



## Study of Propagation of Shear Waves in a Multilayer Medium Including a Fluid-Saturated Porous Stratum

Sonia R. Bansal\* and Kuldeepak\*\*

\*Gian Jyoti Group of Institutions, Shambhukalan, (MP)

\*\*Jaypee University of Engineering and Technology, Guna, (MP)

(Received 17 July, 2011, Accepted 15 August, 2011)

**ABSTRACT :** The propagation of SH-type waves is discussed by considering a double surface layered medium; the upper layer is an impermeable, elastic and isotropic medium, the intermediate layer is a transverse-isotropic porous medium filled with fluid and the lower is a non-homogeneous elastic impermeable half-space, in which the in homogeneity varies with depth. Dispersion equation has been derived.

**Keywords:** SH-waves, wave propagation, porous medius, Love type waves.

### I. INTRODUCTION

The study of propagation of waves in medium with porous layers saturated with fluid has been made in few papers, which are based on Biot's (1956) equation of motion [1-4]. The propagation of Love waves in an isotropic, fluid-saturated porous layer with irregular interface between the layer has been investigated by Chattopadhyay and De (1983) [5]. Chattopadhyay et al. (1986) discussed the propagation of SH-type waves in a porous layer of non-uniform thickness and dispersion equation of love waves in porous layer [6-7]. Chattopadhyay et al. (2007) discussed the propagation of shear waves in an anisotropic medium [8], Dey (1987) studied propagation of longitudinal and shear waves in an elastic medium with void pores [9]. Pardhan, Samal and Mahanti (2002) discussed the propagation of shear waves in the transverse-isotropic fluid-saturated porous plate [10]. Recently, Chattopadhyay and Sharma (2008) studied the propagation of shear waves in an irregular boundary of anisotropic medium [11]. Sonia Rani and V.K. Gaur (2010) discussed Love type waves in a porous layer with irregular interface [12]. V.K. Gaur and Sonia Rani (2010) discussed the surface wave propagation in non-dissipative porous medium [13-14].

In this Paper, the propagation of SH-type waves is discussed by considering a double surface layered medium; the upper layer is an impermeable, elastic and isotropic medium, the intermediate layer is a transverse-isotropic porous medium filled with fluid and the lower is a non-homogeneous elastic impermeable half-space, in which the in homogeneity varies with depth [15].

### II. FORMULATION OF THE PROBLEM

A layered model consisting of a transversely isotropic fluid-saturated porous layer of finite width 'H' resting over an impermeable non-homogeneous elastic solid half-space and lying under an impermeable isotropic elastic homogeneous layer of width 'h' is considered. The Cartesian

coordinate system  $(x_1, x_2, x_3)$  is chosen with  $x_3$  axis taken vertically downward in the half-space with the origin lies at the boundary of the half-space. The  $x_1$ -axis is considered parallel to the layers in the direction of the propagation of the disturbance. Therefore, the medium  $M_1$  represents the region  $-(h + H) \leq x_3 \leq -H$ , which is occupied by the homogeneous, isotropic and elastic solid layer, the medium  $M_2$  occupying the transverse-isotropic fluid-saturated porous layer represents the region  $-H \leq x_3 \leq 0$ , and the lower non-homogeneous elastic half-space describes the medium  $M_3$  as  $0 \leq x_3 < \infty$ .

### III. GOVERNING EQUATIONS

Following Konczak (1989) basic field equations for the different media involved in the model are [15].

#### A. For the medium $M_1$

The equation of motion for the isotropic elastic solid medium, without body forces, is taken as

$$\tau_{ij,j} = \rho_0 \ddot{v}_i, (i, j = 1, 2, 3) \quad \dots (1)$$

where  $v_i$  are the components of the displacement,  $\tau_{ij}$  are the components of the stress tensor and  $\rho_0$  is the density of the medium. The comma denotes the differentiation with respect to position and dot denotes the differentiation with respect to time.

The constitutive equations are taken as

$$\tau_{ij} = \lambda e_{kk} + 2\mu e_{ij} \quad \dots (2)$$

where  $\lambda$  and  $\mu$  are Lamé's constant,  $\delta_{ij}$  is the Kronecker delta and

$$e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \quad e_{kk} = v_{k,k} = \text{div } \vec{v} \quad \dots (3)$$

#### B. For the medium $M_2$

Equation of motion for the fluid saturated porous layer in the absence of body forces are taken as

$$\sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\dot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j) \quad \dots (4)$$

$$\sigma_{ij,i} = \rho_{12}\ddot{u}_i + \rho_{22}\dot{U}_i + b_{ij}(\dot{U}_j - \dot{u}_j) \quad \dots [4(a)]$$

where  $\sigma_{ij}$  are the components of stress-tensor in the solid-skeleton,  $u_i$  are the components of the displacement vector of the solid skeleton and  $U_i$  are that of the fluid skeleton and

$$\sigma = -pf \quad \dots (5)$$

is the reduced pressure of the fluid ( $p$  is the pressure in the fluid, and  $f$  is the porosity of the medium). The parameters  $\rho_{11}$ ,  $\rho_{12}$ ,  $\rho_{22}$  are the dynamic coefficients. These parameters are related to mass densities of the solid  $\rho_s$  and the fluid  $\rho_f$  as

$$\rho_{11} + \rho_{12} = (1 - f)\rho_s \quad \dots (6)$$

$$\rho_{12} + \rho_{22} = f\rho_f \quad \dots [6(a)]$$

and satisfy the inequalities

$$\rho_{11} > 0, \rho_{22} > 0, \rho_{12} \leq 0, \rho_{11}\rho_{22} - \rho_{12}^2 > 0 \quad \dots (7)$$

where  $\rho_{12}$  is the coupling parameter.

The constitutive equations for the transversely isotropic fluid-saturated porous medium are taken as

$$\sigma_{11} = (A \in + ME) + 2Nu_{1,1} + (F - A)u_{3,3} \quad \dots (8)$$

$$\sigma_{22} = (A \in + ME) + 2Nu_{2,2} + (F - A)u_{3,3} \quad \dots [8(a)]$$

$$\sigma_{33} = (F \in + QE) + (2C - F)u_{3,3} \quad \dots [8(b)]$$

$$\sigma_{23} = G (u_{2,3} + u_{3,2}) \quad \dots [8(c)]$$

$$\sigma_{31} = G (u_{3,1} + u_{1,3}) \quad \dots [8(d)]$$

$$\sigma_{12} = N (u_{1,2} + u_{2,1}) \quad \dots [8(e)]$$

$$\sigma = (M \in + RE) + (Q - M)u_{3,3} \quad \dots [8(f)]$$

where  $A, F, C, G, M, Q, N, R$  are the material constants and

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \epsilon = \epsilon_{k,k} = \text{div } \bar{u} \quad \text{and} \quad E = \text{div } \bar{U} \quad \dots (9)$$

### C. For the medium $M_3$

The basic equation of motion, in the absence of body forces, for the elastic half-space medium  $M_3$  are taken as

$$t_{ij,j} = \rho^* \ddot{w}_i \quad \dots (10)$$

where  $t_{ij}$  are the components of the stress-tensor,  $w_i$  are the components of the displacement vector, and  $\rho^*$  is the density of the medium and is a function of  $x_1, x_2, x_3$ .

The stress-strain relations are taken as

$$t_{ij} = \lambda^* \bar{e}_{kk} \delta_{ij} + 2\mu^* \bar{e}_{ij} \quad \dots (11)$$

where  $\lambda^*$  and  $\mu^*$  are Lamé's elastic coefficients and are the functions of  $x_1, x_2, x_3$  and

$$\bar{e}_{ij} = \frac{1}{2}(w_{i,j} + w_{j,i}), \quad \bar{e}_{kk} = \text{div } \bar{w} \quad \dots (12)$$

In this paper, we study mainly the SH-type waves propagating in the  $(x_1, x_2)$  plane. The displacements are parallel to  $x_2$  direction and are independent of  $x_2$  coordinate. Thus

$$v_1 = v_3 = 0, \quad v_2 = v(x_1, x_3, t) \quad \dots (13)$$

$$u_1 = u_3 = 0, \quad u_2 = u(x_1, x_3, t) \quad \dots [13(a)]$$

$$U_1 = U_3 = 0, \quad U_2 = U(x_1, x_3, t) \quad \dots [13(b)]$$

$$w_1 = w_3 = 0, \quad w_2 = w(x_1, x_3, t) \quad \dots [13(c)]$$

The equation of motion (1) for the isotropic elastic medium  $M_1$  with the help of (2), (3) and (13), takes the form of

$$\mu \nabla^2 v = \rho_0 \ddot{v}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \quad \dots (14)$$

The equations of motion (4) and (4a) for the transversely isotropic fluid-saturated porous medium  $M_2$  with the help of (8), (9) and (13a), (13b), become

$$Nu_{,11} + Gu_{,33} = \rho_{11}\ddot{u} + \rho_{12}\dot{U} - b_{11}(\dot{U} - \dot{u}) \quad \dots (15)$$

$$0 = \rho_{12}\ddot{u} + \rho_{22}\dot{U} + b_{11}(\dot{U} - \dot{u}) \quad \dots [15(a)]$$

The equation of motion (10) for the non-homogeneous elastic half-space medium  $M_3$ , by using (11), (12) and (13c), yields

$$\mu^* \nabla^2 w + \mu_{,3}^* w_{,3} = \rho^* \ddot{w} \quad \dots (16)$$

## IV. SOLUTION OF THE PROBLEM

### A. Solution of the basic equations

For the harmonic waves to propagate along  $x_1$ -direction, we assume, the displacements  $v, u, U, w$  in the form

$$(v, u, U, w) = (v_0, u_0, U_0, w_0) \exp[i(kx_1 - \omega t)] \quad \dots (1)$$

where  $v_0, u_0, U_0, w_0$  are the functions of  $x_3$  only,  $\omega$  is the angular frequency,  $k$  is the wave number (in general complex).

With the help of (1), the equation (14) takes the form of

$$\left( \frac{\partial^2}{\partial x_3^2} + \chi_1^2 \right) v_0 = 0 \quad \dots (2)$$

$$\text{where } \chi_1^2 = \frac{\omega^2}{c_{TU}^2} - k^2, \quad c_{TU}^2 = \frac{\mu}{\rho_0} \quad \dots (3)$$

From (15) and (15a), by using (1), we get

$$\left( \frac{\partial^2}{\partial x_3^2} + \chi_2^2 \right) \begin{pmatrix} u_0 \\ U_0 \end{pmatrix} = 0 \quad \dots (4)$$

$$\text{where } \chi_2^2 = \xi^2 - \frac{N}{G} k^2 \quad \dots (5)$$

$$\text{with } \xi^2 = (F + iR) \frac{\omega^2}{c_G^2} \quad \dots (6)$$

$$\text{where } F = \frac{b_{11}^2 + \gamma_{22} C^* \rho^2 \omega^2}{b_{11}^2 + (\rho \gamma_{22} \omega)^2} \frac{\gamma_{22}}{C^*} \quad \dots (7)$$

$$R = \frac{(\gamma_{22} - C^*) b_{11} \rho \omega}{b_{11}^2 + (\rho \gamma_{22} \omega)^2} \frac{\gamma_{22}}{C^*} \quad \dots [7(a)]$$

$$c_G^2 = \frac{G}{\rho_z}, \quad \rho_z = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}, \quad \rho = \rho_{11} + 2\rho_{12} + \rho_{22}$$

$$C^* = \gamma_{11} \gamma_{22} - \gamma_{12}^2, \quad \gamma_{kl} = \frac{\rho_{kl}}{\rho}, \quad (k, l = 1, 2) \quad \dots (8)$$

Similarly, for the non-homogeneous elastic half-space medium  $M_3$ , we take the in homogeneity in the form of

$$\mu^*(x_3) = \mu_0^* \exp(mx_3) \quad \dots (9)$$

$$\rho^*(x_3) = \rho_0^* \exp(mx_3) \quad \dots (10)$$

where  $\mu_0^*$  and  $\rho_0^*$  are the constant values of shear modulus  $\mu^*$  and the mass density  $\rho^*$  at the interface  $x_3 = 0$ , and  $m$  is a constant.

Using (1), (9) and (10) in (16), we get

$$\left( \frac{\partial^2}{\partial x_3^2} + m \frac{\partial}{\partial x_3} - \chi_3^2 \right) \omega_0 = 0 \quad \dots (11)$$

$$\text{where } \chi_3^2 = k^2 - \frac{\omega^2}{c_{TL}^2}, \quad c_{TL}^2 = \frac{\mu_0^*}{\rho_0^*} \quad \dots (12)$$

Here  $c_{TU}$  and  $c_{TL}$  are the velocities of shear waves in the upper layer and the lower half-space, respectively and the velocity in the porous layer is  $c_G / \text{Re}(F + iR)^{\frac{1}{2}}$ .

Since, the quantity  $\xi^2$  defined by (6) is complex, so the quantity  $\chi_2^2$  defined by (5) is complex too. Thus the wave number  $k$  is also complex and all the  $\chi_j (j = 1, 2, 3)$  are complex.

The solutions of the equations (2), (4) and (11) and are obtained as

$$v_0(x_1, x_3, t) = (A_1 \cos \chi_1 x_3 + A_2 \sin \chi_1 x_3) \quad \dots (13)$$

$$u_0(x_1, x_3, t) = (A_3 \cos \chi_2 x_3 + A_4 \sin \chi_2 x_3) \quad \dots (14)$$

$$U_0(x_1, x_3, t) = (\bar{A}_3 \cos \chi_2 x_3 + \bar{A}_4 \sin \chi_2 x_3) \quad \dots [14(a)]$$

$$w_0(x_1, x_3, t) = A_5 \exp(-\eta x_3) \quad \dots (15)$$

where  $A_1, A_2, A_3, A_4, \bar{A}_3, \bar{A}_4, A_5$  are arbitrary constants and

$$\eta = \left( \frac{m}{2} + \sqrt{B + \text{Im} k^2} \right), \quad B = \left\{ \left( \frac{m}{2} \right)^2 + \text{Re} k^2 - \frac{\omega^2}{C_{TL}^2} \right\} \quad \dots (16)$$

Using the solutions (13), (14), (15) and following (1), the wave form solutions are obtained as

$$v(x_1, x_3, t) = (A_1 \cos \chi_1 x_3 + A_2 \sin \chi_1 x_3) \exp[i(kx_1 - \omega t)] \quad (17)$$

$$u(x_1, x_3, t) = (A_3 \cos \chi_2 x_3 + A_4 \sin \chi_2 x_3) \exp[i(kx_1 - \omega t)] \quad (18)$$

$$U(x_1, x_3, t) = (\bar{A}_3 \cos \chi_2 x_3 + \bar{A}_4 \sin \chi_2 x_3) \exp[i(kx_1 - \omega t)] \quad (18a)$$

$$w(x_1, x_3, t) = A_5 \exp(-\eta x_3) \exp[i(kx_1 - \omega t)] \quad \dots (19)$$

The relation between  $A_j$  and  $\bar{A}_j (j = 3, 4)$  are provided by using (18) and (18a) in (15a) and are obtained as

$$\bar{A}_j = (\beta_r + i\beta_i) A_j \quad \dots (20)$$

$$\text{where } \beta_r = \frac{b_{11}^2 - \gamma_{12} \gamma_{22} \rho^2 \omega^2}{b_{11}^2 + (\gamma_{22} \rho \omega)^2}, \quad \beta_i = \frac{(\gamma_{12} + \gamma_{22}) b_{11} \rho \omega}{b_{11}^2 + (\gamma_{22} \rho \omega)^2}$$

## B. Boundary conditions

The appropriate boundary conditions for the equations of motion (14) - (16) are as follows:

(i) At the free surface  $x_3 = -(h + H)$ , the shear stress component vanishes, *i.e.*,

$$\tau_{32}(x_1, x_3 = -(h + H), t) = 0 \quad \dots (21)$$

(ii) At the interface  $x_3 = -H$ , the continuity of shear displacement and stress components, *i.e.*,

$$v(x_1, x_3 = -H, t) = u(x_1, x_3 = -H, t) \quad \dots (22)$$

$$\tau_{32}(x_1, x_3 = -H, t) = \sigma_{32}(x_1, x_3 = -H, t) \quad \dots [22(a)]$$

(iii) At the interface  $x_3 = 0$ , the continuity of shear displacement and stress components, *i.e.*,

$$u(x_1, x_3 = 0, t) = w(x_1, x_3 = 0, t) \quad \dots (23)$$

$$\sigma_{32}(x_1, x_3 = 0, t) = t_{32}(x_1, x_3 = 0, t) \quad \dots [23(a)]$$

The relevant stress components used in the boundary conditions in terms of the solutions of the respective medium, are obtained as follows:

Using (3), (13) and (17) in the equation (2), we obtain

$$\tau_{32} = \mu \chi_1 (-A_1 \sin \chi_1 x_3 + A_2 \cos \chi_1 x_3 + A_2 \cos \chi_1 x_3) \times \exp[i(kx_1 - \omega t)] \quad \dots (24)$$

Further the equation (8c) with the aid of (13a) and (18), gives

$$\sigma_{32} = G\chi_1 (-A_3 \sin \chi_1 x_3 + A_4 \cos \chi_1 x_3) \quad \dots (25)$$

Similarly, the equation (11) with the help of (12), (13c) and (19), takes the form of

$$t_{32} = -\mu\eta A_5 \exp(-\eta x_3) \exp[i(kx_1 - \omega t)] \quad \dots (26)$$

The first boundary condition (21) with the help of (24), gives

$$A_1 \sin \chi_1 (h+H) + A_2 \cos \chi_1 (h+H) = 0 \quad \dots (27)$$

From the boundary condition (22), by using (13), (13a), (17) and (18), we get

$$A_1 \cos \chi_1 H - A_2 \sin \chi_1 H - A_3 \cos \chi_2 H + A_4 \sin \chi_2 H = 0 \dots (28)$$

The boundary condition (22a) with respect to (24) and (25) gives

$$\begin{aligned} \mu\chi_1 [A_1 \sin \chi_1 H + A_2 \cos \chi_1 H] \\ -G\chi_2 [A_3 \sin \chi_2 H + A_4 \cos \chi_2 H] = 0 \dots (29) \end{aligned}$$

Similarly, the boundary conditions (19) and (19a), by using (18), (19) and (25), (26) respectively, yield

$$A_3 - A_5 = 0 \quad \dots (30)$$

$$\text{and } A_4 G\chi_2 + A_5 \eta \mu^* = 0 \quad \dots (31)$$

The non-zero solution of this system of five equations (27) - (31) with five unknowns  $A_j (j = 1, 2 \dots 5)$  is possible if and only if

$$\begin{vmatrix} \sin \chi_1 (h+H) & \cos \chi_1 (h+H) & 0 & 0 & 0 \\ \cos \chi_1 H & -\sin \chi_1 H & -\cos \chi_2 H & \sin \chi_2 H & 0 \\ \mu\chi_1 \sin \chi_1 H & \mu\chi_1 \cos \chi_1 H & -G\chi_2 \sin \chi_2 H & -G\chi_2 \cos \chi_2 H & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & G\chi_2 & \mu^* \eta \end{vmatrix} = 0$$

$$\frac{\mu\chi_1}{G\chi_2} \tan \chi_1 h + \tan \chi_2 H = \frac{\mu^* \eta}{G\chi_2} \left( 1 - \frac{\mu\chi_1}{G\chi_2} \tan \chi_1 h \tan \chi_2 H \right) \quad \dots (32)$$

The equation (32) is referred as a dispersion equation, which relates the phase velocity of propagation  $c = \omega/\text{Re}k$  to the reduced wave number, anisotropic factor and inhomogeneity parameter.

Since,  $k$  is complex, therefore, the values of 1, 2 and obtained in the equations (3), (5) and (16) respectively, can be written as

$$\chi_1 = \lambda_{1r} - i\lambda_{1i} \quad \dots (33)$$

$$\chi_2 = \lambda_{2r} + i\lambda_{2i} \quad \dots (34)$$

$$\text{and } \eta = \xi^* + i\xi_2, \quad \xi^* = \frac{m}{2} + \xi_1 \quad \dots (35)$$

where,

$$\lambda_{1r,1i} = \left\{ \frac{1}{2} \left[ \sqrt{\left( \frac{\omega^2}{c_{TU}^2} - \text{Re} k^2 \right)^2 + \text{Im} k^2} \pm \left( \frac{\omega^2}{c_{TU}^2} - \text{Re} k^2 \right) \right] \right\}^{\frac{1}{2}} \quad (36)$$

$$\lambda_{2r,2i} = \left\{ \frac{1}{2} \left[ \sqrt{\left( \text{Re} \xi^2 - \frac{N}{G} \text{Re} k^2 \right)^2 + \left( \text{Im} \xi^2 - \frac{N}{G} \text{Im} k^2 \right)^2} \pm \left( \text{Re} \xi^2 - \frac{N}{G} \text{Re} k^2 \right) \right] \right\} \quad \dots (37)$$

$$\text{and } \zeta_{1,2} = \left\{ \frac{1}{2} \left( \sqrt{B^2 + (\text{Im} k^2)^2} \pm B \right) \right\}^{\frac{1}{2}} \quad \dots (38)$$

With the help of (33) - (38), the dispersion equation (32) is complex and takes the form

$$\begin{aligned} \frac{\mu}{G} \left( \frac{\lambda_{1r} - i\lambda_{1i}}{\lambda_{2r} + i\lambda_{2i}} \right) \tan(\lambda_{1r} - \lambda_{1i})h + \tan(\lambda_{2r} + i\lambda_{2i})H \\ = \frac{\mu_0^* \eta}{G(\lambda_{2r} + i\lambda_{2i})} \left( 1 - \frac{\mu(\lambda_{1r} - i\lambda_{1i})}{G(\lambda_{2r} + i\lambda_{2i})} \right) \\ \times \tan(\lambda_{1r} - i\lambda_{1i})h \tan(\lambda_{2r} + i\lambda_{2i})H \end{aligned}$$

On rationalization and simplification, we have

$$\begin{aligned} \frac{\mu}{G} (A_r - iA_i) \tan(\lambda_{1r} - i\lambda_{1i})h + \tan(\lambda_{2r} + i\lambda_{2i})H \\ = \frac{\mu_0^*}{G} (D_r + iD_i) \left[ 1 - \frac{\mu}{G} (A_r - iA_i) \tan(\lambda_{1r} - i\lambda_{1i})h \tan(\lambda_{2r} + i\lambda_{2i})H \right] \dots (39) \end{aligned}$$

where,

$$A_r = \frac{\lambda_{1r}\lambda_{2r} - \lambda_{1i}\lambda_{2i}}{\lambda_{2r}^2 + \lambda_{2i}^2} \quad \dots (40)$$

$$A_i = \frac{\lambda_{1i}\lambda_{2r} + \lambda_{2i}\lambda_{1r}}{\lambda_{2r}^2 + \lambda_{2i}^2} \quad \dots [40(a)]$$

$$D_r = \frac{\zeta^* \lambda_{2r} + \zeta_2 \lambda_{2i}}{\lambda_{2r}^2 + \lambda_{2i}^2} \quad \dots [40(b)]$$

$$D_i = \frac{\zeta_2 \lambda_{2r} - \zeta^* \lambda_{2i}}{\lambda_{2r}^2 + \lambda_{2i}^2} \quad \dots [40(c)]$$

The dispersion equation (39) is very complicated and therefore the roots of it can be found by an approximation method only, e.g. by the use of small parameter expansion.

## IV. CONCLUSIONS

The propagation of SH-type waves is discussed by considering a double surface layered medium; the upper layer is an impermeable, elastic and isotropic medium, the intermediate layer is a transverse-isotropic porous medium filled with fluid and the lower is a non-homogeneous elastic impermeable half-space, in which the in homogeneity varies with depth. The dispersion equation, without simplifying assumption concerning the dissipation terms that occur in

the Biot's equation of motion, has been derived, which relates the phase velocity of propagation to the wave number, shear wave speeds of the layers, anisotropy factors and non-homogeneity characteristic of the lower semi-infinite half-space.

## V. ACKNOWLEDGEMENT

The author wishes to acknowledge sincere thanks to Managing director, Satya Prakshan (Journal) and Referees for their careful reading of the manuscript and constructive comments, her family members for their constant inspiration, her Supervisor Dr. V.K. Gaur, Assistant Professor, Govt. Dunder College, Bikaner for their kind support and guidance and Management, Gian Jyoti Group of Institutions, Shambhukalan for providing valuable time and resources for this research work.

## REFERENCES

- [1] Biot, M.A. (1956a); Theory of propagation of elastic waves in a fluid saturated porous solid I: Low frequency range, *J. Acoust. Soc. Am.*, **28**(2), 168-178.
- [2] Biot, M.A. (1956b); Theory of propagation of elastic waves in a fluid saturated porous solid II: High frequency range, *J. Acoust. Soc. Am.* **28**(2), 179-191.
- [3] Biot, M.A. (1956c); General solution of equations of elasticity and consolidation for a porous material, *J. Appl. Mech.*, **23**, 91-96.
- [4] Biot, M.A. (1956d); Propagation of elastic waves in a liquid filled porous solid, *J. Appl. Phys.*, **27**, 459-467.
- [5] Chattopadhyay, A. and De, R.K. (1983); Love type waves in a porous layer with irregular interface, *Int. J. Engg. Sci.*, **21**, 1295-1303.
- [6] Chattopadhyay, A., Chakraborty, M. and Mahata, N. P. (1986); SH waves in a porous layer of non uniform thickness, *Engg. Trans.*, **34**, 3-13.
- [7] Chattopadhyay, A., Chakraborty, M. and Kushwaha, V. (1986); On the dispersion equation of Love waves in a porous layer, *Acta Mech.*, **58**, 125-136.
- [8] Chattopadhyay, A. and Kumari, P. (2007); Propagation of shear waves in an anisotropic medium, *Appl. Math. Sc.*, **55**(1), 2699-2706.
- [9] Dey, S. (1987); Longitudinal and shear waves in an elastic medium with void pores, *Proc. Indian National Sci. Acad., INDIA*, **53**(4), 554-563.
- [10] Pradhan, A., Samal, S.K. and Mahanti, N.C. (2002); *Shear waves in a fluid saturated elastic plate, SADHANA*, **27**(6), 595-604.
- [11] Chattopadhyay, A. and Sharma, V. K. (2008); Propagation of shear waves in an irregular boundary of an anisotropic medium, *Int. J. Engg. Research*, Vol **3**, No 1.
- [12] Sonia Rani, V.K Gaur (2010) ; Study of Love Type Waves in a Porous Layer with Irregular Interface, *International Journal of Educational Administration*, ISSN 0976-5883 Volume **2**, Number 4, pp. 713-726.
- [13] V.K Gaur, Sonia Rani (2010); Surface Wave Propagation in Non-Dissipative Porous Medium, *International Journal of Educational Administration*, ISSN 0976-5883 Volume **2**, Number 2, pp. 443-454.
- [14] Sonia Rani (2011); Surface Wave Propagation in Non-Dissipative Porous Medium, Ph.D Thesis, Singhania University, Rajasthan.
- [15] Konczak, Z. (1989); On propagation of shear waves in a multilayer medium including a fluid saturated porous stratum, *J. Acta Mechanica*, **79**, 169-181.