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Anisotropic Bianchi Type-I Model in a Self Creation Cosmology

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ABSTRACT: Bianchi type-I cosmological model filled with perfect fluid in Barber second self-creation theory is considered by using deceleration parameter to be constant where the metric potentials are taken as function of x and t.. Einstein field equations are considered in the presence of a perfect fluid for a Bianchi type-I universe. Some physical consequences of the models have been discussed in case of Zel'dovich fluid and radiation dominated fluid.

Keywords: Bianchi Type I Universe, Deceleration parameter, Hubble's parameter, Cosmological models.
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I. INTRODUCTION

Cosmology is the scientific study of large scale properties of the universe as a whole. Like any field of science, cosmology involves the formation of theories or hypothesis about the universe which make specific predictions for phenomenon that can be tested with observations. The origin of our universe is one of the greatest cosmological mysteries even today.

Many of the Researchers are involved in predicting these mysteries through the study of various cosmological models. Pimentel [10], Soleng [11,12], Singh [13], Reddy[14], Reddy *et al.* [15,16,17,18], Reddy and Venkateshwarlu[19], Venkateshwarlu and Reddy [20], Shanti and Rao [21], Sanyasiraju and Rao [22], Shri and Singh [23,24], Mohanty et al. [25], Pradhan and Vishwakarma [26,27], Sahu and Panigrahi [28], Venkateshwarlu and Kumar [29], Tiwari and Kumar [30], Pradhan, Agarwal and Singh [31], Akarsu and Kilinc [32] are some of the researchers who have studied various aspects of cosmological models in self-creation theory.

In the early stages of evolution of universe, the forms of matter fields are uncertain. But in the present scenario, the Universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. The large-scale structure of the present Universe is suitably represented by Friedmann-Robertson-Walker (FRW) models which are itself an isotropic and homogeneous in nature.

We are interested in investigating the early stages of the evolution of the Universe, for which models with anisotropic background are more suitable. Bianchi type-I homogeneous models are the simplest anisotropic models of the universe whose spatial sections are flat, but the expansion or contraction rate are directionally dependent. For a simplification and description of the large scale structure and behaviour, of the actual Universe, anisotropic Bianchi type I models have been considered.

In this paper, we study the homogeneous anisotropic Bianchi type-I spacetime in a self creation theory. In order to solve the field equations, we apply a condition of exponential expansion. This condition, together with the Einstein's field equations, leads to the solutions which are discussed for the Zel'dovich fluid, radiation and vacuum dominated cases. In each case physical behaviour of the model is discussed in detail and the nature of initial singularity is clarified.

II. MODEL AND FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type –I space time is described by line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad ...(1)$$

The spatial volume of this model is given by

$$V = ABC \quad ...(2)$$

we define $S=(ABC)^{\frac{1}{3}}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H=\frac{\dot{S}}{S}=\frac{1}{3}\left(H_1+H_2+H_3\right) \qquad \text{… (3)}$$

where a dot stands for ordinary time derivative of the concerned quantity and $H_1=\frac{\dot{A}}{A}$,

$H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$ are directional Hubble factors in the *x, y, z* directions, respectively.

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij}=(\rho+p)\upsilon_i\upsilon_j+pg_{ij} \qquad \text{… (4)}$$

where ρ and p are energy density and thermodynamic pressure and υ_i is the four velocity vector of the fluid satisfying the relation $\upsilon_i\upsilon^i=-1$. We assume that the matter content obeys an equation of state

$$p=\omega\rho.0\le\omega\le 1 \qquad \text{… (5)}$$

The Einstein's field equations in Barber's self- creation theory (1982) are

$$R_{ij}-\frac{1}{2}g_{ij}R=-8\pi\phi^{-1}T_{ij} \qquad \text{… (6)}$$

$$\Box\phi = \frac{8\pi}{3}\eta T \qquad \text{…(7)}$$

Where $\Box\phi^{;k}_{;k}=\phi$ is the invariant d'Alembertian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of the coupling to $|\eta|\le 10^{-1}$. This theory leads to the Einstein's theory in every respect when $\eta \to 0$.

In a commoving system of coordinates the field equations (6) for the metric (1) can be written as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi\phi^{-1}p \qquad \text{…(8)}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi\phi^{-1}p \qquad \text{…(9)}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-8\pi\phi^{-1}p \qquad \text{…(10)}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi\phi^{-1}\rho \qquad \text{…(11)}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{8\pi\eta}{3}\left(\rho-3p\right) \qquad \text{…(12)}$$

If we use the equivalent energy conservation equation of general relativity quantities, we find that

$$\left(\frac{\rho}{\phi}\right)^{\bullet}+\left(\frac{\rho+p}{\phi}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \qquad \text{…(13)}$$

The shear scalar σ is obtained as

$$\sigma^2 = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A}\right)\right] \qquad \ldots(14)$$

III. SOLUTIONS OF FIELD EQUATIONS

The field equations (8) - (12) supply five independent equations in six unknowns A, B, C, ρ, p, ϕ. Extra conditions are needed to solve the system completely. As the early universe experiences a period of accelerated expansion and essentially expands at an exponential rate [36], we can consider

$$S = e^{at} \qquad \ldots(15)$$

On integrating Eqs (8)-(10), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = k_3 e^{-3at} \qquad \ldots(16)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = k_4 e^{-3at} \qquad \ldots(17)$$

where k_3 and k_4 are constants of integration.

From Eqs. (16) and (17), we obtain

$$A(t) = c_1 \exp\left(at - \left(\frac{2k_3 + k_4}{9a}\right)e^{-3at}\right) \qquad \ldots(18)$$

$$B(t) = c_2 \exp\left(at + \left(\frac{k_3 - k_4}{9a}\right)e^{-3at}\right) \qquad \ldots(19)$$

$$C(t) = c_3 \exp\left(at + \left(\frac{k_3 + 2k_4}{9a}\right)e^{-3at}\right) \qquad \ldots(20)$$

Eq. (13) yields

$$\frac{\rho}{\phi} = k_1 e^{-3(\omega+1)at} \qquad \ldots(21)$$

Here Deceleration parameter q is obtained as

$$q = \frac{-S\ddot{S}}{\dot{S}^2} = -1 \qquad \ldots(22)$$

Law (15) gives a negative value of the deceleration parameter. Hence our model represents an accelerating universe.

We obtain the line element (1) in the form

$$ds^2 = -dt^2 + c_1^2 \exp 2\left(at - \left(\frac{2k_3 + k_4}{9a}\right)e^{-3at}\right)dx^2 + c_2^2 \exp 2\left(at + \left(\frac{k_3 - k_4}{9a}\right)e^{-3at}\right)dy^2 + c_3^2 \exp 2\left(at + \left(\frac{k_3 + 2k_4}{9a}\right)e^{-3at}\right)dz^2 \qquad \ldots(23)$$

Eq (12) yields

$$\ddot{\phi} + 3a\dot{\phi} = \frac{8\pi\eta}{3} k_1 (1 - 3\omega) e^{-3(\omega+1)at} \phi \qquad \ldots(24)$$

Now we discuss the model for Zel dovich fluid, radiation dominated case and the vacuum case.

A. Zel Dovich Fluid Distribution

It corresponds to the equation of state $\rho = p$. This equation of state has been widely used in general relativity to obtain stellar and cosmological model for utter dense matter (Zel' dovich 1962).

Equation (24) on integration gives

$$\phi(t) = k_5 Cos\left[\sqrt{\frac{16\pi\eta k_1}{3}}\left(\frac{e^{-3at}}{3a}\right)\right] \quad \ldots(25)$$

Where c_3 and k_1 are constants of integration and η is non- negative.

The energy density and pressure are given by

$$\rho = p = k_1 e^{-6at}\left\{k_5 Cos\left[\sqrt{\frac{16\pi\eta k_1}{3}}\left(\frac{e^{-3at}}{3a}\right)\right]\right\} \quad \ldots(26)$$

For the reality condition ρ> 0 to hold, it is necessary that $k_1 > 0$

Cosmological parameters are given by

The volume of the universe is

$$V = c' e^{3at} \quad \ldots(27)$$

where $c' = c_1 c_2 c_3$

$$\theta = 3a \quad \ldots(28)$$

$$\sigma = \sqrt{\frac{k_3^{\ 2} + k_4^{\ 2} + k_3 k_4}{3e^{6at}}} \quad \ldots(29)$$

$$H_1 = \frac{(2k_3 + k_4)}{3} e^{-3at} + a \quad \ldots(30)$$

$$H_2 = \frac{(k_4 - k_3)}{3} e^{-3at} + a \quad \ldots(31)$$

$$H_3 = \frac{-(k_3 + 2k_4)}{3} e^{-3at} + a \quad \ldots(32)$$

The anisotropy parameter is

$$\overline{A} = \frac{2}{9a^2} e^{-6at}\left[k_4^{\ 2} + k_3^{\ 2} + k_3 k_4\right] \ldots(33)$$

Observations

(i) Volume V→ constant as $t \to 0$ and as $t \to \infty$ the spatial volume V increases. Here expansion scalar remain constant throughout the whole span of evolution, which shows that our universe is expanding uniformly

(ii) Clearly the scalar field remains finite throughout the evolution of universe..

(iii) Energy density $\rho \to k_1\phi$ as $t \to \infty$ i.e. in the absence of field ϕ, energy density tends to zero and field ϕ is only responsible for energy density at the later stages of evolution of the universe.

(iv) The energy density, pressure, anisotropic parameter and directional Hubble's factors tend to zero as $t \to \infty$.

(v) The shear σ tends to zero as $t \to \infty$. Since $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, the model approaches isotropy for large values of t.

(vi) The model represents non shearing, non rotating and expanding universe with uniform rate of expansion. .

B. Radiation Dominated Solutions

Disordered radiation corresponds to the equation $\rho = 3p$. In this case (24) immediately integrates to field

$$\phi(t) = \frac{k_6}{-3a} e^{-3at} + k_7 \quad \ldots(34)$$

$$\rho = 3p = k_1 e^{-4at}\left(\frac{-k_6}{3a} e^{-3at} + k_7\right) \ldots(35)$$

$$V = c' e^{3at} \quad \ldots (36)$$

where $c' = c_1 c_2 c_3$

$$\theta = 3a \quad \ldots(37)$$

$$\sigma = \sqrt{\frac{k_3^{\ 2} + k_4^{\ 2} + k_3 k_4}{3e^{6at}}} \quad \ldots(38)$$

$$H_1 = \frac{(2k_3 + k_4)}{3} e^{-3at} + a \quad \ldots(39)$$

$$H_2 = \frac{(k_4 - k_3)}{3} e^{-3at} + a \quad \ldots(40)$$

Where k_6 and k_7 are constant of integration..
Other cosmological parameter are given by

$$H_3 = \frac{-(k_3 + 2k_4)}{3} e^{-3at} + a \qquad \ldots(41)$$

$$\overline{A} = \frac{2}{9a^2} e^{-6at} \left[k_4^{\ 2} + k_3^{\ 2} + k_3 k_4 \right] \quad \ldots(42)$$

Observations

(i) Volume V→ constant as t→0 and as $t \to \infty$ the spatial volume V increases. Here expansion scalar remain constant throughout the whole span of evolution, which shows that our universe is expanding uniformly.
(ii) At the initial stage of evolution scalar field was finite and started decreasing as t→ ∞.
(iii) Rest of the results are similar as observed in the case of Zel Dovich fluid distribution.
(iv) The model represents non shearing, non rotating and expanding universe with uniform rate of expansion. .

C. Vaccum Solution (ρ = p= 0)
In this case, the behavior of the model is same as in section C.

IV. CONCLUDING REMARKS

In this paper a spatially homogeneous and anisotropic Bianchi type-1 model has been investigated. The field equations have been solved exactly by considering law of exponential expansion. We obtained a non shearing cosmological model of Barber's second self creation theory in a Bianchi type-I model. We observe that as $t \to \infty, V \to \infty$ and $\rho \to 0$ i.e spatial volume (V) increases with time and proper energy density (ρ) decreases with time as expected. The main features of the derived model are as follows:

- Cosmological models are presented Zel'dovich fluid, radiation dominated and vacuum cases.
- Geometrical and kinematical properties of different parameters have been discussed in detail for each phase. The nature of singularities of the models is clarified and explicit forms of scalar factors are obtained.
- Deceleration parameter is negative. This is consistent with the present observational results i.e. our universe is accelerating.
- Our model approaches isotropy at the later time.
- In contrary with the results of paper by Pradhan and Pandey [33], here in both the sections while the anisotropy parameter decreases and tend to null as the universe expands, the energy density of the fluid also decreases to null.
- In our model, the scalar field $\varphi(t)$ and expansion θ doesn't possess any kind of initial singularity irrespective of the results presented by Venkateshwarlu and Reddy [20].
- In general it has been observed that the model represents non shearing, non-rotating and expanding universe.

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