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RESEARCH ARTICLE

# Magnetized Inhomogeneous Plane Symmetric Cosmological Model in Bimetric Theory of Relativity

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## ABSTRACT

Plane symmetric inhomogeneous cosmological model for electromagnetic field is studied in the frame work of Rosen's bimetric theory of relativity and have obtained the vacuum solutions for the model.

**Keyword**: - Electromagnetic field, Cosmological model, Bimetric relativity.

## 1. INTRODUCTION

An alternative theory of gravitation, called the bimetric theory of gravitation, was proposed by Rosen (1973) to modify the Einstein's general theory of relativity by assuming two metric tensors, a Riemannian metric tensor $g_{ij}$ and a background metric tensor $\gamma_{ij}$. The metric tensor $g_{ij}$ determines the Riemannian geometry of the curved space-time, here $g_{ij}$ has the same importance as in the Einstein theory of relativity and it interacts with matter. The background metric tensor $\gamma_{ij}$ refers to the geometry of the empty (free from matter and radiation) universe and describes the inertial forces. This metric tensor $\gamma_{ij}$ has no physical importance but $\gamma_{ij}$ appears in the field equations of Rosen's Bimetric relativity. Therefore $\gamma_{ij}$ interacts with $\gamma_{ij}$ but not directly with matter. One can regard $\gamma_{ij}$ as giving the geometry that would exist if there were no matter. Thus at every point of space - time, there are two line elements.

$$ds^2 = g_{ij}\, dx^i\, dx^j \qquad \text{.......(1.1)}$$

$$d\sigma^2 = g_{ij}\, dx^i\, dx^j \qquad \text{.......(1.2)}$$

The field equations of Rosen's bimetric theory of relativity are

$$K_i^j = N_i^j - \frac{1}{2} NKg_i^j = -8\,\pi k T_i^j \qquad \text{.......(1.3)}$$

Where, $N_i^j = \frac{1}{2}\, \gamma^{\alpha\beta}\left[g^{hj} \quad g_{hi|\alpha}\right]_{|\beta}$ .......(1.4)

$$K = \sqrt{\frac{g}{\gamma}}, \text{ where } g = \det(g_{ij}) \quad \text{...........(1.5)}$$

$$\& \, \gamma = \det(\gamma_{ij}), \; N = N^{\propto}_{\propto} \quad \text{.....................(1.6)}$$

Vertical bar (|) denotes $\gamma$-covariant differentiation with respect to $\gamma_{ij}$ & $T_i^j$ is the energy momentum tensor of the matter field.

Rosen (1973; 1974; 1980); Yilmaz (1975); Israelit (1981); Karade (1980); Mohantty and Sahoo (2003); Katore (2006); Deo (2012; 2003) are some of the eminent authors who have studied several aspects of bimetric theory of relativity.

In particular, Reddy and Venkateswarlu (1988), Reddy and Rao (1998) have established the non-existence of spatially homogenous & isotropic cosmological model of Bianchi types and Kantowski-Sachs models in bimetric theory of relativity when the matter is taken either perfect fluid or cosmic string dust. Now a days, there are a lot of keen interest in studying cosmological models in the context of bimetric theory of relativity The purpose of Rosen's bimetric theory is free from the singularities that occur in general relativity.

It is interesting to note that magnetic field plays a significant role at cosmological model. In this work, we have investigated the inhomogeneous plane symmetric model for electromagnetic field in the context of bimetric theory of relativity & observed that the electromagnetic field does not exist in this theory. Hence resulting space time represents vacuum cosmological model.

## 2. METRIC AND ENERGY MOMENTUM TENSOR

We consider an inhomogeneous plane symmetric metric in the form

$$ds^2 = A(-dt^2 + dz^2) + Bdx^2 + dy^2 \text{...(2.1)}$$

Where A, B are function of (z, t)

The background metric in the form

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \text{.........(2.2)}$$

Here the energy momentum tensor $E_{ij}$ for the electromagnetic field is given by

$$E_{ij} = \frac{1}{4\pi}\left[-F_{ir}F_j^r + \frac{1}{4}g_{ij}F_{ab}F^{ab}\right] \text{.......(2.3)}$$

Where $E_{ij}$ is the electromagnetic field and $F_{ij}$ Maxwell electromagnetic field tensor.

In co-moving coordinate system the magnetic field is taken along X-direction. We assume that the only non-vanishing component element to magnetic field tensor $F_{ij}$ is $F_{23}$ hence, due to assumption of infinite electrical conductivity $F_{23} = F_{24} = F_{34} = 0$ the first set of Maxwell's equation $F_{[ij,k]} = 0$ lead to the $F_{23} = constant = H$, from (2.3) with (2.1)

We obtained $E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{H^2}{8\pi AB}$

In co-moving coordinate the field equation (1.3) with the equation (2.1) & (2.3) we get

$$\left(\frac{A''}{A} - \frac{A'^2}{A^2}\right) - \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) = 2k\,\frac{H^2}{AB} \quad \text{.....(2.4)}$$

$$\left(\frac{A''}{A} - \frac{A'^2}{A^2}\right) - \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) = -2k\,\frac{H^2}{AB} \quad \text{..... (2.5)}$$

$$\left(\frac{B''}{B} - \frac{B'^2}{B^2}\right) - \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = -2k\,\frac{H^2}{AB} \quad \text{..... (2.6)}$$

$$\left(\frac{B''}{B} - \frac{B'^2}{B^2}\right) - \left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = 2k\,\frac{H^2}{AB} \quad \text{..... (2.7)}$$

Where,

$$A' = \frac{\delta A}{\delta Z}, \qquad \dot{A} = \frac{\delta A}{\delta t}$$

$$A'' = \frac{\delta^2 A}{\delta Z^2}, \qquad \ddot{A} = \frac{\delta^2 A}{\delta Z^2} \; etc.$$

Using equation (2.4) to (2.7) we have,

$$H=0$$

$$\text{i.e. } i.e.\; F_{23} = 0 \quad \text{.................... (2.8)}$$

Which gives us inhomogeneous plane symmetric cosmological model does not accommodate electro-magnetic field.

## 3. VACUUM COSMOLOGICAL MODEL

By using (2.8) in equation (2.4) to (2.7), we obtained

$$\left(\frac{A^{"}}{A}-\frac{A^{`2}}{A^2}\right)-\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)=0 \qquad \text{.........(3.1)}$$

$$\left(\frac{B^{"}}{B}-\frac{B^{`2}}{B^2}\right)-\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)=0 \qquad \text{.........(3.2)}$$

From Eqns (3.1) to (3.2), we get

$$A=\exp(k_1 z+ k_2 t) \quad \text{...........................(3.3)}$$

$$B=\exp(k_3 z+ k_4 t) \quad \text{...........................(3.4)}$$

Where $k_i{}'s$ 's are constants of integration.

The corresponding vacuum cosmological model can now be written in the form by using eq. (3.3) and (3.4) in eq. (2.1) we get

$$ds^2 = exp(k_1 z+ k_2 t)(k_1 z+ k_2 t)+ exp(k_1 z+ k_2 t)(k_1 z+ k_2 t) \quad \text{..... (3.5)}$$

And it is interesting to note that the model (3.5) is free from singularity at t = 0.

## CONCLUSION

In the present work, we have investigated the inhomogeneous plane symmetric model for electromagnetic field distribution in the frame work of Rosen's bimetric theory of gravitation.

This clearly represented a vacuum cosmological model, which is free from singularity at t =0.

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## REFERENCES


1) Rosen N. A bimetric theory of gravitation I, *Gen. Rela. Grav*, 1973; 4 (6): 435-477.
2) Rosen N. A theory of gravitation, *Ann.Phys.*, 1974;84(1-2) :455-473.
3) Rosen N. Bimetric general relativity and cosmology", *Gen. Rela. Grav.*, 1980; 12(7): 493-510.
4) Yilmaz H. Inhomogeneous cosmological models in biometric theory of gravitation"*Gen. Relativ. Gravit.,*1975; 6: 269.
5) Isrelit M. Spherically symmetric fields in Rosen's bimetric theories of gravitation" *Gen. Relativ. Gravit*, 1981; 13(7): 681-688.
6) Karade TM. Spherically symmetric space times in bimetric relativity theory" I, *Indian J. Pure-appl. Math*, 1980; 11(9): 1202-1209.
7) Mohanty G, and Sahoo PK Inhomogeneous plane symmetric vacuum model" *journal of Orissa mathematical society*, 2002-2003; 21-22: 143-147.
8) Sahoo PK. Inhomogeneous Plane Symmetric String Cosmological Models in Bimetric Theory*", Int J Theor Phys*, 2009; 48: 2022–2026
9) Mohanty G, Sahoo PK, Mishra B. On Bianchi Type-I Mesonic Cosmological Model In bimetric Theory"(????).
10) Katore SD. Magnetized cosmological models in bimetric theory of gravitation", *journal of physics*, 2006; 67(2): 227-23.
11) Deo SD. Spherically Symmetric Cosmic Strings and Domain Walls in Bimetric Relativity." *International J of Applied Computational Science and Mathematics,* 2012; 2(1): 23-28
12) Deo SD. On Bianchi-I cosmic strings coupled with Maxwell fields in bimetric relativity". *Journal of physics*, 2003; 61(4): 655–658.
13) Reddy DRK and Venkateswarlu R. Magnetized cosmological model in lyra manifold" *Astrophys. Space Sci.* 1988; 149: 287
14) Reddy DRK and Rao NV. On Some Bianchi Type cosmological models in bimetric theory of gravitation", *Astrophys. Space Sci.*, 1998; 257, 293-298.