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The Fourth Main Boundary Value Problem of Dynamics of Thermo-resiliency's Momentum Theory

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Abstract

In the paper is presented the fourth main boundary value problem of Dynamics of Thermo-resiliency's Momentum theory. The problem states to find in the cylinder D_l the regular solution of the system:

$$M(\partial_x)\mathcal{U} - \nu\chi\theta - \chi^0 \frac{\partial^2 \mathcal{U}}{\partial t^2} = \mathcal{H}, \quad \Delta\theta - \frac{1}{\vartheta}\frac{\partial\theta}{\partial t} - \eta\frac{\partial}{\partial t}\, div\, u = \mathcal{H}_7,$$

which satisfies the initial conditions:

$$\forall x \in D: \lim_{t\to 0} U(x,t) = \varphi^{(0)}(x), \lim_{t\to 0} \theta(x,t) = \varphi_7^{(0)}(x), \ \lim_{t\to 0}\frac{\partial \mathcal{U}(x,t)}{\partial t} = \varphi^{(1)}(x)$$

and the boundary conditions:

$$\forall (x,t) \in S_l: \lim_{D\ni x\to y\in S} PU = f, \quad \lim_{D\ni x\to y\in S}\{\theta\}_S^{\pm} = f_7.$$

The uniqueness theorem of the solution is proved for this problem.

Keywords: the main boundary value problem; initial conditions; boundary conditions; the uniqueness theorem of the solution.

Introduction

Let D be a finite or infinite three-dimensional space with the compact boundary S from the class $\Lambda_2(\alpha)$, $(\alpha > 0)$.

Denote by D_l and S_l cylinders $D_l = D \times l$, $S_l = S \times l$, respectively, where $l = [0, \infty)$.

In the problems of Dynamics of Thermo-resiliency's Momentum theory any point of environment is characterized by seven quantities: a movement vector -$u = (u_1, u_2, u_3)$, a rotation vector - $\omega = (\omega_1, \omega_2, \omega_3)$ and a temperature deviation - θ.

The main equations of the Thermo-resiliency's Momentum theory can be written in a matrix form as follows [1], [2]:

$$M(\partial_x)\mathcal{U} - \nu\chi\theta - \chi^0 \frac{\partial^2 \mathcal{U}}{\partial t^2} = \mathcal{H}, \;\; \Delta\theta - \frac{1}{\vartheta}\frac{\partial\theta}{\partial t} - \eta\frac{\partial}{\partial t}\, div\, u = \mathcal{H}_7, \qquad (1)$$

where $M(\partial_x)$ is a matrix differential operator of the Momentum Resilience theory [3] and $\chi = (\partial_{x_1}, \partial_{x_2}, \partial_{x_3}, 0, 0, 0)$, $\chi^0 = \|\chi_{ij}^0\|_{6\times 6}$, $\chi_{ii}^0 = \rho$ for $i = 1, 2, 3$, $\chi_{ii}^0 = \zeta$ for $i = 4, 5, 6$, $\chi_{ij}^0 = 0$ for $i \neq j$, $\mathcal{H} = (-\rho F, -\rho\, \mathcal{Y})$, $\mathcal{H}_7 = -\frac{1}{\vartheta}\, Q$, $\mathcal{U} = (u, \omega)$.

Let $\varphi^{(i)} = (\overset{1}{\varphi}(i), \overset{2}{\varphi}(i))$ for $i = 0, 1$, where $\overset{k}{\varphi}(i) = (\varphi_1^{k(i)}, \varphi_2^{k(i)}, \varphi_3^{k(i)})$ for $k = 1, 2$ and $\varphi_7^{(i)}$ for $i = 0, 1$ be functions given in the area $\overline{D}$, while $f = (f^{(1)}, f^{(2)})$, $f^{(i)} = (f_1^{(i)}, f_2^{(i)}, f_3^{(i)})$ for $i = 1, 2$ and f_7 are functions given on S_l.

Definition

Vector-function $U = (u, \omega, \theta)$ is called as regular in the area D_l^+ if $U \in C^1(\overline{D}_l^+) \cap C^2(D_l^+)$ for $\forall t \in l$ and $B(\partial x, \partial t)$U is integrable in the area D^+.

Analogously, vector-function $U = (u, \omega, \theta)$ is called as regular in the area D_l^- if $U \in C^1(\overline{D}_l^-) \cap C^2(D_l^-)$ for $\forall t \in l$ and $B(\partial x, \partial t)$U is integrable in the area $D^- \cap \mathcal{M}(0, \delta)$ for any number $\delta > 0$ and

$$|U(x,\tau)| \leq \frac{c(t)}{1+|x|^2}, \left|\frac{\partial U(x,\tau)}{\partial t}\right| \leq \frac{c(t)}{1+|x|^2}, \left|\frac{\partial U(x,\tau)}{\partial x_i}\right| \leq \frac{c(t)}{1+|x|^2}, \qquad (2)$$

where $B(\partial x, \partial t)$ is an operator standing on the left side of the system (1) and is written in the form of a matrix differential operator.

In the paper is studied the following problem of Dynamics of Thermo-resiliency's Momentum theory: to find in the cylinder D_l the regular solution of the system (1) which satisfies the initial conditions:

$$\forall x \in D: \lim_{t\to 0} U(x,t) = \varphi^{(0)}(x), \lim_{t\to 0} \theta(x,t) = \varphi_7^{(0)}(x), \;\; \lim_{t\to 0}\frac{\partial \mathcal{U}(x,t)}{\partial t} = \varphi^{(1)}(x)$$

and the boundary conditions:

$$\forall (x,t) \in S_l: \lim_{D\ni x\to y\in S} PU = f, \;\; \lim_{D\ni x\to y\in S} \{\theta\}_S^{\pm} = f_7.$$

Here, $P = P(\partial x, n)$ is an operator of thermo-momentary voltage:

$$P(\partial x, n)U = T(\partial x, n)\mathcal{U} - ve\theta,$$

where $T(\partial x, n)$ is an operator of momentary voltage [3], $e = (n_1, n_2, n_3, 0, 0, 0)$, $\mathcal{U} = (u, \omega)$ and n(n_1, n_2, n_3) is a normal of the surface S.

The main result

The following uniqueness theorem is true:

Theorem. In the cylinder $D_l^{\pm}$ the regular solution of the homogenous problem, corresponding to the above stated problem, is identical to 0**.**

The proof of the theorem. Let $U = (\mathcal{U}, \theta)$ be a regular solution in D_l^+ of the homogenous equation corresponding to (1). Then, the following formula is true:

$$\frac{\partial}{\partial t}\int_{D^+}\left\{\frac{1}{2}\sum_{i=1}^{6}\chi_{ii}^0\left|\frac{\partial \mathcal{U}_i}{\partial t}\right|^2 + \frac{1}{2}E(\mathcal{U},\mathcal{U}) + \frac{\nu}{2\vartheta\eta}|\theta|^2\right\}dx +$$

$$+\frac{\nu}{\eta}\int_{D^+}|grad\ \theta|^2 dx = \int_S \left\{\frac{\partial \mathcal{U}}{\partial t}PU - \frac{\nu}{\eta}\theta\frac{\partial\theta}{\partial n}\right\}dS, \qquad (3)$$

where $E(\mathcal{U}, \mathcal{U})$ is a positively defined form [3].

For the regular solution of the homogenous problem in D_l^+ the right side of (3) is equal to 0. Hence, the left side of it is also equal to 0, from which follows that $\mathcal{U} = 0$, $\theta = 0$. So, $U = 0$.

Now, let $U = (\mathcal{U}, \theta)$ be a regular solution of the homogenous problem in D_l^-, corresponding to the system (1). We can write (3) for $D^- \cap \mathcal{M}(0, z)$ as follows:

$$\frac{\partial}{\partial t}\int\limits_{D^-\cap\mathcal{M}(0,z)}\left\{\frac{1}{2}\sum_{i=1}^{6}\chi_{ii}^{0}\left|\frac{\partial \mathcal{U}_i}{\partial t}\right|^2+\frac{1}{2}E(\mathcal{U},\mathcal{U})+\frac{\nu}{2\vartheta\eta}|\theta|^2\right\}dx+$$

$$+\frac{\nu}{\eta}\int\limits_{D^-\cap\mathcal{M}(0,z)}|grad\ \theta|^2dx=\int\limits_{C(0,z)}\left\{\frac{\partial \mathcal{U}}{\partial t}PU+\frac{\nu}{\eta}\theta\frac{\partial\theta}{\partial n}\right\}dS,$$

where z is a sufficiently large number.

Considering the conditions (2) and taking the limit of the above equation as $z\to\infty$, we get that

$$\frac{\partial}{\partial t}\int\limits_{D^-}\left\{\frac{1}{2}\sum_{i=1}^{6}\chi_{ii}^{0}\left|\frac{\partial \mathcal{U}_i}{\partial t}\right|^2+\frac{1}{2}E(\mathcal{U},\mathcal{U})+\frac{\nu}{2\vartheta\eta}|\theta|^2\right\}dx+\frac{\nu}{\eta}\int\limits_{D^-}|grad\ \theta|^2dx=0,$$

from which, using the homogeneity of the initial conditions, we have:

$$U=0.$$

Thus, the theorem is proved.

Conclusion

The main task was to prove the uniqueness theorem of the solution of the fourth main boundary value problem of Dynamics of Thermo-resiliency's Momentum theory. In the cylinder D_l was found the regular solution of the system:

$$M(\partial_x)\mathcal{U}-\nu\chi\theta-\chi^0\frac{\partial^2\mathcal{U}}{\partial t^2}=\mathcal{H},\ \ \Delta\theta-\frac{1}{\vartheta}\frac{\partial\theta}{\partial t}-\eta\frac{\partial}{\partial t}\ div\ u=\mathcal{H}_7,$$

which satisfies the following initial and boundary conditions:

$$\forall x\in D:\lim_{t\to 0}U(x,t)=\varphi^{(0)}(x),\lim_{t\to 0}\theta(x,t)=\varphi_7^{(0)}(x),\ \lim_{t\to 0}\frac{\partial\mathcal{U}(x,t)}{\partial t}=\varphi^{(1)}(x);$$

$$\forall(x,t)\in S_l:\lim_{D\ni x\to y\in S}PU=f,\quad \lim_{D\ni x\to y\in S}\{\theta\}_S^{\pm}=f_7.$$

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