

SIMPLE MODEL FOR THE INPUT IMPEDANCE OF RECTANGULAR MICROSTRIP ANTENNA

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ABSTRACT

A very simple model for the input impedance of a coax-fed rectangular microstrip patch antenna is presented. It is based on the cavity model and the equivalent resonant circuits. The theoretical input impedance results obtained from this model are in good agreement with the experimental results available in the literature. This model is well suited for computer-aided design (CAD).

Key Words : Microstrip antenna, Rectangular, Input impedance

DİKDÖRTGEN MİKROŞERİT ANTENLERİN GİRİŞ EMPEDANSI İÇİN BASİT BİR MODEL

ÖZET

Koaksiyel-besleme dikdörtgen mikroşerit yama antenin giriş empedansı için çok basit bir model sunulmuştur. Bu modelin temeli, rezonatör modeline ve eşdeğer rezonans devresine dayanmaktadır. Bu modelden elde edilen teorik giriş empedansı sonuçları, literatürdeki mevcut deneysel sonuçlarla uyumluluk içindedir. Bu model, bilgisayar yardımı ile tasarım için çok uygundur.

Anahtar Kelimeler : Mikroşerit anten, Dikdörtgen, Giriş empedansı

1. INTRODUCTION

Microstrip antennas are attractive due to their low profile and conformality, potential low cost, and the ease with which they can be integrated with printed feed networks and active circuits (Lo et al., 1977 ; Carver and Coffey, 1979; Lo et al., 1979; Bahl and Bhartia, 1980; Carver and Mink, 1981; Deshpande and Bailey, 1982; Pozar, 1982; Abboud et al., 1988; Gupta and Benalla, 1988; Richards, 1988; James and Hall 1989; Lo et al., 1989; Bhartia et al., 1991; Hirasawa and Haneishi, 1992; Damiano and Papiernik, 1993; Güney, 1993; Güney, 1993a; Güney, 1994; Güney, 1994a; Güney, 1994b; Güney, 1994c; Güney, 1994d; Güney, 1995; Güney, 1995a; Zürcher and Gardiol, 1995; Pozar and Schaubert, 1995; Sağıroğlu and Güney, 1997;) These attractive features have increased the applications of

microstrip antennas recently and stimulated greater effort to investigate their performance. Microstrip antennas have been used in various configurations: square, rectangular, circular, triangular, trapezoidal, elliptical etc. In rectangular microstrip antenna designs, it is important to determine the input impedance of the antenna accurately because the input impedance directly affects the efficiency of energy transfer to or from the antenna. It depends strongly on the dimensions of the patch, the location of the feed point, the excitation frequency, the permittivity of the substrate and its thickness. Owing to its effects on the efficiency of energy transfer, it should also be determined accurately in order to provide a good match between the patch and the feed.

A number of methods (Lo et al., 1977; Carver and Coffey, 1979; Lo et al., 1979; Bahl and Bhartia,

1980; Richards et al., 1981; Carver and Mink, 1981; Deshpande and Bailey, 1982; Pozar, 1982; Gupta and Benalla, 1988; Richards, 1988; Abboud et al., 1988; James and Hall, 1989, Lo et al., 1989; Aksun et al., 1990; Bhartia et al., 1991; Hirasawa and Haneishi, 1992; Damiano and Papiernik, 1993; Zürcher and Gardiol, 1995; Pozar and Schaubert, 1995), have been developed for the determination of the input impedance of rectangular microstrip antenna with probe excitation fed from a coaxial line. These methods have different levels of complexity, require vastly different computational efforts, and can generally be divided into two groups: simple analytical methods and rigorous numerical methods. Simple analytical methods offer both simplicity and physical insight. They yield results accurate enough for many engineering purposes. However, exact mathematical formulations in rigorous methods involve extensive numerical procedures, resulting in round-off errors, and may also need final experimental adjustments to the theoretical results. They are also time consuming and not easily included in a computer-aided design system, and provide little physical insight.

In this paper, an analytical expression for the input impedance of rectangular microstrip patch antenna excited by a coaxial probe is presented using the cavity model and the equivalent resonant circuits. This expression depends firmly on the resonant frequency, the radiation quality factor, and the resonant resistance. In previous works (Güney, 1994a, Güney, 1994b; Güney, 1994c), Güney proposed very simple formulas to determine accurately the resonant frequency, the radiation quality factor, and the resonant resistance. It was shown by Güney (Güney, 1994a; Güney, 1994b; Güney, 1994c) that the theoretical results obtained from these simple formulas are in very good agreement with the experimental results. In this study, the same formulas are used in calculating the input impedance. Thus, the theoretical input impedance results obtained in this article are in good agreement with the experimental results reported elsewhere (Lo et al., 1977; Carver and Coffey, 1979; Deshpande and Bailey, 1982).

2. ANALYSIS

2. 1. Input Impedance

Consider a rectangular patch of width W and length L over a ground plane with a substrate of thickness h and a dielectric constant ϵ_r , as shown in Figure 1. Carver and Coffey (1979) showed that the lumped-

element model of a single-port single-mode cavity can be used to determine the input impedance, resonant frequency, and efficiency of rectangular microstrip patches.

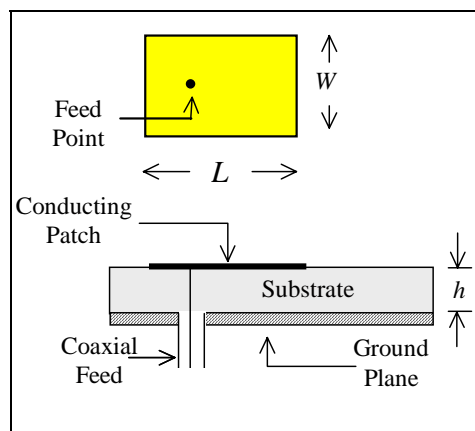


Figure 1. Geometry of a rectangular microstrip antenna

It has been shown that the equivalent circuit for a single spectrally-isolated resonant mode of the microstrip antenna can be represented by a parallel RLC lossy resonant circuit which is in series with an inductive term representing both the influence of the feed and also the residual net magnetic energy associated with all the higher order modes. The residual effects of evanescent and higher-order modes, together with any error in the calculated effective dielectric constant are accounted for in the edge extension term. It is also possible to consider either the dominant mode or the complete spectrum of modes. The rectangular microstrip patch antenna can be considered in the fundamental mode, modelled by a simple resonant parallel RLC circuit, as shown in Figure 2. In order to take the coax-feed probe into account, it is necessary to modify the input impedance by an inductive reactance term (Deshpande and Bailey, 1982):

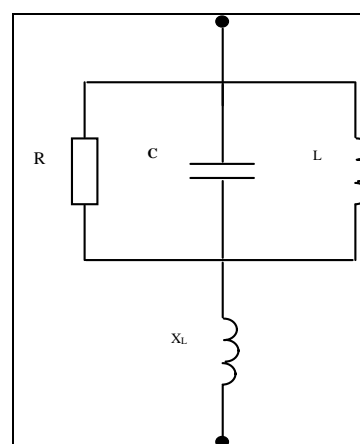


Figure 2. Equivalent resonant parallel RLC circuit

$$X_L = \frac{377fh}{c} \ln\left(\frac{c}{\pi f d_0 \sqrt{\epsilon_r}}\right) \quad (1)$$

where c is the velocity of electromagnetic waves in free space, and d_0 is the diameter of the probe. The following equation can be used to calculate the input impedance of rectangular microstrip antenna

$$Z(f) = \frac{R}{1 + Q_T^2 \left[\frac{f}{f_r} - \frac{f_r}{f} \right]^2} + j \left[X_L - \frac{RQ_T \left[\frac{f}{f_r} - \frac{f_r}{f} \right]}{1 + Q_T^2 \left[\frac{f}{f_r} - \frac{f_r}{f} \right]^2} \right] \quad (2)$$

where R is the resonant resistance of the resonant parallel RLC circuit, f_r is the resonant frequency, and Q_T is the total quality factor associated with system losses including radiation, the loss due to heating in the conducting elements and the ground plane, and the loss due to heating within the dielectric medium. It is clear from equation 2 that the input impedance depends strongly on the resonant frequency, the quality factor, and the resonant resistance. In the following sections, the formulas proposed by Güney (Güney, 1994a; Güney, 1994b; Güney, 1994c) for these parameters are given.

2. 2. Resonant Frequency

In general, the resonant frequency of rectangular patch antennas is calculated by using resonant length transmission line or cavity models, together with equations for the effective dielectric constant and edge extension from the literature. The resonant frequency f_{mn} of a rectangular patch of width W and length L , both comparable to $\lambda_s/2$, where λ_s is the wavelength in the substrate, is given by

$$f_{mn} = f_r = \frac{c}{2(\epsilon_e)^{1/2}} \left[(m/L_e)^2 + (n/W_e)^2 \right]^{1/2} \quad (3)$$

where ϵ_e is the effective relative dielectric constant for the patch, m and n take integer values, and L_e and W_e are the effective dimensions. To calculate the resonant frequency of a rectangular patch antenna driven at its fundamental TM_{10} mode, equation 3 is written as

$$f_{10} = f_r = \frac{c}{2(\epsilon_e)^{1/2} L_e} \quad (4)$$

The effective length L_e can be defined as follows:

$$L_e = L + 2\Delta L \quad (5)$$

The effects of the non-uniform medium and the fringing fields at each end of the patch are accounted for by the effective relative dielectric constant, ϵ_e , and the edge extension, ΔL , being the effective length to which the fields fringe at each end of the patch. The following effective dielectric constant formula proposed by Hammerstad and Jensen (1980) is used in equation 4

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10h}{W} \right)^{-ab} \quad (6)$$

where

$$a = 1 + \frac{1}{49} \ln \left\{ \frac{(W/h)^4 + W^2/(52h)^2}{(W/h)^4 + 0.432} \right\} + \frac{1}{18.7} \ln \left\{ 1 + \left(\frac{W}{18.1h} \right)^3 \right\} \quad (7a)$$

$$b = 0.564 \left(\frac{\epsilon_r - 0.9}{\epsilon_r + 3} \right)^{0.053} \quad (7b)$$

The accuracy of the results given by equation 6 with 7a and 7b is claimed to be better than 2.5 % for the range of normalized widths $0.01 \leq W/h \leq 100$ and $\epsilon_r \leq 50$.

The following closed-form expressions proposed by Güney (1994b) are used for the edge extension

$$\frac{\Delta L}{h} = \frac{21.4075 + k_0 h (184.6614 - 1.1475\epsilon_r + 8.5k_0 h) - 1.35\epsilon_r}{18(1 + 10.85k_0 h + 8.5k_0^2 h^2)} \quad (8a)$$

for $h/\lambda_s \leq 0.11$

$$\frac{\Delta L}{h} = \frac{6.8955 + k_0 h (61.062 - 0.3315\epsilon_r + 8.5k_0 h) - 0.39\epsilon_r}{5.2(1 + 10.85k_0 h + 8.5k_0^2 h^2)} \quad (8b)$$

for $h/\lambda_s > 0.11$

where $k_0 = 2\pi/\lambda_0$ and $\lambda_0 = 2.08 L \sqrt{\epsilon_r}$. If ϵ_r is used for the calculation of free-space wavelength, this wavelength can be calculated from $\lambda_0 = 2.08 L \sqrt{\epsilon_r}$. This equation is also used by Pozar (1985) for the computer-aided design of rectangular microstrip antennas. The resonant frequency is then obtained by substituting equations. 5-8 into equation 4.

2. 3. Resonant Resistance

The resonant resistance can be evaluated from the following formula proposed by Güney (1994c)

$$R = R_r + R_c + R_d \quad (9)$$

where R_r , R_c , and R_d represent, respectively, the radiation resistance, the equivalent resistance of the copper loss, and the equivalent resistance of the dielectric loss. In equation 9, the surface wave effect is assumed to be negligible. The equivalent resistances for the copper loss R_c and dielectric loss R_d can be calculated from the equations given by Carver and Coffey (1979):

$$R_c = 0.00027 \sqrt{f_r} \frac{L}{W} Q_r^2 \quad (f_r \text{ in gigahertz}), \quad (10)$$

$$R_d = \frac{30 \tan \delta}{\epsilon_r} \frac{h \lambda_0}{LW} Q_r^2 \quad (11)$$

where $\tan \delta$ is the dielectric loss tangent to the substrate and Q_r is the radiation quality factor.

The radiation resistance R_r associated with the radiation quality factor Q_r can be written as follows:

$$R_r = \frac{Q_r}{2\pi f_r C} \quad (12)$$

where C is the capacitance of the TM_{10} mode. It is given by

$$C = \frac{\epsilon_e \epsilon_0 WL}{2h} \cos^{-2} \frac{\pi x_0}{L} \quad (13)$$

where x_0 is the distance of the feed point from the edge of the patch.

It is apparent from equations 10-12 that the resonant resistance depends firmly on the radiation quality factor and the resonant frequency.

2. 4. Quality Factor

The total quality factor is given as,

$$Q_T = Q_r \frac{R}{R_r} \quad (14)$$

where R and R_r are given by equations 9 and 12, respectively. The following radiation quality factor expression presented by Güney (1994c) is used in equation 14

$$Q_r = \frac{c\sqrt{\epsilon_e}}{4hf_r} - \frac{\epsilon_e \Delta L}{h} \quad (15)$$

The radiation quality factor expression given by equation 15 is also used in equations 10-12. The

input impedance is then calculated from equation 2 using equations 1, 4, 9 and 14.

3. NUMERICAL RESULTS DISCUSSIONS AND CONCLUSIONS

In order to check the accuracy of the method proposed in this article, the numerical results for the input impedance obtained from the method proposed here are compared in Figures 3-5 with previously presented measurements (Lo et al., 1977; Carver and Coffey, 1979; Deshpande and Bailey, 1982) and numerical results (Pozar, 1982; Abboud et al., 1988).

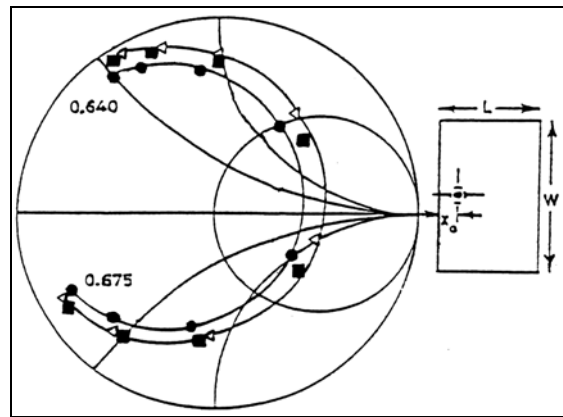


Figure 3. Input impedance of rectangular microstrip antenna with $W = 20.45$ cm; $L = 13.97$ cm; $h = 0.1588$ cm; $d_0 = 0.127$ cm; $\epsilon_r = 2.59$; $\tan \delta = 0.003$; $m = 1$, $n = 0$; $x_0 = 0.635$ cm; • measured by Lo et al. (1977); Δ calculated by Pozar (1982); present model

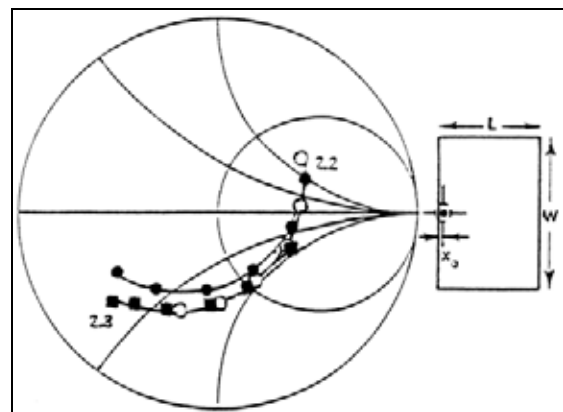


Figure 4. Input impedance of rectangular microstrip antenna with $W = 6.858$ cm; $L = 4.140$ cm; $h = 0.1524$ cm; $d_0 = 0.127$ cm; $\epsilon_r = 2.50$; $\tan \delta = 0.002$; $m = 1$, $n = 0$; $x_0 = 0.0$ cm; • measured by

Carver and Coffey (1979); o calculated by Abboud et al. (1988); present model

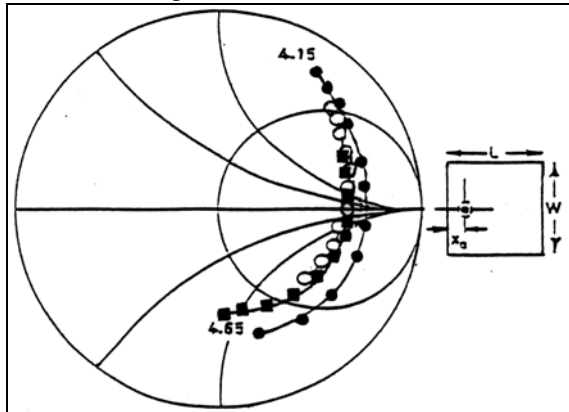


Figure 5. Input impedance of rectangular microstrip antenna with $W = 2.01$ cm; $L = 2.01$ cm; $h = 0.159$ cm; $d_0 = 0.127$ cm; $\epsilon_r = 2.55$; $\tan \delta = 0.002$; $m = 1$, $n = 0$; $x_0 = 0.13$ cm; • measured by Deshpande and Bailey (1982); o calculated by Abboud et al. (1988); present model

As shown in Figures 3-5, our results agree somewhat better with the experimental results over the frequency range of interest, than do the previously presented results. Furthermore, the formulas given here are more simpler than those given by Pozar (1982) and Abboud et al. (1988), and are very useful for practical designs of microstrip antennas. The good agreement between the measured and our computed input impedance values supports the validity of the method presented in this work.

In this study, the resonant frequency, the radiation quality factor and the resonant resistance of a rectangular microstrip antenna are calculated from the equations proposed by Güney (1994a, 1994b, 1994c) which are given in Section-2. This is because the resonant frequency and the resonant resistance results obtained from these equations are in very good agreement with the experimental results which, in turn, leads to good accuracy in the calculation of the input impedance.

Since the formulas presented in this work have good accuracy and requires no complicated mathematical functions, it can be very useful for the development of fast CAD algorithms. These CAD formulas, capable of accurately predicting the input impedance of rectangular microstrip antennas, is also very useful to antenna engineers. Using these formulas, one can calculate accurately, by a hand calculator, the input impedance of rectangular patch antennas, without any background knowledge of microstrip antennas. For engineering applications, the simple

formulas are very useful. Thus, the formulas proposed in this work can also be used for many engineering applications and purposes.

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