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Computer Simulation of Streamflows with GAR(1)-Monthly and GAR(1)-Fragments Models

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Abstract—in streamflow simulation, the first-order gamma autoregressive (GAR(1)) model [2] has been found to be very effective for annual data. This paper presents some attempts to apply the GAR(1) model to the simulation of monthly streamflows. To this aim, we propose two models, namely the GAR(1)-Monthly and GAR(1)-Fragments models that will be compared with the popular Thomas-Fiering model. Based on actual data of monthly streamflows at three stations and generated series of monthly data for 1000 years, it was found that both GAR(1)-Monthly and GAR(1)-Fragments models can reproduce very well all statistical descriptors, namely mean value, standard deviation and skewness coefficient, of the historical monthly series. Moreover, the GAR(1)-Fragments model was found to perform very well in reproducing those statistical descriptors of historical annual flows also**.**

Keywords-GAR(1) model; GAR(1)-Monthly model; GAR(1)-Fragments model; Streamflows simulation.

I. INTRODUCTION

The simulation of seasonal streamflows has been extensively studied in previous and recent years. Various parametric and nonparametric models have been suggested in literature as in [4],[9],[10],[13]. For stochastic generation of streamflows, the parametric model uses the statistical descriptors of historical data while nonparametric model does not. So far, no any work has evaluated the effectiveness between parametric and non-parametric models.

Using the parametric model and based on monthly historical data, streamflow simulation methods have been used to generate series of monthly data having the same characteristics for use in the analysis and designs of Water Resources (including Irrigation) projects. Starting with the assumption that annual streamflows follows a dependent and skew distribution, Fernandez and Salas[2] employed a parametric model: the First Order Autoregressive Model with Gamma Variables (GAR(1)) in computer simulation and satisfactory results were obtained. The applicability of the GAR(1) model to the simulation of monthly streamflows is investigated in the present study and based on generated monthly data to obtain the annual data has not been investigated, and, that is the subject of this paper.

II. LITERATURE REVIEW

A. The Gamma Distribution

A continuous random variable X is said to have a three parameter gamma distribution if its density can be expressed as

$$f(x) = \frac{(x-c)^{a-1}e^{-(x-c)/b}}{b^a\Gamma(a)} \tag{1}$$

Where $a>0$, $b>0$, $c>0$, $x\geq c$ and a, b, c are respectively the shape, scale, and location parameters. The gamma function is defined by

$$\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt, \quad a>0$$

this function satisfies the following recursive formula

$$\Gamma(a+1) = a\Gamma(a)$$

and, for a= k (a positive integer), we have :

$$\Gamma(k) = (k-1)! = 1*2*\dots(k-1)$$

when $c = 0$ we have the two-parameter gamma distribution, and, when $c = 0$ and $b = 1$we have the one-parameter gamma distribution.The statistical descriptors of the three-parameter gamma distribution are given by the following formulas:

Expected value: $E(X) = ab + c$

Variance: $Var(X) = ab^2$

Skewness coefficient: $g = 2/\sqrt{a}$

B. First Oder Autoregressive Model with Gamma Variables

The model by Lawrance and Lewis[8] has the following form

$$X_i = \Phi X_{i-1} + e_i \quad (2)$$

where X_i *is* the random variable representing the dependent processes at time *i*, Φ *is* autoregressive coefficient and e_i is an independent variable to be specified X_i has a marginal distribution given by a three- parameter gamma density function defined as Eq.1. The process defined by Eq.2 is denoted as the GAR(1) model. To simulate the process, the parameters of the model must be known and e_i can be generated by certain generators. When the shape parameter *a* is an integer, the following generation scheme for e_i of Eq.2 may be used:

$$e_i = \frac{c(1-\phi)}{a} + \sum_{j=1}^{a} Z_j$$

$Z_j = 0$, with probability Φ

$Z_j =$ E, with probability $1 - \Phi$

and E is an exponentiallly distributed random variable with expected value *b*. When *a* is nonintegral, based on the shot-noise process used by Weiss[14] the following scheme may be used:

$$e_i = c(1-\Phi) + Z \quad (3)$$

where $$Z = 0 \text{ if } Q = 0 \quad (4)$$

and $$Z = \sum_{j=1}^{Q} Y_j \Phi^{U_j} \text{ if} Q > 0 \quad (5)$$

In Eq.4 and Eq.5, Q is an integer random variable with Poisson distribution of mean value $-a\ln(\Phi)$ the U_j are identical and independently distribution random variables with uniform distribution in (0,1) and the Y_j are independent, identically distribution exponential variables with mean value *b*.

C. Estimation of Model Parameters

Fernandez and Salas[2] have presented a procedure for bias correction based on computer simulation studies, applicable for the parameters of GAR(1) model. When used in conjunction with this procedure, the GAR(1) model is an attractive alternative for synthetic streamflow simulation, is simple to use, and does not require any transformation of the original data. The stationary linear GAR(1) process of Eq.2 has four parameters, namely *a*, *b*, *c* and Φ. By using the method of moments, these parameters and the population moments of the variable X_i have the following relationships:

$$M = c + ab \quad (6)$$

$$S^2 = ab^2 \quad (7)$$

$$G = 2/\sqrt{a} \quad (8)$$

$$R = \Phi \quad (9)$$

where M, S^2, G, R are the mean, variance, skewness coefficient, and the lag-one autocorrelation coefficient, respectively. These population statistical can be estimated based on a sample $\{X_1, X_2, \ldots, X_N\}$ by:

$$m = \frac{1}{N}\sum_{i=1}^{N} X_i \quad (10)$$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - m)^2 \quad (11)$$

$$g = \frac{N}{(N-1)(N-2)s^3}\sum_{i=1}^{N}(X_i - m)^3 \quad (12)$$

$$r = \frac{1}{(N-1)s^2}\sum_{i=1}^{N-1}(X_i - m)(X_{i+1} - m) \quad (13)$$

where *m*, *s*, *g* and *r* are estimators of *M*,*S*,*G* and *R* respectively and *N* is sample size. As the variables are dependent and nonnormal, some of these estimators are biased. Hence some correction needs to be made as follows:

$$M^* = m \quad (14)$$

$$R^* = \frac{rN+1}{N-4} \quad (15)$$

$$S^{*2} = \frac{N-1}{N-K}s^2 \quad (16)$$

$$K = \frac{N(1-R^{*2}) - 2R^{*2}(1-R^{*N})}{N(1-R^*)^2}$$

and s^2, R^* are given by Eq.11 and Eq.15, respectively

$$G^* = \frac{T}{f} \quad (17)$$

where $$T = \frac{pg(A + B\left(\frac{p^2}{N}\right)g^2)}{\sqrt{N}}$$

here,*g* is given by Eq.12 and *p* has been given by Kirby[7]

$$p = \frac{N-2}{\sqrt{N-1}}$$

A and *B* have been given by Bobee and Robitaille[1]

$$A = 1 + 6.51N^{-1} + 20.2N^{-2}$$

$$B = 1.48N^{-1} + 6.77N^{-2}$$

finally the expression of *f* in the Eq.17 is:

$$f = 1 - 3.12R^{*3.7}N^{-0.49}$$

By the correction we obtain M^*, R^*, S^*, and G^* which are unbiased estimators of *M, R, S* and *G*. Once all these values are computed, Eqs.6-9 are used to estimate the set of model parameters *a*, *b*, *c* and Φ, respectively.

D. Thomas-Fiering Model (TF Model)

Phien and Ruksasilip[10] studied several models as compared to the Thomas-Fiering model for generation of monthly streamflows, in terms of preserving of historical parameters, namely the mean value, standard deviation and skewness coefficient of historical series. The Thomas-Fiering model has been confirmed by Singh and Lonnquist[5] to be quite popular in monthly streamflow studies. The basic model is:

$$Q_{i,j} = Q_j^* + b_j\left(Q_{i,j-1} - Q_{j-1}^*\right) + s_j(1 - r_j^2)^{\frac{1}{2}}t_j \quad (18)$$

where $Q_{i,j}$ is the monthly flow in month *j* of year *i*; b_j is the regression coefficient for estimating the flow in month *j* from that in month *j-1*; Q^*_j and s_j are respectively the mean and standard deviation of the historical flows in month *j*; r_j is the correlation coefficient between historical flow sequences in months *j* and *j-1* and t_j is a random variable with zero mean and unit variance. The parameters of the historical series of monthly flows (ie. the mean value, standard deviation, skewness coefficient) are computed according to Eqs.10-12 while the correlation coefficient is as in [5]:

$$r_j = \frac{1}{(N-1)s_j s_{j-1}} \sum_{i=1}^{N} (X_{i,j} - m_j)(X_{i,j-1} - m_{j-1})$$

$$j = 2,..,12$$

$$r_1 = \frac{1}{(N-2)s_1 s_{12}} \sum_{i=2}^{N} (X_{i,1} - m_1)(X_{i-1,12} - m_{12}) \quad j = 1$$

E. Method of Fragments

Svanidze[12] presented a method in which the monthly flows are standardized year by year so that the sum of the monthly flows in any year equals unity. This is done by dividing the monthly flows in a year by the corresponding annual flow. By doing so, from a record of *N* years, one will have *N* fragments of twelve monthly flows. The annual flows obtained from an annual model can be disaggregated by selecting the fragments at random. Since the monthly parameters were not preserved well, Srikanthan and McMahon[11] suggested a way to improve this preservation by selecting the appropriate fragment for each flow in the annual flow series. This was done as follows: The annual flows from the historical record were ranked according to increasing magnitude, and *N* classes were formed. The first class has the lower limit at zero while class *N* has no upper limit. The intermediate class limits are obtained by averaging two successive annual flows in the ranked series. The corresponding fragments were then assigned to each class. That is, the fragment obtained from the monthly flows corresponding to the smallest annual flow was assigned to class 1, the fragment obtained from the monthly flows corresponding to the second smallest annual flows was assigned to class 2 and so on. The annual flows were then checked one by one for the class to which they belong and disaggregated using the corresponding fragment.

III. THE PROPOSED MODELS

A. Gar(1)-Monthly model (GAR(1)-M)

The GAR(1) model has been found to be very good in the case of annual data [2]. For the case of monthly data, each sequence of data of the same month, say *j*, of *N* years long forms a sequence of data in month *j*, and the GAR(1) model can be applied to simulate these monthly data. So the GAR(1)-Monthly model is as follows:

$$X_{i,j} \;=\; \Phi_j X_{i-1,j} + e_i, \quad j = 1..12 \qquad (19)$$

where: $X_{i,j}$ is the random variable representing the dependent processes at time *i* of month *j*, Φ_j *is* autoregressive coefficient of month *j* and e_i is an independent variable to be specified. Each sequence of dependent gamma variable represents a sequence of data of same month over years. The system of equations in (19) constitutes a model for use to simulate monthly streamflows.

In reality, the correlation coefficient between monthly flows into consecutive years may be negative and this may give rise to a negative value of the autoregressive coefficient, therefore a modification of the correlation coefficient of month *j* is needed to make the GAR(1) model applicable:

$$r_j = -r_j \text{ if } r_j < 0.$$

B. Gar(1)-Fragments model (GAR(1)-F)

This model is obtained by a combination of the GAR(1) model with the fragments method. From the historical record of monthly data (of *N* years long), the historical record of annual flows with *N* years, the classes and the fragments are formed. The annual flows obtained from the GAR(1) model will be disaggregated to obtain the monthly flows by using the corresponding fragments. Based on historical record of monthlyflows, the GAR(1)-fragments model generates monthly flows in the following algorithm:

1:Seperate the historical series becomes *N* classes, each class is one year of history.

2:Sort *N* classes according to increasing magnitude of historical annual streamflow A_i

$$A_i = \sum_{j=1}^{12} A_{ij}$$

$A_{i,j}$ is the monthly streamflow in month *j* of year *i*, after sorting A_1 corresponding to smallest annual flow, A_N corresponding to largest annual flow.

3:Compute the upper bound U_i of two successive classes:

$U_i=(A_i+A_{i+1})/2$, i=1,2,..*N*-1. U_N has arbitrary large value.

4:Compute the parameters: shape, scale, location and autoregressive coefficient of GAR(1) model based on the historical annual streamflow.

5:Generate a random number X_1 has three-parameter gamma distribution (the parameters were computed as in Step 4).

6:Select the class has the smallest upper bound is greater than or equal to X_1 (so called i[th] class).

7:Compute $Q_{1,j} = M_{i,j} * X_1$,$Q_{1,j}$ is the monthly streamflow in month *j* of year 1; $M_{i,j} = A_{i,j}/A_i$, $M_{i,j}$ is the fragment of historical monthly streamflow in month *j* of year i.

8:Compute $Q_{k,j}$: *k=2,..n* (*n*: number of years to genarate), use GAR(1) model to generate e_k and compute X_k, *k=2,..,n.* Select the class having the smallest upper bound greater than or equal to X_k (so called i[th] class), then

$$Q_{k,j} = M_{i,j} * X_k .$$

IV. COMPUTER SIMULATIOM

The simulator is the set of programs which were coded in C++. To generate the GAR(1) variables, we can use the most suitable algorithms which have been proposed for generating the needed random variables.To generate Poisson variates, the efficient method of Kemp and Kemp[6] was used while exponential variates were obtained by inversion. Gamma variates were obtained by Marsaglia and Tsang[3].

To verify the proposed models, with the GAR(1)-Fragments model, first the annual runoffs are computed from the monthly streamflow records, then the GAR(1) model is used to generate sequences of annual flows which are used for the disaggregation to obtain the generated monthly flows. The values of the shape parameters obtained from the historical monthly data were used by GAR(1)-Monthly and Thomas-Fiering models for generating the sequence of monthly streamflows. For evaluating the preservation of the mean, standard deviation and the skewness coefficient, no statistical tests were employed. Insteaded, the parameters of the historical and generated sequences were tabulated for visual inspection. Monthly and annual flow data at three stations (*source: Institute of Meteorology and Hydrology*) were used to estimate the statistical descriptors and model parameters for use in the simulation study. For each model and at each station, a moderate sample of 1000 years of data was generated on computer using the referred algorithms.The reproduction of monthly and annual descriptors corresponding to three stations is investigated. The results are as follows:

TABLE I. LISTOF STATIONS WHERE DATA WERE EMPLOYED

Station	River	Location	Period of record
Nong Son	Thu Bon	Quang Nam, VietNam	1980-2010
Thanh My	Vu Gia	Quang Nam, VietNam	1980-2010
Yen Bai	Thao	Yen Bai, VietNam	1958-2010

TABLE II. PARAMETERS AT STATIONS OBTAINED FROM THE CORRECTION PROCEDURE AND USED IN GAR(1)-FRAGMENTS MODEL

Parameters	Nong Son	Thanh My	Yen Bai
Shape	4.84	2.96	3.59
Scale	473.56	300.08	960.62
Location	1178.97	689.03	5344.71
Autoregressive	0.24	3.18	0.18

TABLE III. SHAPE PARAMETERS FOR GENERATING THE SEQUENCE OF MONTHLY STREAMFLOWS BY GAR(1)-M AND THOMAS-FIERING MODELS

Month	Nong Son	Thanh My	Yen Bai
1	1.03	6.42	7.46
2	2.24	4.11	47.34
3	3.63	3.90	2.22
4	0.81	2.65	3.52
5	4.44	1.42	3.79
6	4.69	1.04	2.83
7	12.60	0.87	4.29
8	1.01	0.41	0.72
9	0.16	0.17	14.11
10	54.53	10.34	6.67
11	6.08	3.56	1.83
12	3.91	3.06	1.25

TABLE IV. CORRELATION COEFFICIENTS OF THE HIS. MONTHLY RECORDS

Month	Nong Son	Thanh My	Yen Bai
1	- 0.04	0.07	0.09
2	0.16	0.26	0.15
3	0.02	0.13	- 0.05
4	0.07	0.25	- 0.03
5	0.20	0.31	- 0.14
6	0.01	0.03	0.05
7	0.27	0.20	0.13
8	0.08	0.16	0.19
9	- 0.02	0.02	0.04
10	- 0.33	- 0.33	0.03
11	0.05	0.10	- 0.11
12	0.17	0.14	0.14

TABLE V. MEAN VALUE AT NONG SON STATION

Month	Historical	GAR(1)-M	GAR(1)-F	TF Model
1	248.96	245.40	220.25	267.63
2	138.21	137.85	136.53	147.64
3	94.05	93.01	94.06	101.39
4	76.45	76.84	66.42	87.16
5	107.30	106.38	97.66	121.01
6	94.54	94.15	93.68	101.73
7	70.33	71.44	74.95	74.84
8	85.02	85.60	91.32	93.60
9	195.59	195.30	174.61	94.19
10	697.19	705.26	778.81	754.37
11	1041.81	1039.30	1074.54	1116.12
12	619.97	622.19	559.19	659.08

TABLE VI. MEAN VALUE AT THANH MY STATION

Month	Historical	GAR(1)-M	GAR(1)-F	TF Model
1	116.05	116.72	101.25	116.52
2	71.03	71.29	66.39	71.66
3	50.73	50.51	47.96	50.89
4	45.03	44.92	37.92	44.96
5	58.50	58.19	53.76	58.29
6	56.09	56.36	56.69	55.18
7	46.80	46.31	46.63	45.37
8	59.03	58.79	55.97	57.02
9	113.24	114.31	85.78	115.42
10	301.76	308.59	347.83	302.87
11	403.93	404.16	405.80	413.26
12	255.31	255.11	243.10	258.62

TABLE VII. MEAN VALUE AT YEN BAI STATION

Month	Historical	GAR(1)-M	GAR(1)-F	TF Model
1	311.09	310.70	308.49	296.54
2	264.81	263.61	262.53	264.18
3	236.31	232.17	233.56	244.88
4	269.61	269.16	268.31	258.46
5	433.35	429.57	434.60	346.55
6	851.98	848.37	847.83	803.96
7	1340.41	1354.18	1355.75	1281.54
8	1699.61	1682.67	1689.19	1711.18
9	1353.24	1336.17	1358.96	1303.05
10	976.46	976.90	983.76	1075.25
11	655.91	650.63	640.44	776.64
12	403.48	400.92	398.18	480.87

TABLE VIII. STANDARD DEVIATION AT NONG SON STATION

Month	Historical	GAR(1)-M	GAR(1)-F	TF Model
1	110.97	104.54	87.42	79.22
2	46.07	45.50	37.07	34.23
3	33.30	32.67	30.37	24.61
4	39.32	40.82	34.25	29.29
5	60.89	63.72	53.22	45.05
6	39.63	38.2	32.01	29.01
7	25.65	26.07	29.32	19.35
8	48.82	49.52	71.14	36.02
9	174.70	178.68	88.39	18.56
10	354.16	376.42	438.79	244.56
11	549.65	544.42	534.59	401.98
12	329.72	334.52	311.34	235.41

TABLE IX. STANDARD DEVIATION AT THANH MY STATION

Month	Historical	GAR(1)-M	GAR(1)-F	TF Model
1	45.38	45.52	42.36	46.49
2	23.88	24.38	24.74	24.40
3	16.73	16.13	16.34	16.34
4	17.86	17.82	17.83	17.26
5	28.40	28.52	24.26	27.75
6	27.23	27.06	27.46	26.01
7	17.16	17.10	17.29	16.16
8	31.67	31.00	32.57	31.07
9	90.08	92.33	44.23	90.30
10	159.87	158.20	164.95	159.19
11	236.99	240.94	215.25	236.50
12	128.07	127.88	110.51	129.15

TABLE X. STANDARD DEVIATION AT YEN BAI STATION

Month	Historical	GAR(1)-S	GAR(1)-F	TF Model
1	76.00	75.22	83.20	75.41
2	58.90	69.28	58.08	57.61
3	73.98	73.87	76.44	71.29
4	80.73	84.47	85.85	78.45
5	182.15	182.56	191.76	181.82
6	351.79	342.62	317.07	342.55
7	453.46	450.88	424.31	469.40
8	632.72	621.31	502.87	626.62
9	422.36	438.64	421.78	431.73
10	291.48	316.70	258.66	291.06
11	275.57	264.99	214.89	268.58
12	132.14	135.29	155.43	127.62

TABLE XI. SKEWNESS COEFFICIENT AT NONG SON STATION

Month	Historical	GAR(1)-S	GAR(1)-F	TF Model
1	1.54	1.53	1.51	0.67
2	1.09	1.23	0.95	0.57
3	0.87	1.20	0.73	0.43
4	1.70	1.98	2.18	0.48
5	0.79	1.00	0.78	0.35
6	0.77	0.80	0.93	0.34
7	0.47	0.64	1.32	0.22
8	1.55	1.76	3.44	0.62
9	3.08	5.17	2.32	1.73
10	0.23	-0.01	-0.12	0.22
11	0.68	0.66	1.66	0.42
12	0.84	1.12	0.96	0.55

TABLE XII. SKEWNESS COEFFICIENT AT THANH MY STATION

Month	Historical	GAR(1)-S	GAR(1)-F	TF Model
1	0.66	0.65	1.39	0.55
2	0.81	0.89	1.34	0.46
3	0.84	0.98	1.26	0.40
4	1.00	1.21	1.33	0.43
5	1.31	1.45	1.53	0.47
6	1.53	1.57	1.20	0.70
7	1.65	1.67	1.69	0.73
8	2.21	2.78	3.90	0.98
9	3.02	4.28	1.63	1.65
10	0.51	0.55	0.43	0.49
11	0.88	0.98	0.52	0.56
12	0.94	1.14	0.79	0.71

TABLE XIII. SKEWNESS COEFFICIENT AT YEN BAI STATION

Month	Historical	GAR(1)-S	GAR(1)-F	TF Model
1	0.66	0.81	1.24	0.42
2	0.26	0.32	0.48	0.13
3	1.18	1.21	2.66	0.63
4	0.95	1.00	1.29	0.42
5	0.92	0.93	2.06	0.54
6	1.06	1.24	0.58	0.48
7	0.86	0.91	1.38	0.33
8	1.96	2.13	1.44	0.92
9	0.48	0.41	0.60	0.25
10	0.70	0.70	0.65	0.25
11	1.30	1.29	0.96	0.85
12	1.54	2.05	2.24	0.66

TABLE XIV. STATISTICAL PARAMETERS OF ANNUAL DATA AT NONG SON

Parameters	Historical	GAR(1)-M	GAR(1)-F	TF Model
Mean value	3469.72	3454.17	3467.92	3588.66
Standard Dev.	1030.77	729.03	1025.29	664.64
Skew. Coeff.	0.76	0.32	0.78	0.08

TABLE XV. STATISTICAL PARAMETERS OF ANNUAL DATA AT THANH MY

Parameters	Historical	GAR(1)-M	GAR(1)-F	TF Model
Mean value	1577.48	1583.05	1572.32	1558.69
Standard Dev.	507.77	347.30	508.08	453.43
Skew. Coeff.	0.95	0.51	1.15	0.24

TABLE XVI. STATISTICAL PARAMETERS OF ANNUAL DATA AT YEN BAI

Parameters	Historical	GAR(1)-M	GAR(1)-F	TF Model
Mean value	8796.28	8825.09	8732.18	8695.73
Standard Dev.	1813.37	1312.66	1803.24	1700.69
Skew. Coeff.	0.94	0.75	0.93	0.04

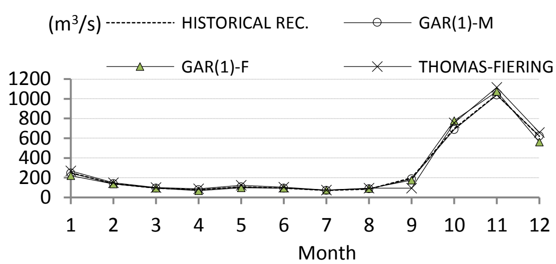

Figure 1. Mean value at Nong Son station

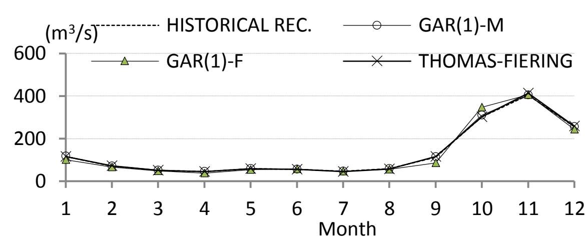

Figure 2. Mean value at Thanh My station

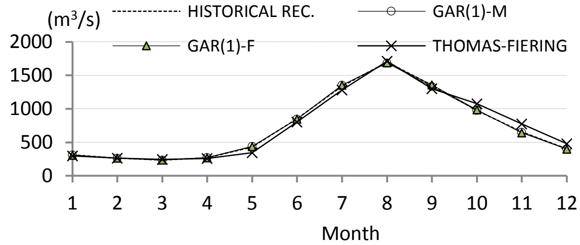

Figure 3. Mean value at Yen Bai station

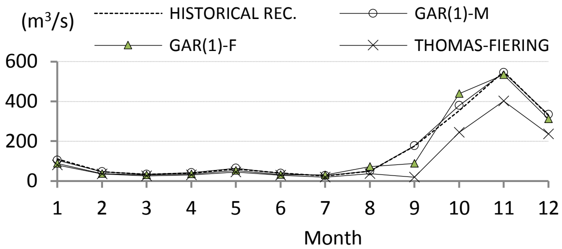

Figure 4. Standard Deviation at Nong Son station

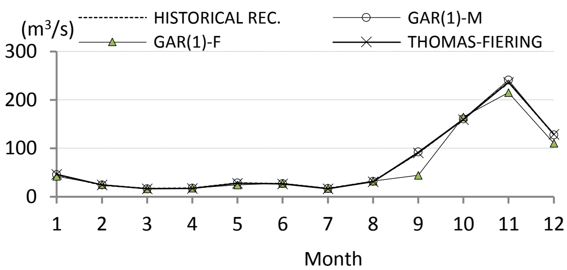

Figure 5. Standard Deviation at Thanh My station

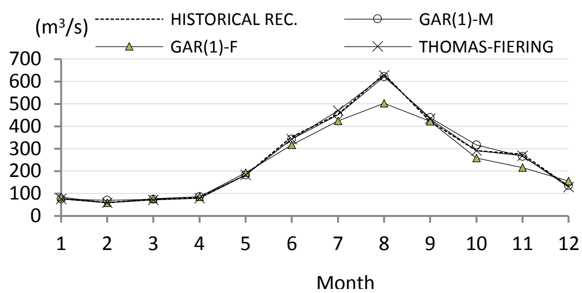

Figure 6. Standard Deviation at Yen Bai station

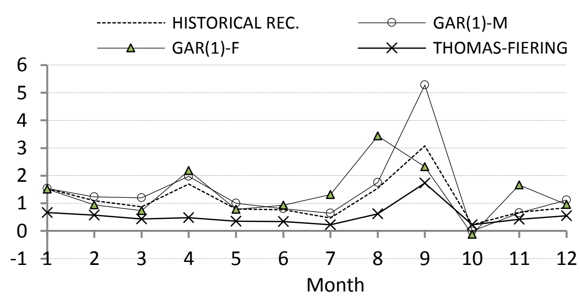

Figure 7. Skewness coefficient at Nong Son station

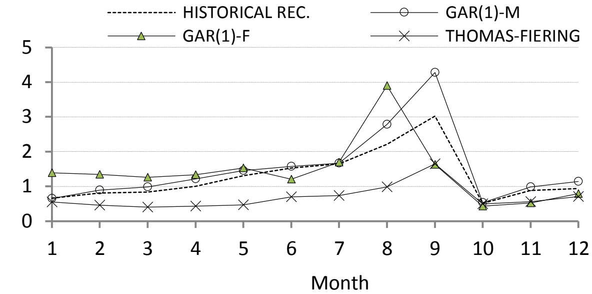

Figure 8. Skewness coefficient at Thanh My station

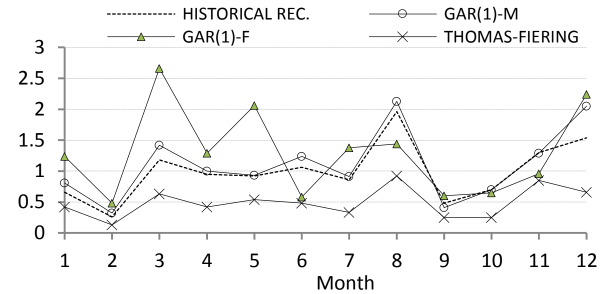

Figure 9. Skewness coefficient at Yen Bai station

V. CONCLUSION

The following conclusions are drawn from this study:

- For monthly data:

- Theoretically, the GAR(1)-Monthly model cannot apply in the case of negative autoregressive coefficient Φ_j, but in reality, for monthly flows, this coefficient can take on negative values (table IV), so the proposed modication of the correlation coefficient is really needed to release any restriction on the use of the GAR(1)-Monthly model.
- The mean and the standard deviation of the historical sequences were preserved very well by three models under consideration (tables V-X and figs. 1-6), whereas GAR(1)-Fragments and Thomas-Fiering models do not preserve the skewness coefficient well (tables XI-XIII and figs. 7-9).
- The mean, standard deviation and the skewness coefficient obtained from monthly data generated by the GAR(1)-Monthly model are closer to their historical values than those obtained by GAR(1)-Fragments and Thomas-Fiering models.
- The statistical values obtained by GAR(1)-Fragments and Thomas-Fiering models are very close to each other.

- For annual data:

In this study, generated annual data were obtained from generated monthly data by taking the sum of twelve monthly values in a year. Then the mean, standard deviation and skewness coefficient can be estimated

- It was found that these statistical descriptors can be reproduced very well by the GAR(1)-Fragments model, much better than those by the GAR(1)-Monthly and Thomas-Fiering models (tables XIV-XVI).

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