Tire* longitudinal stiffness, for different vertical load – experimental data [1]

| Vertical load $F_z$, N | Longitudinal stiffness $C_s$, N |
|---|---|
| 2000 | 40000 |
| 4000 | 75000 |
| 6000 | 130000 |
| 8000 | 160000 |

Analyzing results, it can be noted that tire longitudinal stiffness changes linearly with the change of the tire vertical load (Figure 3).

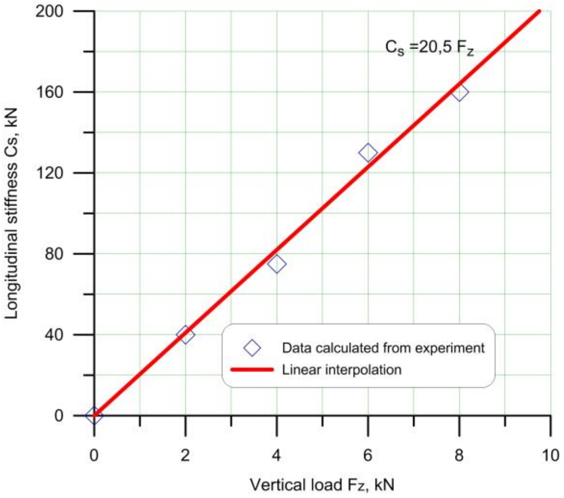

***Figure 3*** *Tire longitudinal stiffness as function of vertical load*

Adopting the assumption of linear dependence of the longitudinal stiffness of the tire vertical load, the relation can be established

$$C_s = K_1 F_z \tag{13}$$

where for the experimental results the coefficient $K_1$ = 20,5.

Tire lateral stiffness is defined as the slope of the curve of the lateral force at zero slip angle:

$$C_\alpha = \left.\frac{dF_y}{d\alpha}\right|_{\alpha=0} \tag{14}$$

For experimental data presented in [1], tire lateral stiffness for different values of vertical loads can be calculated (Table 2).

***Table 2*** *Tire* cornering stiffness, for different vertical load – experimental data [1]

| Vertical load $F_z$, N | Cornering stiffness $C_\alpha$, N/° |
|---|---|
| 2000 | 40000 |
| 4000 | 65000 |
| 6000 | 75000 |
| 8000 | 80000 |

Analyzing results, it can be noted that with the change of tire vertical load the tire lateral stiffness can be modeled as a parabolic function (Figure 4).

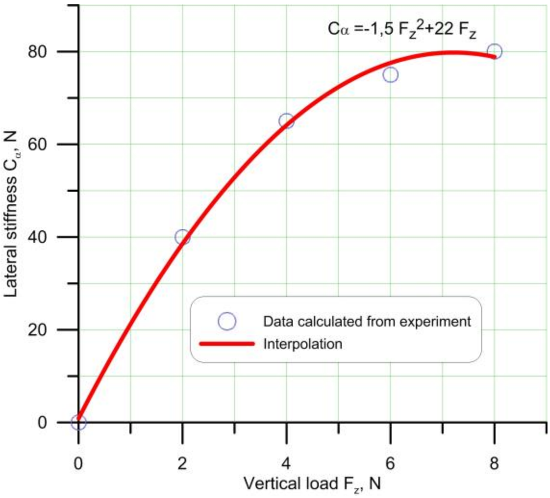

***Figure 4*** *Tire cornering stiffness as function of vertical load*

Adopting the assumption of a square dependence of the lateral stiffness of tire vertical load, the relation can be established

$$C\alpha = K_2 F_z^2 + K_3 F_z \tag{15}$$

Where for the given experimental results the coefficient $K_2$ = -1,5, and coefficient $K_3$ = 22.

### *Tire friction coefficient*

Another influential parameter that figures in physical tire models is friction coefficient between tire and road. Using a study developed by Dugoff [2], and based on experimental results, it can be shown that friction coefficient is a function of slip velocity, and it can be presented in the following form:

$$\mu = \mu_0 \left(1 - M_1 v_s - M_2 v_s^2\right) \tag{16}$$

or

$$\mu = \mu_0 \left(1 - M_1^* S - M_2^* S^2\right) \tag{17}$$

where $\mu_0$ is friction coefficient at zero slip, $M_1$ i $M_2$ reduced friction factors, $v_s$ slip velocity, and $S$ is tire slip.

Analyzing the influence of the vertical load on the longitudinal and lateral forces at the contact of tire and road, it can be seen that the friction coefficient changes with the tire vertical load. To take into account the influence of tire vertical load on friction coefficient a modification of the equation (17) for the friction coefficient is made, whereby linear effects of tire vertical load is assumed. Taking into account the mentioned, it can be established the following relation for friction coefficient:

$$\mu = C_1 S^2 + C_2 S + C_3 + C_4 F_z \tag{18}$$

where $S$ is longitudinal slip $S_S$ (for friction coefficient in longitudinal direction), or slip angle $\alpha$ (for friction coefficient in lateral direction), and $C_1$ to $C_4$ coefficients, and $\mu$ friction coefficient in longitudinal $\mu_x$ or lateral direction $\mu_y$.

Analyzing experimental results for longitudinal force presented in [1], friction coefficient in longitudinal direction for different vertical load can be presented by equation (18), where the corresponding coefficients have values:

$$C_{1,x} = 3 \cdot 10^{-5},\ C_{2,x} = -0{,}007,\ C_{3,x} = 1{,}27,\ C_{4,x} = -0{,}037$$

Experimentally determined and calculated values of friction coefficients in longitudinal direction are shown in the Figure 5.

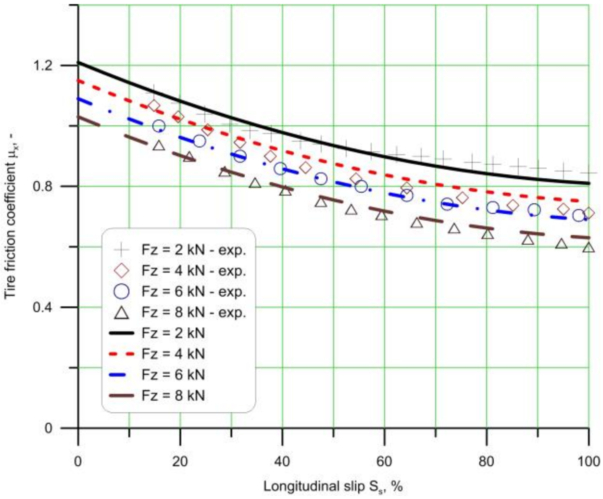

***Figure 5** Longitudinal tire friction coefficient as function of vertical load and longitudinal slip - experimental data [1] and calculation*

Analysis of the experimental results for lateral force presented in [1], shows that the friction coefficient in lateral direction for different vertical load can also be presented by equation (18), where corresponding coefficients have values:

$$C_{1,y} = -0{,}65 \cdot 10^{-3},\ C_{2,y} = 0{,}017,\ C_{3,y} = 0{,}89,\ C_{4,y} = -0{,}029$$

Experimentally determined and calculated values of friction coefficients in lateral direction are shown in the Figure 6.

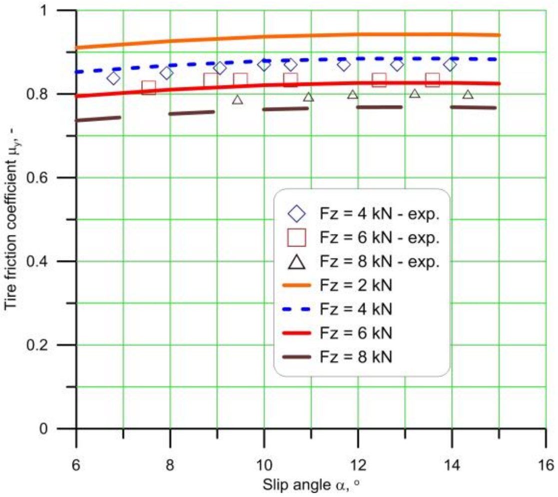

***Figure 6*** *Lateral tire friction coefficient as function of vertical load and slip angle - experimental data [1] and calculation*

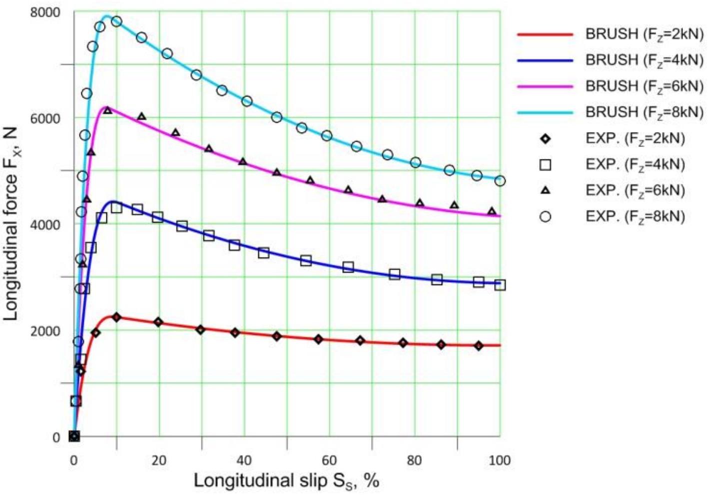

***Figure 7*** *Tire longitudinal force – experimental data [1] and Brush model calculation*

## TIRE DATA CALCULATION

Entering values for longitudinal and lateral stiffness and friction coefficient of tire, based on experimental data from [1], in the equation for the Brush tire model, the values of longitudinal and lateral forces at contact of tire and road are obtained.

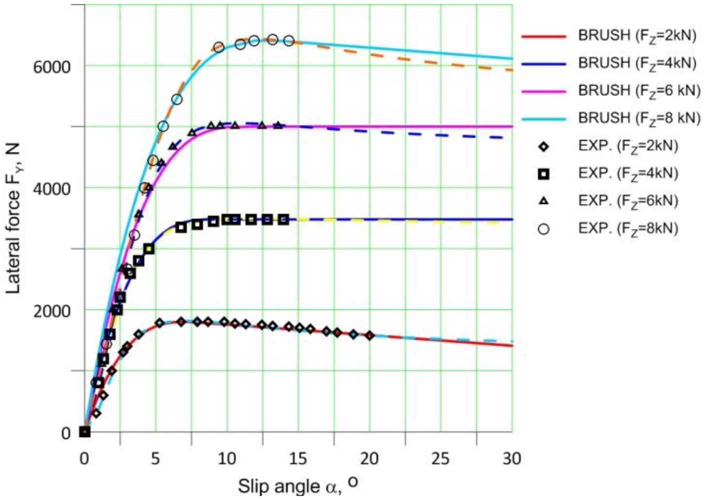

***Figure 7*** *Tire lateral force – experimental data [1] and Brush model calculation*

As seen on the above diagrams, for correctly determined values of stiffness and friction coefficient, Brush tire model shows exceptionally good match with the experimental results.

## CONCLUSIONS

In the physical tire models, which are often used in vehicle dynamics simulations, tire stiffness and friction coefficient between tire and road are used as influential parameters. Correct choice of their values greatly affects the accuracy of those models and their applicability to vehicle dynamics simulation models.

Using experimental data available in literature, the paper analyzes the influence of tire slip and vertical load on the values of tire longitudinal and lateral stiffness, and friction coefficient. The influence of tire vertical load on the tire longitudinal and lateral stiffness is shown. The relations which show the dependence of the friction coefficient in function of tire slip are proven, and the assumptions about the effects of vertical load on friction coefficient in the longitudinal and lateral direction are given. For the available experimental data the equations and the corresponding coefficients are determined. By using this equation it is possible to calculate tire stiffness and friction coefficient, for different values of tire vertical load. For complete confirmation of obtained results further analysis of even greater number of experimental data is necessary.

With defined values of stiffness and friction coefficient, the calculation of tire longitudinal and lateral forces is carried out, using one of the physical tire models (Brush model with a parabolic distribution of contact pressure). The results show good match with experimental data. This way, it was shown that simple physical tire models provide quite satisfactory prediction of the tire forces values for the analysis of vehicle dynamics, but correctly determined values of stiffness and friction coefficient.

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