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Fixed Point and Common Fixed Point Theorem in 2-Banach Space Taking Rational Expression for 1, 2, 3 Mapping

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ABSTRACT : In the present paper we will establish some fixed point and common fixed point theorem in 2-Banach space taking rational expression for 1,2,3 mappings. Our result is extended form of many known results taking particular inequality.

Keywords : Fixed point, Common fixed point, Banach space.

I. INTRODUCTION

In recent years, nonlinear analysis have attracted much attention .The study of non contraction mapping concerning the existence of fixed points draw attention of various authors in non linear analysis. It is well known that the differential and integral equations that arise in physical problems are generally nonlinear, therefore fixed point methods especially Banach contraction principle provide powerful tool for obtaining the solution of these equations which are very difficult to solve by other method. Recently Verma [24] described about the application of Banach contraction principle [2].

Browder [4] was the first mathematician to study non expansive mappings. Mean while Browder [4] and Ghode [6] have independently proved a fixed point theorem for non expansive mapping.

Many other Mathematicians have done the generalization of non-expansive mappings as well as non-contraction mappings Kirk [15, 16 & 17] gives the comprehensive survey concerning fixed point theorems for non expansive mappings.

Ghalar [10] introduced the concept of 2-Banach spaces. Recently Yadava, Rajput, Chaudhary, Bhardwaj [28, 29] and Dwivedi, Bhardwaj, Shrivastava [6] worked for this space; motivated by them we are proving for different rational inequality.

Before start the main result we write some definitions.

II. PRELIMINARIES

Definition 1: Gahler [10] defined a linear 2-normed space. Let L be a linear space and $\|,\|$ is nonnegative, real valued function define on L such that for all x, y, z $\in L$ and $\alpha \in R$ or C.

(*i*) $\|x, y\| = 0 \Leftrightarrow x, y$ are linearly dependent

(*ii*) $\|x, y\| = \|y, x\|$

(*iii*) $\|x, \alpha y\| = |\alpha|\|x, y\|$

(*iv*) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$

Then $\|,\|$ is called 2-norm and $(L, \|,\|)$ is called 2-normed linear space.

Definition 2: A sequence in a 2-normed linear space L, is called Cauchy sequence if

$$\lim_{x\to\infty} \| x_n - x, y \| = 0 \text{ for all } y \in L$$

Definition 3: A sequence in a 2-normed linear space L, is called Cauchy sequence if

$$\lim_{x\to\infty} \| x_n - x_m, y \| = 0$$

Definition 4: A 2-normed linear space in which every Cauchy sequence is convergent is called 2-Banach space.

III. MAIN RESULT

Theorem 1:

Let F be mapping of a 2- Banach space X into it self. If F satisfies the following conditions:

(*i*) $F^2 = I$, where I is identity mapping.

(*ii*) $\|F(X) - F(Y), a\|$

$$\leq \alpha\left[\frac{\|X-F(X),a\|\|X-Y,a\|+\|Y-F(Y),a\|\|Y-F(X),a\|+\|X-Y,a\|^2}{\|X-F(X),a\|+\|X-Y,a\|}\right]$$

$$+\beta\left[\frac{\|Y-F(Y),a\|\|X-Y,a\|+\|X-F(X),a\|\|X-F(Y),a\|+\|X-Y,a\|^2}{\|Y-F(Y),a\|+\|X-Y,a\|}\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\left[\|X-F(Y),a\|+\|Y-F(X),a\|\right]+\eta\,\|X-Y,a\|$$

For every x, $y \in X$, where $\alpha,\beta,\gamma,\delta,\eta>0$ and $5\alpha+5\beta+4\gamma+2\delta+\eta<2$, then F has a fixed point. If $\alpha+\beta+2\delta+\eta<1$, then F has a unique fixed point.

Proof:

Suppose X is a point in the Banach space X,

Taking $Y=\frac{1}{2}(F+I)(X), Z=F(Y)$ and $u=2Y-Z$ we have

$$\|Z-X,a\|=\|F(Y)-F^2(X),a\|=\|F(Y)-F[F(X),a]\|$$

$$\leq \alpha\left[\frac{\|Y-F(Y),a\|\|Y-F(X),a\|+\|F(X)-F^2(X),a\|\|F(X)-F(Y),a\|+\|Y-F(X),a\|^2}{\|Y-F(Y),a\|+\|Y-F(X),a\|}\right]$$

$$+\beta\left[\frac{\|F(X)-F^2(X),a\|\|Y-F(X),a\|+\|Y-F(Y),a\|\|Y-F^2(X),a\|+\|Y-F(X),a\|^2}{\|F(X)-F^2(X),a\|+\|Y-F(X),a\|}\right]$$

$$+\gamma\left[\|Y-F(Y),a\|+\|F(X)-F^2(X),a\|\right]+\delta\left[\|Y-F^2(X),a\|+\|F(X)-F(Y),a\|\right]+\eta\left[\|Y-F(X),a\|\right]$$

$$=\alpha\left[\frac{\|Y-F(Y),a\|\|Y-F(X),a\|+\|F(X)-X,a\|\|F(X)-F(Y),a\|+\|Y-F(X),a\|^2}{\|Y-F(Y),a\|+\|Y-F(X),a\|}\right]$$

$$+\beta\left[\frac{\|F(X)-X,a\|\|Y-F(X),a\|+\|Y-F(Y),a\|\|Y-X,a\|+\|Y-F(X),a\|^2}{\|F(X)-X,a\|+\|Y-F(X),a\|}\right]$$

$$+\gamma\left[\|Y-F(Y),a\|+\|F(X)-X,a\|\right]+\delta\left[\|Y-X,a\|+\|F(X)-F(Y),a\|\right]+\eta\left[\|Y-F(X),a\|\right]$$

$$=\alpha\left[\frac{\|Y-F(Y),a\|\|Y-F(X),a\|+\|F(X)-X,a\|\|F(X)-F(Y),a\|+\|Y-F(X),a\|^2}{\|F(X)-F(Y),a\|}\right]$$

$$+\beta\left[\frac{\|F(X)-X,a\|\|Y-F(X),a\|+\|Y-F(Y),a\|\|Y-X,a\|+\|Y-F(X),a\|^2}{\|Y-X,a\|}\right]$$

$$+\gamma\left[\|Y-F(Y),a\|+\|F(X)-X,a\|\right]+\delta\left[\|Y-X,a\|+\|F(X)-F(Y),a\|\right]+\eta\left[\|Y-F(X),a\|\right]$$

$$= \alpha \left[\frac{\|Y - F(Y), a\| \|\frac{1}{2}(F+I)(X) - F(X), a\| + \|F(X) - X, a\| \|F(X) - F[\frac{1}{2}(F+I)(X)], a\| + \|\frac{1}{2}(F+I)(X) - F(X), a\|^2}{\|F(X) - F[\frac{1}{2}(F+I)(X)], a\|} \right]$$

$$+ \beta \left[\frac{\|F(X) - X, a\| \|\frac{1}{2}(F+I)(X) - F(X), a\| + \|Y - F(Y), a\| \|[\frac{1}{2}(F+I)(X) - X], a\| + \|\frac{1}{2}(F+I)(X) - F(X), a\|^2}{\|\frac{1}{2}(F+I)(X) - X, a\|} \right]$$

$$+ \gamma \left[\|Y - F(Y), a\| + \|F(X) - X, a\| \right] + \delta \left[\|\tfrac{1}{2}(F+I)(X) - X, a\| + \|F(X) - F[\tfrac{1}{2}(F+I)(X)], a\| \right]$$

$$+ \eta \|\tfrac{1}{2}(F+I)(X) - F(X), a\|$$

$$= \alpha \left[\frac{\|Y - F(Y), a\| \frac{1}{2}\|F(X) - X, a\| + \|F(X) - X, a\| \frac{1}{2}\|F(X) - X, a\| + \frac{1}{4}\|F(X) - X, a\|^2}{\frac{1}{2}\|F(X) - X, a\|} \right]$$

$$+ \beta \left[\frac{\|F(X) - X, a\| \frac{1}{2}\|F(X) - X, a\| + \|Y - F(Y), a\| \frac{1}{2}\|F(X) - X, a\| + \frac{1}{4}\|F(X) - X, a\|^2}{\frac{1}{2}\|F(X) - X, a\|} \right]$$

$$+ \gamma \left[\|Y - F(Y), a\| + \|F(X) - X, a\| \right] + \delta \left[\tfrac{1}{2}\|F(X) - X, a\| + \tfrac{1}{2}\|F(X) - X, a\| \right] + \eta \tfrac{1}{2}\|F(X) - X, a\|$$

$$= \alpha \left[\|Y - F(Y), a\| + \|F(X) - X, a\| + \tfrac{1}{2}\|F(X) - X, a\| \right] + \beta \left[\|F(X) - X, a\| + \|Y - F(Y), a\| + \tfrac{1}{2}\|F(X) - X, a\| \right]$$

$$+ \gamma \left[\|Y - F(Y), a\| + \|F(X) - X, a\| \right] + \delta \|F(X) - X, a\| + \frac{\eta}{2}\|F(X) - X, a\|$$

$$= \alpha \left[\|Y - F(Y), a\| + \tfrac{3}{2}\|F(X) - X, a\| \right] + \beta \left[\tfrac{3}{2}\|F(X) - X, a\| + \|Y - F(Y), a\| \right]$$

$$+ \gamma \left[\|Y - F(Y), a\| + \|F(X) - X, a\| \right] + \delta \|F(X) - X, a\| + \frac{\eta}{2}\|F(X) - X, a\|$$

$$= \left[\frac{3}{2}\alpha + \frac{3}{2}\beta + \gamma + \delta + \frac{\eta}{2} \right] \|F(X) - X, a\| + [\alpha + \beta + \gamma] \|Y - F(Y), a\|$$

$$\|Z - X, a\| \le \left[\frac{3}{2}\alpha + \frac{3}{2}\beta + \gamma + \delta + \frac{\eta}{2} \right] \|F(X) - X, a\| + [\alpha + \beta + \gamma] \|Y - F(Y), a\|$$

Also

$$\|u - X, a\| = \|2Y - Z - X, a\| = \|(F+I)(X) - Z - X, a\| = \|F(X) + X - Z - X, a\| \qquad \dots (1)$$

$$= \|F(X) - Z, a\| = \|F(X) - F(Y), a\|$$

$$\le \alpha \left[\frac{\|X - F(X), a\| \|X - Y, a\| + \|Y - F(Y), a\| \|Y - F(X), a\| + \|X - Y, a\|^2}{\|X - F(X), a\| + \|X - Y, a\|} \right]$$

$$+\beta\left[\frac{\|Y-F(Y),a\|\|X-Y,a\|+\|X-F(X),a\|\|X-F(Y),a\|+\|X-Y,a\|^2}{\|Y-F(Y),a\|+\|X-Y,a\|}\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\left[\|X-F(Y),a\|+\|Y-F(X),a\|\right]+\eta\|X-Y,a\|$$

$$=\alpha\left[\frac{\|X-F(X),a\|\|X-Y,a\|+\|Y-F(Y),a\|\|Y-F(X),a\|+\|X-Y,a\|^2}{\|Y-F(X),a\|}\right]$$

$$+\beta\left[\frac{\|Y-F(Y),a\|\|X-Y,a\|+\|X-F(X),a\|\|X-F(Y),a\|+\|X-Y,a\|^2}{\|X-F(Y),a\|}\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\left[\|X-F(Y),a\|+\|Y-F(X),a\|\right]+\eta\|X-Y,a\|$$

$$=\alpha\left[\frac{\|X-F(X),a\|\|X-\left[\tfrac{1}{2}(F+I)(X)\right],a\|+\|Y-F(Y),a\|\|\tfrac{1}{2}(F+I)(X)-F(X),a\|+\|X-\left[\tfrac{1}{2}(F+I)(X),\right],a\|^2}{\|\tfrac{1}{2}(F+I)(X)-F(X),a\|}\right]$$

$$+\beta\left[\frac{\|Y-F(Y),a\|\|X-\left[\tfrac{1}{2}(F+I)(X)\right],a\|+\|X-F(X),a\|\|X-F\left[\tfrac{1}{2}(F+I)(X)\right],a\|+\|X-\left[\tfrac{1}{2}(F+I)(X)\right],a\|^2}{\|X-F\left[\tfrac{1}{2}(F+I)(X)\right],a\|}\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]$$

$$+\delta\left[\|X-F\left[\tfrac{1}{2}(F+I)(X),a\right]\|+\|\tfrac{1}{2}(F+I)(X)-F(X),a\|\right]+\eta\left[\|X-\left[\tfrac{1}{2}(F+I)(X)\right],a\|\right]$$

$$=\alpha\left[\frac{\|X-F(X),a\|\tfrac{1}{2}\|X-F(X),a\|+\|Y-F(Y),a\|\tfrac{1}{2}\|X-F(X),a\|+\tfrac{1}{4}\|X-F(X),a\|^2}{\tfrac{1}{2}\|X-F(X),a\|}\right]$$

$$+\beta\left[\frac{\|Y-F(Y),a\|\tfrac{1}{2}\|X-F(X),a\|+\|X-F(X),a\|\tfrac{1}{2}\|X-F(X),a\|+\tfrac{1}{4}\|X-F(X),a\|^2}{\tfrac{1}{2}\|X-F(X),a\|}\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\left[\tfrac{1}{2}\|X-F(X),a\|+\tfrac{1}{2}\|X-F(X),a\|\right]+\eta\left[\tfrac{1}{2}\|X-F(X),a\|\right]$$

$$=\alpha\left[\|X-F(X),a\|+\|Y-F(Y),a\|+\tfrac{1}{2}\|X-F(X),a\|\right]$$

$$+\beta\left[\|Y-F(Y),a\|+\|X-F(X),a\|+\tfrac{1}{2}\|X-F(X),a\|\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\left[\|X-F(X),a\|\right]+\frac{\eta}{2}\|X-F(X),a\|$$

$$=\alpha\left[\tfrac{3}{2}\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\beta\left[\|Y-F(Y),a\|+\tfrac{3}{2}\|X-F(X),a\|\right]$$

$$+\gamma\left[\|X-F(X),a\|+\|Y-F(Y),a\|\right]+\delta\|X-F(X),a\|+\frac{\eta}{2}\|X-F(X),a\|$$

$$=\left[\frac{3}{2}\alpha+\frac{3}{2}\beta+\gamma+\delta+\frac{\eta}{2}\right]\|X-F(X),a\|+[\alpha+\beta+\gamma]\|Y-F(Y),a\|$$

$\therefore$ $$\|u-X,a\|\le\left[\frac{3}{2}\alpha+\frac{3}{2}\beta+\gamma+\delta+\frac{\eta}{2}\right]\|X-F(X),a\|+[\alpha+\beta+\gamma]\|Y-F(Y),a\| \quad \text{... (2)}$$

Now,

$$\|Z-u,a\|\le\|Z-X,a\|+\|X-u,a\|$$

$$=\left[\frac{3}{2}\alpha+\frac{3}{2}\beta+\gamma+\delta+\frac{\eta}{2}\right]\|X-F(X),a\|+[\alpha+\beta+\gamma]\|Y-F(Y),a\|$$

$$+\left[\frac{3}{2}\alpha+\frac{3}{2}\beta+\gamma+\delta+\frac{\eta}{2}\right]\|X-F(X),a\|+[\alpha+\beta+\gamma]\|Y-F(Y),a\|$$

$$=2\left[\frac{3}{2}\alpha+\frac{3}{2}\beta+\gamma+\delta+\frac{\eta}{2}\right]\|X-F(X),a\|+2[\alpha+\beta+\gamma]\|Y-F(Y),a\|$$

$$=[3\alpha+3\beta+2\gamma+2\delta+\eta]\|X-F(X),a\|+[2\alpha+2\beta+2\gamma]\|Y-F(Y),a\|$$

$$\|Z-u,a\|=[3\alpha+3\beta+2\gamma+2\delta+\eta]\|X-F(X),a\|+[2\alpha+2\beta+2\gamma]\|Y-F(Y),a\| \quad \text{... (3)}$$

Also

$$\|Z-u,a\|=\|F(Y)-(2Y-Z),a\|$$

$$=\|F(Y)-2Y+Z,a\|$$

$$=2\|F(Y)-Y,a\|$$

$\therefore$ From (3)

$\therefore$ $$2\|Y-F(Y),a\|=[3\alpha+3\beta+2\gamma+2\delta+\eta]\|X-F(X),a\|+[2\alpha+2\beta+2\gamma]\|Y-F(Y),a\|$$

$\therefore$ $$\|Y-F(Y),a\|\le q\|X-F(X),a\|$$

where $q=\dfrac{3\alpha+3\beta+2\gamma+2\delta+\eta}{2-(2\alpha+2\beta+2\gamma)}<1$

since $5\alpha+5\beta+4\gamma+2\delta+\eta<2$

Let $G=\frac{1}{2}(F+I)$ then for every $x\in X$

$$\|G^2(X)-G(X),a\|=\|G(Y)-Y,a\|$$

$$=\left\|\frac{1}{2}(F+I)(Y)-Y,a\right\|$$

$$=\frac{1}{2}\|Y-F(Y),a\|$$

$$< \frac{q}{2}\left\|X - F\left(X\right), a\right\|$$

By the definition of q, we claim that $\{G^n(X)\}$ is a Cauchy sequence in X.

By the completeness, $\{G^n(X)\}$ converges to some element X_0 in X.

i.e. $\lim_{n\to\infty} G^n(X) = X_0$

Which implies that $G(X_0) = X_0$.

Hence $F(X_0) = X_0$

i.e. X_0 is a fixed point of F.

For the uniqueness, if possible let $Y_0 (\neq X_0)$ be another fixed point of F then

$$\|X_0 - Y_0, a\| = \|F(X_0) - F(Y_0), a\|$$

$$\leq \alpha\left[\frac{\left\|X_0 - F\left(X_0\right), a\right\|\left\|X_0 - Y_0, a\right\| + \left\|Y_0 - F\left(Y_0\right), a\right\|\left\|Y_0 - F\left(X_0\right), a\right\| + \left\|X_0 - Y_0, a\right\|^2}{\left\|X_0 - F\left(X_0\right), a\right\| + \left\|X_0 - Y_0, a\right\|}\right]$$

$$+\beta\left[\frac{\left\|Y_0 - F\left(Y_0\right), a\right\|\left\|X_0 - Y_0, a\right\| + \left\|X_0 - F\left(X_0\right), a\right\|\left\|X_0 - F\left(Y_0\right), a\right\| + \left\|X_0 - Y_0, a\right\|^2}{\left\|Y_0 - F\left(Y_0\right), a\right\| + \left\|X_0 - Y_0, a\right\|}\right]$$

$$+\gamma\left[\left\|X_0 - F\left(X_0\right), a\right\| + \left\|Y_0 - F\left(Y_0\right), a\right\|\right] + \delta\left[\left\|X_0 - F\left(Y_0\right), a\right\| + \left\|Y_0 - F\left(Y_0\right), a\right\|\right] + \eta\left\|X_0 - Y_0, a\right\|$$

$$= \alpha\frac{\left\|X_0 - Y_0, a\right\|^2}{\left\|X_0 - Y_0, a\right\|} + \beta\frac{\left\|X_0 - Y_0, a\right\|^2}{\left\|X_0 - Y_0, a\right\|} + 2\delta\left\|X_0 - Y_0, a\right\| + \eta\left\|X_0 - Y_0, a\right\|$$

$$= \alpha\left\|X_0 - Y_0, a\right\| + \beta\left\|X_0 - Y_0, a\right\| + 2\delta\left\|X_0 - Y_0, a\right\| + \eta\left\|X_0 - Y_0, a\right\|$$

$$= \left[\alpha + \beta + 2\delta + \eta\right]\left\|X_0 - Y_0, a\right\|$$

$$\therefore \quad \left\|X_0 - Y_0, a\right\| \leq \left[\alpha + \beta + 2\delta + \eta\right]\left\|X_0 - Y_0, a\right\|$$

since $\alpha + \beta + 2\delta + \eta < 1$

$$\left\|X_0 - Y_0, a\right\| = 0$$

$$\therefore \quad X_0 = Y_0$$

This completes the proof.

Theorem 2:

Let K be closed and convex subject of a 2-Banach space X. Let $F : K \rightarrow, G : K \rightarrow K$ satisfy the following conditions :

(*i*) F and G commute

(*ii*) $F^2 = I$ and $G^2 = I$, where I denotes identify mapping

(*iii*) $\|F(X) - F(Y), a\|$

$$\leq \alpha\left[\frac{\|G(X)-F(X),a\|\|G(X)-G(Y),a\|+\|G(Y)-F(Y),a\|\|G(Y)-F(X),a\|+\|G(X)-G(Y),a\|^2}{\|G(X)-F(X),a\|+\|G(X)-G(Y),a\|}\right]$$

$$+\beta\left[\frac{\|G(Y)-F(Y),a\|\|G(X)-G(Y),a\|+\|G(X)-F(X),a\|\|G(X)-F(Y),a\|+\|G(X)-G(Y),a\|^2}{\|G(Y)-F(Y),a\|+\|G(X)-G(Y),a\|}\right]$$

$$+\gamma\left[\|G(X)-F(X),a\|+\|G(Y)-F(Y),a\|\right]+\delta\left[\|G(X)-F(Y),a\|+\|G(Y)-F(X),a\|\right]+\eta\|G(X)-G(Y),a\|$$

For every $X,Y,\in X, 0\leq \alpha,\beta,\gamma,\delta,\eta$ and $5\alpha+5\beta+4\gamma+2\delta+\eta<2$. Then there exist at least one fixed point, $X_0\in X$ such that $F(X_0)=G(X_0)=X_0$. Further if $\alpha+\beta+2\delta+\eta<1$ then *X* is the unique fixed point of *F* and *G*.

Proof:

From (*i*) and (*ii*) it follows that $(FG)^2 = I$ and (*ii*) and (*iii*) imply.

$$\|FGG(X)-FGG(Y),a\|=\|FG^2(X)-FG^2(Y),a\|$$

This can be proved easily by theorem 1.

Theorem 3:

Let *K* be a closed and convex subset of a 2-Banach space *X*. Let *F*, *G* and *H* be three mappings of *X* into itself such that

(*i*) $FG = GF$, $GH = HG$ and $FH = HF$

(*ii*) $F^2 = I$, $G^2 = I$, $H^2 = I$, where *I* denotes the identify mapping

(*iii*) $\|F(X) - F(Y), a\|$

$$\leq \alpha\left[\frac{\|GH(X)-F(X),a\|\|GH(X)-GH(Y),a\|+\|GH(Y)-F(Y),a\|\|GH(Y)-F(X),a\|+\|GH(X)-GH(Y),a\|^2}{\|GH(X)-F(X),a\|+\|GH(X)-GH(Y),a\|}\right]$$

$$+\beta\left[\frac{\|GH(Y)-F(Y),a\|\|GH(X)-GH(Y),a\|+\|GH(X)-F(X),a\|\|GH(X)-F(Y),a\|+\|GH(X)-GH(Y),a\|^2}{\|GH(Y)-F(Y),a\|+\|GH(X)-GH(Y),a\|}\right]$$

$$+\gamma\left[\|GH(X)-F(X),a\|+\|GH(Y)-F(Y),a\|\right]+\delta\left[\|GH(X)-F(Y),a\|+\|GH(Y)-F(X),a\|\right]$$

$$+\eta\|GH(X)-GH(Y),a\|$$

For every $X,Y\in K$ and $0\leq\alpha,\ \beta,\ \gamma,\ \delta,\ \eta$ such that $5\alpha+5\beta+4\gamma+2\delta+\eta<2$ then there exist at least one fixed point $X_0\in X$ such that

$$F(X_0)=GH(X_0)\text{ and }FG(X_0)=H(X_0)$$

Further if $\alpha+\beta+2\delta+\eta<1$ then *F* has a unique fixed point.

Proof:

From (*i*) and (*ii*) it follows that $FGH^2 = I$ where *I* is the identify mapping, from (*ii*) and (*iii*) we have

$$\|FGH\cdot G(X)-FGH\cdot G(Y),a\|=\|F\cdot GHG(X)-F\cdot GHG(Y),a\|$$

Proof can be done as theorem 1.

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