International Journal of Theoretical and Applied Science **3**(1): 9-12(2011) **ISSN : 0975-1718**

Sub-compatibility and fixed point theorem in intuitionistic Menger space

Abhishek Sharma,* Arihant Jain** and Sanjay Choudhari*

**Department of Mathematics, Govt. Narmada P.G. College, Hoshangabad (M.P.)*
***Department of Applied Mathematics, Shri Guru Sandipani Institute of Technology and Science, Ujjain (M.P.)*

(Received 10 Jan., 2011, Accepted 15 Feb., 2011)

ABSTRACT : In this paper, the new concepts of subcompatibility and subsequential continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity in intuitionistic Menger space has been applied to prove a common fixed point theorem. We extend the result of [1] from metric space to intuitionistic Menger space. Also we cited examples in support our results.

Keywords : Common fixed points, intuitionistic Menger space, subcompatibility and subsequential continuity.

I. INTRODUCTION

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [3]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [4, 5] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [6] obtained a generalization of Banach Contraction Principle on a complete Menger space which is a milestone in developing fixed point theory in Menger space.

Recently, Park [8] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces. Kutukcu et. al. [2] introduced the notion of intuitionistic Menger spaces with the help of t-norms and t-conorms as a generalization of Menger space due to Menger [3]. Recently in 2009, using the concept of subcompatible maps, Bouhadjera et. al. [1] proved common fixed point theorems in metric space. Using the concept of weakly compatible maps in intuitionistic Menger space, Pant *et. al.* [7] proved a common fixed point theorem for six self maps without appeal to continuity.

In this paper, we introduce the new concepts of subcompatibility and subsequential continuity which are respectively weaker than occasionally weak compatibility and reciprocal continuity in intuitionistic Menger space and establish a common fixed point theorem. We extend the result of [1] from metric space to intuitionistic Menger space. Also we cited examples in support our results.

II. PRELIMINARIES

Definition 1. [7] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a *t-norm* if $*$ satisfying the following conditions :

(1) $*$ is commutative and associative.
(2) $*$ is continuous,
(3) $a * 1 = a$, for all $a \in [0, 1]$
(4) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for all $a, b, c, d \in [0, 1]$.

Definition 2. [7] A binary operation $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a *t*-conorm if $\Diamond$ is satisfying the following conditions :

(1) $\Diamond$ is commutative and associative.
(2) $\Diamond$ is continuous,
(3) $a \Diamond 0 = a$, for all $a \in [0, 1]$
(4) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$, for all $a, b, c, d \in [0, 1]$.

Remark 1. [7] The concept of triangular norms (*t*-norms) and triangular conorms (*t*-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersection and union respectively. These concepts were originally introduced by Menger [1] in his study of statistical metric spaces.

Definition 3. [7] A distance distribution function is a function $F : R \to R^+$ which is left continuous on R, non-decreasing and $\inf_{t\in r} F(t) = 0$, $\sup_{t\in r} F(t) = 1$. We will denote by D the family of all distance distribution function and by H a special of D defined by

$$H(t) = \begin{cases} 0, & if \quad t \le 0 \\ 1, & if \quad t > 0 \end{cases}.$$

If X is a non-empty set, $F : X \times X \to D$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by $F_{x,y}$.

Definition 4. [7] A non-distance distribution function is a function $L : R \to R^+$ which is right continuous on R, non-increasing and $\inf_{t\in r} L(t) = 1$, $\sup_{t\in r} L(t) = 0$. We will denote by E the family of all distance distribution function and by G a special of E defined by

$$G(t) = \begin{cases} 1, & if \quad t \le 0 \\ 0, & if \quad t > 0 \end{cases}.$$

If X is an non-empty set, $L : X \times X \to E$ is called a probabilistic non-distance on X and $L(x, y)$ is usually denoted by $L_{x,y}$.

Definition 5. [7] A 5-tuple $(X, F, L, *, \Diamond)$ is said to be an intuitionistic Menger space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is continuous t-conorm, F is a probabilistic distance and L is a probabilistic non-distance on X satisfying the following conditions : for all $x, y, z \in X$ and $t, s \geq 0$,

(1) $F_{x,y}(t) + L_{x,y}(t) \leq 1$,
(2) $F_{x,y}(0) = 0$,
(3) $F_{x,y}(t) = H(t)$ if and only if $x = y$,
(4) $F_{x,y}(t) = F_{y,x}(t)$,
(5) if $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t + s) = 1$,
(6) $F_{x,z}(t + s) \geq F_{x,y}(t) * F_{y,z}(s)$,
(7) $L_{x,y}(0) = 1$,
(8) $L_{x,y}(t) = G(t)$ if and only if $x = y$,
(9) $L_{x,y}(t) = L_{y,x}(t)$,
(10) if $L_{x,y}(t) = 0$ and $L_{y,z}(s) = 0$, then $L_{x,z}(t + s) = 0$,
(11) $L_{x,z}(t + s) \leq L_{x,y}(t) \Diamond L_{y,z}(s)$.

The function $F_{x,y}(t)$ and $L_{x,y}(t)$ denotes the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Example 1. [7] Let (X, d) be a usual metric space. Then the metric d induces a distance distribution function F defined by $F_{x,y}(t) = H(t - d(x, y))$ and non-distance distribution function L defined by $L_{x,y}(t) = G(t - d(x, y))$ for all $x, y \in$ X and $t \geq 0$. Then (X, F, L) is an intuitionistic probabilistic metric space. We call this intuitionistic probabilistic metric space induced by a metric d the induced intuitionistic probabilistic metric space. If t-norm $*$ is $a * b = \min\{a, b\}$ and t-conorm $\Diamond$ is $a \Diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$ then $(X, F, L, * \Diamond)$ is an intuitionistic Menger space.

Definition 6. Suppose A and S be self mappings of an intuitionistic Menger space $(X, F, L, * \Diamond)$. A point x in X is called a coincidence point of A and S if and only if $Ax = Sx$. In this case, $w = Ax = Sx$ is called a point of coincidence of A and S.

Definition 7. Self maps A and S of an intuitionistic Menger space $(X, F, L, *, \Diamond)$ are said to be *occasionally weakly compatible (owc)* if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

In this paper, we weaken the above notion by introducing a new concept called subcompatibility just as defined by Bouhadjera *et. al.* [1] in metric space as follows :

Definition 8. Self mapping A and S of an intuitionistic Menger space $(X, F, L, *, \Diamond)$ are said to be *subcompatible* if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, z \in X$ and satisfy $\lim_{n\to\infty} F_{ASx_n, SAx_n}(t) = 1$ and $\lim_{n\to\infty} L_{ASx_n, SAx_n}(t) = 0$.

Obviously, two owc maps are subcompatible, however the converse is not true in general as shown in the following example.

Example 2. Let $X = [0, \infty)$ for each $t \in (0, \infty)$ and $x, y \in X$. Define (F, L) by

$$F_{x,y}(t) = \begin{cases} \dfrac{t}{t + |x - y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

$$L_{x,y}(t) = \begin{cases} \dfrac{|x - y|}{t + |x - y|}, & \text{if } t > 0 \\ 1, & \text{if } t = 0 \end{cases}$$

Define self maps A and B as follows :

$$A(x) = x^2 \text{ and } B(x) = \begin{cases} x + 6, & \text{if } x \in [0, 9] \cup (16, \infty) \\ x + 72, & \text{if } x \in (9, 16] \end{cases}$$

Now consider a sequence $x_n = 3 + \dfrac{1}{n}$ for $n = 1, 2, 3, \ldots$. then

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = 9$, $9 \in X$ and

$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} BAx_n = 81$.

Then $\lim_{n\to\infty} F_{ABx_n, BAx_n}(t) = 1$ and $\lim_{n\to\infty} L_{ABx_n, BAx_n}(t) = 0$.

Thus, (A, B) is sub-compatible.

On the other hand, $Ax = Bx$ if and only if $x = 3$ and $AB(3) = 81$, $BA(3) = 15$, therefore, $AB(3) \neq BA(3)$.

Hence, A and B are not occasionally weakly compatible maps.

Thus, A and B are sub-compatible continuous.

Definition 9. Self mappings A and S of an intuitionistic Menger space $(X, F, L, *, \Diamond)$ are said to be *reciprocal continuous* if $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n = St$ for some $t \in X$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X$.

Definition 10. Self mappings A and S of an intuitionistic Menger space $(X, F, L, *, \Diamond)$ are said to be *subsequentially continuous* if and only if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, z \in X$ and satisfy $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n = Sz$.

Clearly, if A and S are continouous or reciprocally continuous then they are obviously subsequentially continuous. However, the converse is not true in general.

Now we give an example which shows that there exists a subsequential continuous pair of maps which is neither continuous nor reciprocal continuous.

Example 3. Let $X = [0, \infty]$ for each $t \in (0, \infty)$ and $x, y \in X$. Define (F, L) by

$$F_{x,y}(t) = \begin{cases} \dfrac{t}{t + |x - Y|} & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

$$L_{x,y}(t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|} & \text{if } t > 0 \\ 1, & \text{if } t = 0 \end{cases}$$

Define self maps A and B as follows :

$$A(x) = \begin{cases} 2+x, & \text{if } x \in [0, 2] \\ x, & \text{if } x \in (2, \infty) \end{cases} \text{ and}$$

$$B(x) = \begin{cases} 2-x, & \text{if } x \in [0, 2) \\ 2x-2, & \text{if } x \in [2, \infty) \end{cases}$$

A and B are discontinuous at $x = 2$.

Now consider a sequence $x_n = \dfrac{1}{n}$ for $n = 1,2, 3, \ldots$.

Then $\lim_{n\to\infty} Ax_n = 2$, $\lim_{n\to\infty} Bx_n = 2$, $2 \in X$ and

$\lim_{n\to\infty} ABx_n = 4 = A(2)$ and $\lim_{n\to\infty} BAx_n = 2 = B(2)$.

Thus, A and B are sub-sequential continuous

Let $x_n = 2 + \dfrac{1}{n}$ for $n = 1, 2, 3, \ldots$.

Then $\lim_{n\to\infty} Ax_n = 2$, $\lim_{n\to\infty} Bx_n = 2$ and

$\lim_{n\to\infty} ABx_n = 2 \neq A(2)$.

Thus, A and B are not reciprocal continuous.

Lemma 1. [7] Let $(X, F, L, *, \lozenge)$ be an intuitionistic Menger space with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$ and

$$F_{x,y}(kt) \leq F_{x,y}(t) \text{ and } L_{x,y}(kt) \leq L_{x,y}(t).$$

Then $x = y$.

III. MAIN RESULT

Theorem 1. Let A, B, P and S be four self maps on a intuitionistic Menger space with continuous t-norm $\circledast$ and continuous t-conorm $\lozenge$ defined by $t \circledast t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$. If the pairs (A, P) and (B, S) are sub-compatible and sub-sequential continuous then

(*i*) A and P have a coincidence point.

(*ii*) B and S have a coincidence point.

(*iii*) There exists $k \in (0, 1)$ such that for every, $x, y \in X$ and $t > 0$

$$F_{Ax,By}(kt) \geq F_{Px,Sy}(t) * F_{Ax,Px}(t) * F_{By,Sy}(t) * F_{Ax,Sy}(t).$$

and $$L_{Ax,By}(kt) \leq L_{Px,Sy}(t) \lozenge L_{Ax,Px}(t) \lozenge L_{By,Sy}(t) \lozenge L_{Ax,Sy}(t).$$

Then A, B, S and P have a unique common fixed point X.

Proof : Since the pairs (A, P) and (B, S) are subcompatible and subsequentially continuous, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that.

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Px_n = z_1, z_2 \in X$ and satisfy

$\lim_{n\to\infty} F_{APx_n, PAx_n}(t) = F_{Az_1, Pz_1}(t) = 1$ and

$\lim_{n\to\infty} L_{APx_n, PAx_n}(t) = L_{Az_1, Pz_1}(t) = 0$.

Also $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Sy_n = z_2, z_2, \in X$ and which satisfy

$\lim_{n\to\infty} F_{BSy_n, SBy_n}(t) = F_{Bz_2, Sz_2}(t) = 1$ and

$\lim_{n\to\infty} L_{BSy_n, SBy_n}(t) = L_{Bz_2, Sz_2}(t) = 0$.

Therefore, $A_{z_1} = P_{z_1}$ and $B_{z_2} = S_{z_2}$; that is, z_1 and z_2 is a coincidence point of (A, P) and (B, S).

Now, we prove $z_1 = z_2$.

Put $x = x_n$ and $y = y_n$ in inequality (*iii*), we get

$$F_{Ax_n, By_n}(kt) \geq F_{Px_n, Sy_n}(t) * F_{Ax_n, Px_n}(t) * F_{By_n, Sy_n}(t) * F_{Ax_n, Sy_n}(t).$$

and $$L_{Ax_n, By_n}(kt) \leq L_{Px_n, Sy_n}(t) \lozenge L_{Ax_n, Px_n}(t) \lozenge L_{By_n, Sy_n}(t) \lozenge L_{Ax_n, Sy_n}(t).$$

Taking the limit as $n \to \infty$, we get

$$F_{z_1,z_2}(kt) \geq F_{z_1,z_2}(t) * F_{z_1,z_2}(t) * F_{z_2,z_2}(t) * F_{z_1,z_2}(t)$$
$$F_{z_1,z_2}(kt) \geq F_{z_1,z_2}(t) * 1 * 1 * F_{z_1,z_2}(t)$$
$$F_{z_1,z_2}(kt) \geq F_{z_1,z_2}(t)$$

and $$L_{z_1,z_2}(kt) \leq L_{z_1,z_2}(t) \lozenge L_{z_1,z_1}(t) \lozenge L_{z_2,z_2}(t) \lozenge L_{z_1,z_2}(t)$$
$$L_{z_1,z_2}(kt) \leq L_{z_1,z_2}(t) \lozenge 0 \lozenge 0 \lozenge L_{z_1,z_2}(t)$$
$$L_{z_1,z_2}(kt) \leq L_{z_1,z_2}(t).$$

By lemma 1, we have

$$z_1 = z_2.$$

We claim that $Az_1 = z_1$.

Substitute $x = z_1$ and $y = y_n$ in inequality (*iii*), we get

$$F_{Az_1, By_n}(kt) \geq F_{Pz_1, Sy_n}(t) \circledast F_{Az_1, Pz_1}(t) \circledast F_{By_n, Sy_n}(t) \circledast F_{Az_1, Sy_n}(t)$$

and $$L_{Az_1, By_n}(kt) \leq L_{Pz_1, Sy_n}(t) \lozenge L_{Az_1, Pz_1}(t) \lozenge L_{By_n, Sy_n}(t) \lozenge L_{Az_1, Sy_n}(t)$$

Taking the limit as $n \to \infty$, we get

$$F_{Az_1, z_2}(kt) \geq F_{Pz_1, z_2}(t) \circledast F_{Az_1, Pz_1}(t) \circledast F_{z_2, z_2}(t) \circledast F_{Az_1, z_2}(t)$$
$$F_{Az_1, z_1}(kt) \geq F_{Az_1, z_1}(t) \circledast 1 \circledast 1 \circledast F_{Az_1, z_1}(t)$$
$$F_{Az_1, z_1}(kt) \geq F_{Az_1, z_1}(kt)$$

and

$$L_{Az_1, z_2}(kt) \leq L_{Pz_1, z_2}(t) \lozenge L_{Az_1, Pz_1}(t) \lozenge L_{z_2, z_2}(t) \lozenge L_{Az_1, z_2}(t)$$
$$L_{Az_1, z_1}(kt) \leq L_{Az_1, z_1}(t) \lozenge 0 \lozenge 0 \lozenge L_{Az_1, z_1}(t)$$
$$L_{Az_1, z_1}(kt) \leq L_{Az_1, z_1}(t)$$

By lemma 1, we have

$$Az_1 = z_1$$

Thus $Az_1 = z_1 = z_2$

Now we claim $Az_1 = Bz_2$.

Taking $x = z_1$ and $y = z_2$ in inequality (*iii*), we get

$$F_{Az_1, Bz_2}(kt) \geq F_{Pz_1, Sz_2}(t) \circledast F_{Az_1, Pz_1}(t) \circledast L_{Bz_2, Sz_2}(t) \circledast L_{Az_1, Sz_2}(t)$$

$$F_{Az_1, Bz_2}(kt) \geq F_{Az_1, Bz_2}(t) \circledast 1 \circledast 1 \circledast F_{Az_1, Bz_2}(t)$$

$$F_{Az_1, Bz_2}(kt) \geq F_{Az_1, Bz_2}(t)$$

and

$$L_{Az_1, Bz_2}(kt) \leq L_{Pz_1, Sz_2}(t) \lozenge L_{Az_1, Pz_1}(t) \lozenge L_{Bz_2, Sz_2}(t) \lozenge L_{Az_1, Sz_2}(t)$$

$$L_{Az_1, Bz_2}(kt) \leq L_{Az_1, Bz_2}(t) \lozenge 0 \lozenge 0 \lozenge L_{Az_1, Bz_2}(t)$$

$$L_{Az_1, Bz_2}(kt) \leq L_{Az_1, Bz_2}(t)$$

By lemma 1, we have

$Az_1 = Bz_2$. Since $z_1 = z_2$. Therefore $Az_1 = Bz_1$. So $Az_1 = Pz_1 = Bz_1 = Sz_1 = z_1$ *i.e.* z_1 is a common fixed point of A, B, S and P.

Uniqueness : Let u be another common fixed point of A, P, B and S.

Then $Au = Pu = Bu = Su = u$.

Taking $x = z_1$ and $y = u$ in inequality (*iii*), we get

$$F_{Az_1, Bu}(kt) \geq F_{Pz_1, Su}(t) * F_{Az_1, Pz_1}(t) * F_{Bu, Su}(t) * F_{Az_1, su}(t)$$

$$F_{z_1, u}(kt) \geq F_{z_1, u}(t) * 1 * 1 * F_{z_1, u}(t)$$

$$F_{z_1, u}(kt) \geq F_{z_1, u}(t)$$

and

$$L_{Az_1, Bu}(kt) \leq L_{Pz_1, Su}(t) \lozenge L_{Az_1, Pz_1}(t) \lozenge L_{Bu, Su}(t) \lozenge L_{Az_1, su}(t)$$

$$L_{z_1, u}(kt) \leq L_{z_1, u}(t) \lozenge 0 \lozenge 0 \lozenge L_{z_1, u}(t)$$

$$L_{z_1, u}(kt) \leq L_{z_1, u}(t)$$

Hence, by lemma 1, we have

$$z_1 = u.$$

Therefore, uniqueness follows.

If we take $B = A$ and $S = P$ in theorem 1, we get the following result :

Corollary 1. Let A and P be two self maps of an intuitionistic Menger space with continuous t-norm $\circledast$ and continuous t-conorm $\lozenge$ defined by $t \circledast t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$. If the pairs (A, P) are sub-compatible and sub-sequential continuous then

(*i*) A and P have a coincidence point.

(*ii*) There exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$F_{Ax, Ay}(kt) \geq F_{Px, Py}(t) \circledast F_{Ax, Px}(t)\, F_{Ay, Py}(t) \circledast F_{Ax, Py}(t)$$

and $$L_{Ax, Ay}(kt) \leq L_{Px, Py}(t) \lozenge L_{Ax, Px}(t) \lozenge L_{Ay, Py}(t) \lozenge L_{Ax, Py}(t)$$

Then A and P have a unique common fixed point in X.

The following example illustrates corollary 1.

Example 1. Let $X = [0, \infty)$ with metric defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$ define

$$F_{x, y}(t) = \begin{cases} \dfrac{t}{t + |x - Y|} & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

$$L_{x, y}(t) = \begin{cases} \dfrac{|x - y|}{t + |x - y|}, & \text{if } t > 0 \\ 1, & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$. Clearly $(X, F, L, \circledast, \lozenge)$ is an intuitionistic Menger space where $\circledast$ is defined by $t \circledast t \geq t$ and $\lozenge$ is defined by $(1 - t) \lozenge (1 - t) \leq (1 - t)$.

Define A and $P : X \rightarrow X$ by

$$A(x) = \begin{cases} x & \text{if } x \leq 1 \\ 3x + 1, & \text{if } x > 1 \end{cases} \quad \text{and}$$

$$P(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1 \\ 5x - 1, & \text{if } x > 1 \end{cases}.$$

Consider sequence $\{x_n\} = 1 - \dfrac{1}{n}$.

Clearly, Ax_n and $Px_n \rightarrow 1$.

Also, $APx_n \rightarrow 1$ and $PAx_n \rightarrow 1$.

Therefore, $\lim_{n\to\infty} F_{APx_n, PAx_n}(t) = 1$ and $\lim_{n\to\infty} L_{APx_n, PAx_n}(t) = 0$.

Also, $\lim_{n\to\infty} APx_n = A(1)$ and $\lim_{n\to\infty} PAx_n = P(1)$.

Thus, (A, P) is sub-compatible and sub-sequential continuous. Also, conditions (*i*) and (*ii*) of corollary 1 is satisfied and 1 is unique common fixed point of A and P.

REFERENCES

[1] Bouhadjera, H. and Godet-Thobie, C., Common fixed point theorems for pairs of subcompatible maps, arXiv:0906.3159v1 [math.FA] **17:** (2009).

[2] Kutukcu, S., Tuna, A. and Yakut, A.t., Generalized contraction mapping principle in intuitionistic Menger space and application to differential equations, *Appl. Math. and Mech.*, **28:** 799-809 (2007).

[3] Menger, K., Statistical metrics, Proc. Nat. Acad. Sci. *USA*. **28:** 535-537(1942).

[4] Schweizer, B. and Sklar, A., Statistical metric spaces, *Pacific J. Math*. **10:** 313-334 (1960).

[5] Schweizer, B. and Sklar, A., Probabilistic metric spaces, Elservier, *North-Holland, New York*, (1983).

[6] Sehgal, V.M. and Bharucha-Reid, A.T., Fixed points of contraction maps on probabilistic metric spaces, *Math. System Theory* **6:** 97-102 (1972).

[7] Pant, B.D., Chauhan, S. and Pant, V., Comon fixed point theorems in intuitionistic Menger spaces, *Journal of Advanced Studies in Topology*. **1:** 54-62 (2010).

[8] Park, J.H., Intuitionistic fuzzy metric spacs, *Chos, Solitions and Fractals*, **22:** 1039-1046 (2004).