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Some Fixed Point Theorems for Mappings under General Contractive Condition of Intregral Type in Rational Form

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ABSTRACT: In this paper we prove a fixed point theorem for a pair of mappings, the mapping involved here generalizes various type of contractive mappings in integral setting in rational form.

Keywords: Fixed point, Complete metric Space, General contractive condition, Integral type

AMS Subject Classification-47H10, 54H25.

I. INTRODUCTION AND PRELIMINARIES

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first important result on fixed point for contractive type mapping was the much celebrated Banach's contraction principle by S.Banach [1] in 1992. In the general setting of complete metric space, this theorem runs as follows (see theorem 2.1, [4] or theorem 1.2.2, [13]). After following theorem 1.1 to get classical result. Kannah [5] gave a substantially new contractive mapping to prove fixed point theorem. Since then a number of mathematicians have been working on fixed point theory dealing with mappings satisfying various type of contractive condition [6], [8-12] and [14].

In 2002 theorem 1.2 A Branciari [2] analyzed the existence of fixed point for mapping f defined on a complete metric space(x, d) satisfying a general contractive condition of integral type. After theorem 1.2 we get a lot of research works have been carried out on generalizing contractive conditions of integral type for different contractive mapping satisfying various known properties. A fine work has been done by Rhodes [7] extending the result of theorem by replacing the condition (1.3) by the following

$$\int_0^{d(fx,fy)} \psi(t)dt \le c\int_0^{\max\{d(x,y),d(x,fx),d(y,fy),\frac{[d(x,fx),d(y,fx)]}{2}\}} \psi(t)dt \quad \ldots (1.1)$$

Theorem 1.1: Let(X, d) be a complete metric space, $c \in (0,1)$ *and* $f{:}X \to X$ be a mapping such that for each $x, y \in X$

$$d\left(fx, fy\right) \le cd(x, y) \quad \ldots (1.2)$$

Then f has a unique fixed point $a \in x$, such that for each

$x \in X, \quad \lim_{n\to\infty} f^n x = a$

Theorem 1.2: Let(X, d) be a complete metric space $c\in(0,1)$ and $f:X\to X$ be a mapping such that for each $x, y\in X$

$$\int_0^{d(fx,fy)}\psi(t)dt \le c\int_0^{d(x,y)}\psi(t)dt \qquad \text{…(1.3)}$$

Where $\psi:[(0, +\infty)\to(0, +\infty)]$ is a lesbesgue integrable mapping which is summable (i.e with finite integral) on each compact subset of [(0, +∞)] non-negative and such that for each $\in >0$, $\int_0^{\varepsilon}\psi(t)dt>0$ then f has a unique fixed point $a\in x$. Such that $x\in X$, $\lim_{n\to\infty} f^n x=a$

The aim of this paper is to generalize some mixed type of contractive condition to the mapping and then a pair of mappings satisfying a general contractive condition of integral type which includes several known contractive mapping, such as Kannan type [5], Chatterjea type [3]

,Zamfiresu type [14] etc.

II. MAIN RESULTS

Theorem 2.1: Let f be a self mapping of a complete metric space (X, d) satisfying the following condition:

$$\begin{aligned}\int_0^{d(fx,fy)}\psi(t)dt &\le \alpha\int_0^{d[(x,fx)+d(y,fy)]}\psi(t)dt \\ &+\beta\int_0^{d[(x,fy)+d(y,fx)]}\psi(t)dt+\gamma\int_0^{d(x,y)}\psi(t)dt \\ &+\delta\int_0^{max[d(x,fx),d(y,fx)]}\psi(t)dt \\ &+\eta\int_0^{\frac{d(x,fy)+d(y,fx)+d(x,fx)}{1+(x,fy)d(y,fx)d(x,fx)}}\psi(t)dt \\ &+\mu\int_0^{\frac{d(x,fx)+d(y,fy)+d(x,fy)}{1+(x,fy)d(y,fx)d(x,fx)}}\psi(t)dt \qquad \text{…(2.1)}\end{aligned}$$

For each x,y $\in X$ with non negative reals $\alpha,\beta,\gamma,\delta,\eta,\mu$ Such that $2\alpha+2\beta+\gamma+2\delta+3\eta+4\mu<1$ where $\psi:R^+\to R^+$ is a lesbesgue integrable mapping which is summable on each compact subset of R^+ non negative and such that

For each $\in >0$, $\int_0^{\varepsilon}\psi(t)dt >0.$ …(2.2)

Then f has a unique fixed point $z\in X$ and for each $x\in X$, $\lim_{n\to\infty} f^n x=z$

Proof: Let $x_0 \in X$ and for brevity, define $x_n = fx_{n-1}$ for each integer $n \geq 1$, from (2.1) we get

$$\int_0^{d(x_n, x_{n+1})} \psi(t)dt \leq \int_0^{d(fx_{n-1}, fx_n)} \psi(t)dt$$

$$\leq \alpha \int_0^{d[(x_{n-1}, x_n) + d(x_n, x_{n+1})]} \psi(t)dt$$

$$+ \beta \int_0^{d[(x_{n-1}, x_{n+1}) + d(x_n, x_n)]} \psi(t)dt$$

$$+ \gamma \int_0^{d(x_{n-1}, x_n)} \psi(t)dt + \delta \int_0^{max[d(x_{n-1}, x_{n+1}), d(x_n, x_n)]} \psi(t)dt$$

$$+ \eta \int_0^{\frac{d(x_{n-1}, x_{n+1}) + d(x_n, x_n) + d(x_{n-1}, x_n)}{1 + (x_{n-1}, x_{n+1}) d(x_n, x_n) d(x_{n-1}, x_n)}} \psi(t)dt$$

$$+ \mu \int_0^{\frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + d(x_{n-1}, x_{n+1})}{1 + d(x_n, x_{n+1}) d(x_n, x_n) d(x_{n-1}, x_n)}} \psi(t)dt$$

$$\leq \alpha \int_0^{d[(x_{n-1}, x_n) + d(x_n, x_{n+1})]} \psi(t)dt$$

$$+ \beta \int_0^{d[(x_{n-1}, x_{n+1})]} \psi(t)dt$$

$$+ \gamma \int_0^{d(x_{n-1}, x_n)} \psi(t)dt + \delta \int_0^{d(x_{n-1}, x_{n+1})} \psi(t)dt$$

$$+ \eta \int_0^{d(x_{n-1}, x_{n+1}) + d(x_{n-1}, x_n)} \psi(t)dt$$

$$+ \mu \int_0^{2[d(x_{n-1}, x_n) + d(x_n, x_{n+1})]} \psi(t)dt$$

$$\leq (\alpha + \beta + \gamma + \delta + 2\eta + 2\mu) \int_0^{d(x_{n-1}, x_n)} \psi(t)dt + (\alpha + \beta + \gamma + \delta + \eta + 2\mu) \int_0^{d(x_n, x_{n+1})} \psi(t)dt$$

Which implies that

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le \left(\frac{\alpha+\beta+\gamma+\delta+2\eta+2\mu}{1-\alpha-\beta-\delta-\eta-2\mu}\right)\int_0^{d(x_{n-1},x_n)}\psi(t)dt.$$

And so

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le k\int_0^{d(x_{n-1},x_n)}\psi(t)dt$$

Where

$$K=\frac{\alpha+\beta+\gamma+\delta+2\eta+2\mu}{1-\alpha-\beta-\delta-\eta-2\mu}<1 \qquad \text{….(2.3)}$$

Thus by routine calculation

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \le k^n\int_0^{d(x_0,x_1)}\psi(t)dt \qquad \text{….(2.4)}$$

Taking limit of (2.4) as $n\to\infty$ we get

$$\lim_n \int_0^{d(x_n,x_{n+1})}\psi(t)dt = 0$$

Which from (2.2) implies that

$$\lim_n d(x_n,x_{n+1})=0 \qquad \text{…(2.5)}$$

We now show that $\{x_n\}$ is a Cauchy sequence suppose that it is not. Then there exist $\epsilon>0$,and subsequences $\{m(p)\}$ and $\{n(p)\}$ such that $d(x_m(p), x_n(p))\ge \in$,

$d(x_m(p), x_n(p)\text{-}1)>0$………………(2.6)

$d(x_m(p)\text{-}1, x_n(p)\text{-}1)\le d(x_m(p)\text{-}1, x_{,m}(p))+ d(x_m(p), x_n(p)\text{-}1)$

$$\le d(x_m(p)\text{-}1, x_m(p))+ \in \qquad \text{...(2.7)}$$

hence

$$\lim_{p} \int_0^{d(x_m(p)-1,x_n(p)-1)} \psi(t)dt \leq \int_0^{\epsilon} \psi(t)dt \qquad \text{….(2.8)}$$

Using (2.3),(2.6) and(2.8) we get

$$\int_0^{\epsilon} \psi(t)dt \leq \int_0^{d(x_m(p),x_n(p))} \psi(t)dt$$

$$\leq k\int_0^{d(x_m(p)-1,x_n(p)-1)} \psi(t)dt$$

$$\leq k\int_0^{\epsilon} \psi(t)dt$$

Which is contradiction, since $k \in (0, 1)$, therefore $\{x_n\}$ is Cauchy sequence, hence convergent call the limit Z.

From (2.1) we get

$$\int_0^{d(fz,x_{n+1})} \psi(t)dt \leq \alpha\int_0^{[d[(z,fz)+d(x_n,x_{n+1})]} \psi(t)dt$$

$$+\beta\int_0^{[d[(z,x_{n+1})+d(x_n,fz)]} \psi(t)dt$$

$$+\gamma\int_0^{d(z,x_n)} \psi(t)dt+\delta\int_0^{max[d[(z,x_{n+1}),d(x_n,fz)]} \psi(t)dt$$

$$+\eta\int_0^{\frac{d(z,x_{n+1})+d(x_n,fz)+d(z,fz)}{1+d(z,x_{n+1})d(x_n,fz)d(z,fz)}} \psi(t)dt$$

$$+\mu\int_0^{\frac{d(z,fz)+d(x_n,x_{n+1})+d(z,x_{n+1})}{1+d[(z,x_{n+1})d(x_n,fz)d(z,fz)}} \psi(t)dt$$

Taking limit as $n \to \infty$ we get

$$\int_0^{d(fz,z)} \psi(t)dt \leq (\alpha+\beta+\delta+2\eta+\mu)\int_0^{d(z,fz)} \psi(t)dt$$

$$2\alpha + 2\beta + \gamma + 2\delta + 3\eta + 4\mu < 1$$

$$\int_0^{d(z,fz)} \psi(t)dt = 0$$

Which is from (2.2) implies that *(fz,z)=0 or fz=z*

Next suppose that *w(≠z)* be another fixed point of *f*

Then (2.1) we have

$$\int_0^{d(z,w)} \psi(t)dt = \int_0^{d(fz,fw)} \psi(t)dt$$

$$\int_0^{d(z,w)} \psi(t)dt \le \alpha \int_0^{[d[(z,fz)+d(w,fw)]} \psi(t)dt$$

$$+\beta \int_0^{[d[(z,fw)+d(w,fz)]} \psi(t)dt$$

$$+\gamma \int_0^{d(z,w)} \psi(t)dt + \delta \int_0^{max[d[(z,fw),d(w,fz)]} \psi(t)dt$$

$$+\eta \int_0^{\frac{d(z,fw)+d(w,fz)+d(z,fz)}{1+d(z,fw)d(w,fz)d(z,fz)}} \psi(t)dt$$

$$+\mu \int_0^{\frac{d(z,fz)+d(w,fw)+d(z,fw)}{1+d(z,fw)d(w,fz)d(z,fz)}} \psi(t)dt$$

$$\le (2\beta + \gamma + \delta + 2\eta + \mu) \int_0^{d(z,w)} \psi(t)dt$$

Since $2\beta + \gamma + \delta + 2\eta + \mu < 1$ This implies that $\int_0^{d(z,w)} \psi(t)dt = 0$ which from (2.2) implies that *d(z, w)=0 or z=w* and so the fixed point is unique.

Remarks: - from condition (2.1) of integral type several contractive mappings of integral type can be obtained

1. $\beta = \gamma = \delta = \eta = \mu = 0$ *And* $\alpha \in (0, 1/2)$ Gives Kannan mapping of integral type

2. $\alpha = \gamma = \delta = \eta = \mu = 0$ And $\beta \in (0, 1/2)$ at least one of the following condition hold.

3. $\alpha = \beta = \gamma = \eta = \mu = 0$ And $\delta \in (0, 1/2)$ gives Ramakant Bhardwaj mapping of integral type

Theorem 2.3: let *f* and *g* be self mappings of a complete metric space *(X,d)* satisfying the following condition.

$$\int_0^{d(fx,gy)}\psi(t)dt \le \alpha\int_0^{d[(x,fx)+d(y,gy)]}\psi(t)dt$$
$$+\beta\int_0^{d[(x,gy)+d(y,fx)]}\psi(t)dt$$
$$+\gamma\int_0^{d(x,y)}\psi(t)dt+\delta\int_0^{max[d[(x,gy),d(y,fx)]}\psi(t)dt$$
$$+\eta\int_0^{\frac{d(x,gy)+d(y,fx)+d(x,fx)}{1+d(x,gy)d(y,fx)d(x,fx)}}\psi(t)dt$$
$$+\mu\int_0^{\frac{d(x,fx)+d(y,gy)+d(x,gy)}{1+d[(x,gy)d(y,fx)d(x,fx)}}\psi(t)dt$$

…(2.9)

For each x , $y \in X$ with non negative reals $\alpha,\beta,\gamma,\delta,\eta,\mu$ such that $2\alpha+2\beta+\gamma+2\delta+3\eta+4\mu<1$ where $\psi:R^+\to R^+$ is lesbesgue integrable mapping which is summable(i.e. with finite integral) on each compact subset of R^+ ,non negative and such that for each

$\in>0$, $$\int_0^{\in}\psi(t)dt>0$$ ...(2.10)

Then *f* and *g* have a unique common fixed point $z\in X$

Proof: - let $x_o\in X$ and for brevity define $x_{2n+1} = fx_{2n}$ and $x_{2n+2}=gx_{2n+1}$. For each integer $n\ge 0$, from (2.9) We get

$$\int_0^{d(x_{2n+1},x_{2n+2})}\psi(t)dt \le \int_0^{d(fx_{2n},gx_{2n+1})}\psi(t)dt$$
$$\le\alpha\int_0^{[d[(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})]}\psi(t)dt$$
$$+\beta\int_0^{[d[(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1})]}\psi(t)dt$$
$$+\gamma\int_0^{d(x_{2n},x_{2n+1})}\psi(t)dt+\delta\int_0^{max[d(x_{2n},x_{2n+2}),d(x_{2n+1},x_{2n+1})]}\psi(t)dt$$
$$+\eta\int_0^{\frac{d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1})+d(x_{2n},x_{2n+1})}{1+d(x_{2n},x_{2n+2})d(x_{2n+1},x_{2n+1})d(x_{2n},x_{2n+1})}}\psi(t)dt$$
$$+\mu\int_0^{\frac{d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})+d(x_{2n},x_{2n+2})}{1+d(x_{2n},x_{2n+2})d(x_{2n+1},x_{2n+1})d(x_{2n},x_{2n+1})}}\psi(t)dt$$

$$\leq (\alpha+\beta+\gamma+\delta+2\eta+2\mu)\int_0^{d(x_{2n},x_{2n+1})}\psi(t)dt$$

$$+(\alpha+\beta+\delta+\eta+2\mu)\int_0^{d(x_{2n+1},x_{2n+2})}\psi(t)dt$$

Which implies that

$$\int_0^{d(x_{2n+1},x_{2n+2})}\psi(t)dt \leq \frac{(\alpha+\beta+\gamma+\delta+2\eta+2\mu)}{1-\alpha-\beta-\delta-\eta-2\mu}\int_0^{d(x_{2n},x_{2n+1})}\psi(t)dt$$

And so

$$\int_0^{d(x_{2n+1},x_{2n+2})}\psi(t)dt \leq k\int_0^{d(x_{2n},x_{2n+1})}\psi(t)dt$$

Where k= $\frac{(\alpha+\beta+\gamma+\delta+2\eta+2\mu)}{1-\alpha-\beta-\delta-\eta-2\mu}$ <1 ...(2.11)

Similarly

$$\int_0^{d(x_{2n},x_{2n+1})}\psi(t)dt \leq k\int_0^{d(x_{2n-1},x_{2n})}\psi(t)dt \quad \text{…(2.12)}$$

This is general, for all n=1, 2…..

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \leq k\int_0^{d(x_{n-1},x_n)}\psi(t)dt \quad \text{…(2.13)}$$

Thus by routine calculation, we have

$$\int_0^{d(x_n,x_{n+1})}\psi(t)dt \leq k^n\int_0^{d(x_0,x_1)}\psi(t)dt$$

Taking limit as $n\to\infty$, we get

$$\lim_{n}\int_0^{d(x_n,x_{n+1})}\psi(t)dt = 0$$

Which from (2.10)

$$\lim_{n} d(x_n, x_{n+1}) = 0 \qquad \ldots(2.14)$$

We now show that $\{x_n\}$ is a Cauchy sequence, suppose that it is not.

Then there exist $\epsilon>0$ and subsequences

$\{m(p)\}$ *and* $\{n(p)\}$ such that $d(x_m(p), x_n(p)) \geq \in$,

$$d(x_m(p), x_n(p)-1)>0 \qquad \ldots(2.15)$$

$$d(x_m(p)-1, x_n(p)-1) \leq d(x_m(p)-1, x_{,m}(p)) + d(x_m(p), x_n(p)-1)$$

$$\leq d(x_m(p)-1, x_m(p)) + \in \qquad \ldots(2.16)$$

Now

$$d(x_{2m}(p),x_{2n}(p)) \leq d(x_{2m}(p),x_{2n}(p)-2) + d(x_{2n}(p)-2,x_{2n}(p)-1) + d(x_{2n}(p)-1,x_{2n}(p))$$

$$< \epsilon + d(x_{2n}(p)-2, x_{2n}(p)-1) + d(x_{2n}(p)-1, x_{2n}(p)) \qquad \ldots(2.17)$$

hence

$$\lim_{n} \int_0^{d(x_{2m}(p),x_{2n}(p))} \psi(t)dt \leq \int_0^{\in} \psi(t)dt$$

Then by (2.13) we get

$$\int_0^{d(x_{2m}(p),x_{2n}(p))} \psi(t)dt \leq k \int_0^{d(x_{2m}(p)-1,x_{2n}(p)-1)} \psi(t)dt$$

$$\leq k \left[\begin{array}{l} \int_0^{d(x_{2m}(p)-1,x_{2m}(p))} \psi(t)dt + \int_0^{d(x_{2m}(p),x_{2n}(p))} \psi(t)dt \\ + \int_0^{d(x_{2n}(p)-1,x_{2n}(p))} \psi(t)dt \end{array} \right].$$

Taking limit as $p \to \infty$ we get

$$\int_0^{\in} \psi(t)dt \leq k \int_0^{\in} \psi(t)dt$$

Which is contradiction, since $k \in (0,1)$. Therefore $\{x_n\}$ is Cauchy, hence convergent, call the limit Z.

From (2.9) we get

$$\int_0^{d(fz,x_{2n+2})} \psi(t)dt \le \int_0^{d(fz,gx_{2n+1})} \psi(t)dt$$

$$\le \alpha \int_0^{[d[(z,fz)+d(x_{2n+1},x_{2n+2})]} \psi(t)dt$$

$$+\beta \int_0^{[d[(z,x_{2n+2})+d(x_{2n+1},fz)]} \psi(t)dt$$

$$+\gamma \int_0^{d(z,x_{2n+1})} \psi(t)dt + \delta \int_0^{max[d(z,x_{2n+2}),d(x_{2n+1},fz)]} \psi(t)dt$$

$$+\eta \int_0^{\frac{d(z,x_{2n+2})+d(x_{2n+1},fz)+d(z,fz)}{1+d(z,x_{2n+2})d(x_{2n+1},fz)d(z,fz)}} \psi(t)dt$$

$$+\mu \int_0^{\frac{d(z,fz)+d(x_{2n+1},x_{2n+2})+d(z,x_{2n+2})}{1+d(z,x_{2n+2})d(x_{2n+1},fz)d(z,fz)}} \psi(t)dt$$

Taking limit as $n \to \infty$ we get

$$\int_0^{d(fz,z)} \psi(t)dt \le (\alpha+\beta+\delta+2\eta+\mu)\int_0^{d(z,fz)} \psi(t)dt$$

as

$$(2\alpha+2\beta+\gamma+2\delta+3\eta+4\mu)<1$$

$$\int_0^{d(z,fz)} \psi(t)dt = 0$$

Which from (2.10) implies that $d(fz, z) = 0$ or $fz=z$.

Similarly it can be shown that $gz=z$. so f and g have a common fixed point $z\epsilon X$. we now so that z is the unique fixed point of f and g. If not then let $w(\neq z)$ be another fixed point of f and g then from (2.9) we have

$$\int_0^{d(z,w)} \psi(t)dt \le \int_0^{d(fz,gw)} \psi(t)dt$$

$$\int_0^{d(z,w)} \psi(t)dt \le \alpha\int_0^{[d[(z,fz)+d(w,gw)]} \psi(t)dt$$

$$+\beta\int_0^{[d[(z,gw)+d(w,fz)]} \psi(t)dt$$

$$+\gamma\int_0^{d(z,w)} \psi(t)dt + \delta\int_0^{max[d[(z,gw),d(w,fz)]} \psi(t)dt$$

$$+\eta\int_0^{\frac{d(z,gw)+d(w,fz)+d(z,fz)}{1+d(z,gw)d(w,fz)d(z,fz)}} \psi(t)dt$$

$$+\mu\int_0^{\frac{d(z,fz)+d(w,gw)+d(z,gw)}{1+d(z,gw)d(w,fz)d(z,fz)}} \psi(t)dt$$

$$\le (2\beta+\gamma+\delta+2\eta+\mu)\int_0^{d(z,w)} \psi(t)dt$$

Since $2\beta+\gamma+\delta+2\eta+\mu$ <1. this implies that $\int_0^{d(z,w)} \psi(t)dt$ =0

Which from (2.10) implies that $d(z, w) = 0$ *or* $z=w$ and so the fixed point is unique.

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