Vol. 05, No. 1 (2023) 15-30, doi: 10.24874/PES05.01.002

Proceedings on Engineering Sciences

www.pesjournal.net

ESTIMATION OF POPULATION PROCESS CAPABILITY INDEX WITH CONFIDENCE

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Received 12.05.2022.
Accepted 11.10.2022.
UDC – 005.642.2: 519.876.5

Keywords:

Average width, Coverage probability, Modified Intervals, Process capability index, Symmetric distribution, Skewed distributions, Simulation Study

A B S T R A C T

The process capability index (Cp) measures the amount of dispersion a process involves relative to the limits of specification. This paper considers sixteen different available confidence intervals for estimating the population process capability index. A simulation study under different conditions has been conducted to compare the performance of the estimators. Our vast simulation records reveal that both augmented large sample (ALS) and modified augmented large sample (MALS) intervals have better coverage probability and shorter average width in all simulation conditions. We expect that the results of this paper will contribute to the literature on process capability and will guide the researchers to select an interval estimator when they are interested to estimate the population process capability index.

1. INTRODUCTION

The purpose of many manufacturing industries is to monitor and control the statistical processes. One of the important tools is the process capability index. Recently, statistical inference of this index has drawn considerable attention to the theoretical as well as the applied researchers. It is a convenient measure because it reduces complex information about a process to a single number (Maiti and Saha (2012)). For example, if the value of the process capability index surpasses one, it implies that the process is acceptable or capable. A process capability index uses both the process variability and the process specifications to decide whether the process is capable. This index equates the output of an in-control procedure to the specification limits using capability indices. The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by six standard deviation units, known as the process width. We are often required to compare the output of a stable process with the process specification and make a statement about how well the process meets the specification. To do this, we compare the natural variability of a stable process with the process specification limits. There are various process capability indices exist in the literature. The most commonly applied index is Cp (Kane, 1986, Zhang, 2010). In this paper, we focus only on the process capability index Cp, defined by Kane (1986) as,

$$Cp = (USL-LSL)/6\sigma \qquad (1.1)$$

where USL is the upper specification limit, LSL is the lower specification limit and σ is the process standard deviation. The numerator of Cp gives the size of the range over which the process measurements can vary. The denominator gives the size of the range over which the process actually varies (Kotz and Lovelace (1998)). Due to the fact that the process standard deviation is unknown, it must be estimated from the sample data. The sample

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standard deviation S is used to estimate the unknown parameter σ in the above equation. The estimator of the process capability index Cp is therefore defined as,

$\hat{C}p$= (USL-LSL)/6S (1.2)

Although the point estimator of the capability index Cp shown in the above equation can be a useful measure, it varies from sample to sample. In this case, the confidence interval is a more useful method because it provides much more information about the population characteristic of interest than does a point estimate (e.g. Smithson (2001); Thompson (2002); Steiger (2004)). The confidence interval for the capability index Cp is constructed by using a pivotal quantity Q that follows a chi-square distribution when the data is normally distributed. When the data is normally distributed, the coverage probability of this confidence interval is close to a nominal value of 1-□. However, the underlying process distributions are not normal in many industrial processes (e.g., Chen and Pearn (1997); Bittanti and Moiraghi (1998); Wu and Messimer (1999); Ding (2004)). In these situations, the coverage probability of the confidence interval can be considerably below 1-□. More on process capability indices we refer Kotz and Johnson (2002) and for confidence interval on Cp, we refer to Peng (2010), Abu-Shawiesh et al. (2020a,b), and Kibria and Chen (2021) and very recently Somkhueanl and Wongkhao (2022) among others.

In this paper, we consider sixteen different available confidence intervals for estimating the population Cp. Since different researchers have considered different confidence intervals and compared them under different simulation conditions, the performance of these interval estimators is not comparable as a whole. The objective of this paper is to compare all sixteen interval estimators under the same simulation condition and compare their performances. Using a simulation study, we want to recommend some good interval estimators for Cp under both skewed and symmetric distribution conditions. More on confidence interval and simulation study, we refer to George and Kibria (2011), Banik and Kibria (2016) and Abu-Shawiesh et al (2018) among others, The structure of the paper is as follows. In Section 2, we review and propose various confidence intervals for the process capability index. Simulations are undertaken in section 3 to see how the confidence intervals perform under different conditions. Conclusions are presented in the final Section 4.

2. STATISTICAL METHODOLOGY

2.1 Classical confidence interval

Suppose $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$, then a (1-α)100% classical confidence interval (CI) for Cp is constructed using a pivotal quantity: $Q = (n-1)S^2/\sigma^2$ and is given as follows:

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6S}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL} = \frac{\text{USL}-\text{LSL}}{6S}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}} \quad (2.1)$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are the α/2 and 1−α/2 quantiles of the central chi-squared distribution with n-1 degrees of freedom respectively.

The CI for *Cp* in (2.1) is to be used for normal distribution data. When the data have normally distributed the CI in (2.1) is close to a nominal value. However, the underlying process distributions are non-normal in many industrial processes (see e.g., Chen and Pearn (1997), Bittanti and Moiraghi (1998), Wu and Messimer (1999), Ding (2004) and others). In these situations, the coverage probability of CI can be below the nominal level. Balamurali and Kalyanasundaram (2002) found that for non-normal data the bootstrap method can help to improve CI for *Cp*. The aim of this paper is to review and propose some new CIs for *Cp* that is based on different robust estimators of the scale parameter *σ*.

The robust methods are one of the most commonly used statistical methods when the underlying normality assumption is violated. These methods offer a useful and viable alternative to traditional statistical methods and can provide more accurate results (Abu-Shawiesh, 2008). A robust estimator is one that is resistant to departures from normality and the presence of outliers. Also, an estimator is said to be robust if it is fully efficient or nearly so for an assumed distribution but maintains high efficiency for plausible alternatives (Tiku and Akkaya, 2004). Now, we will review and propose some modified CIs for estimating *Cp* for non-normal distributions based on the robust methods as follows:

2.2 CI based on Sps

Sps is based on the interquartile range (IQR) and is defined as follows:

$$\text{Sps} = \frac{\text{IQR}}{1.349} \quad (2.2)$$

A (1-α)100% **C**I for *Cp* based on Sps is defined as

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6\text{Sps}}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL} = \frac{\text{USL}-\text{LSL}}{6\text{Sps}}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}} \quad (2.3)$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are $\alpha/2^{th}$ and $1-\alpha/2^{th}$ quintiles of the chi-squared distribution with *n*-1 df.

2.3 CI based on AADM

The average absolute deviation from the median (AADM) is a very robust scale estimator that measures the deviation of the data from MD, which is less influenced by outliers. It is defined as follows:

$$\text{AADM} = \frac{\sqrt{\pi/2}}{n}\sum_{i=1}^{n}\left|X_i - \text{MD}\right| \quad (2.4)$$

MD is best known for being insensitive to outliers and has a maximal 50% breakdown point (Rousseeuw and Croux, 1993). As stated in Gastwirth (1982), AADM is a consistent estimate of σ and is asymptotically normally distributed. A (1-α)100% CI based on AADM is defined as

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6\,\text{AADM}}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL}= \frac{\text{USL}-\text{LSL}}{6\,\text{AADM}}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}}\,. \quad (2.5)$$

2.4 CI based on MAD

The MAD was first introduced by Hampel (1974) and is widely used in various applications as an alternative to *S*. MAD for a random sample is defined as follows:

$$MAD = 1.4826MD\{|X_i - MD|\}, i = 1,2,3,\ldots,n \quad (2.6)$$

The 1.4826 factor given in MAD adjusts the scale for maximum efficiency when the data comes from a normal distribution. A $(1-\alpha)100\%$ CI based on MAD is defined as

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6\,\text{MAD}}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL}= \frac{\text{USL}-\text{LSL}}{6\,\text{MAD}}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}}\,. \quad (2.7)$$

2.5 CI based on GMD

The GMD was developed by the Italian mathematician Professor Corrado Gini (Gini, 1912) for measuring the variability of non-normal data. It is defined as follows:

$$\text{GMD} = \frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left|X_i - X_j\right|_{i<j}}{\binom{n}{2}} = \frac{2}{n(n-1)}\left[\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left|X_i - X_j\right|\right]$$

(2.8)

The GMD is more efficient than S if the normal distribution is contaminated by a small fraction (Gerstenberger and Vogel, 2014). A $(1-\alpha)100\%$ CI for on GMD is defined as

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6\,\text{GMD}}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL}= \frac{\text{USL}-\text{LSL}}{6\,\text{GMD}}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}} \quad (2.9)$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are $\alpha/2^{th}$ and $1-\alpha/2^{th}$ quintiles of the chi-squared distribution with n-1 df.

2.6 CI based on Sn

The S_n estimator was proposed by Rousseeuw and Croux (1993) and is defined as follows:

$S_n = 1.1926MD_i\{MD_j|X_i - X_j|\}, i = 1,2,3,\ldots,n, j = 1,2,3,\ldots,n$ (2.10)

A $(1-\alpha)100\%$ CI based on Sn is defined as

$$\text{LCL} = \frac{\text{USL}-\text{LSL}}{6\,\text{Sn}}\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and UCL}= \frac{\text{USL}-\text{LSL}}{6\,\text{Sn}}\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}} \quad (2.11)$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are $\alpha/2^{th}$ and $1-\alpha/2^{th}$ quintiles of the chi-squared distribution with *n*-1 df.

2.7 CI based on degrees of freedom (DF)

Hummel and Hettmansperger (2005) proposed an estimate for the degrees of freedom using the method of matching. It depends on the fact that the sample variance is a sum of squares and, for sufficiently large samples, is approximated as a chi-square estimate with the appropriate degrees of freedom. They matched the first two moments of the distribution of sample variance with that of a random variable X, which is distributed as $c\chi_r^2$. The solution for r and c is solved using the following systems of equations: 1) $\sigma^2 = cr$ and 2) $\frac{\sigma^4}{n}\left(\kappa - \frac{n-3}{n-1}\right)=2rc^2$, where κ is the kurtosis of the distribution.

A (1-α)100% CI for the *Cp* based on the CI for variance by adjusting the degrees of freedom of the Chi-square distribution is given by

$$LCL = \widehat{cp}\sqrt{\frac{\chi^2_{\alpha/2,(r-1)}}{\hat{r}}} \text{ and } UCL = \widehat{cp}\sqrt{\frac{\chi^2_{1-\alpha/2,(r-1)}}{\hat{r}}} \quad (2,12)$$

where $\hat{r} = \frac{2n}{\hat{\gamma}+2n/(n-1)}$ and

$\hat{\gamma} = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\frac{\sum_{i=1}^{n}(X_i-\bar{X})^4}{S^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$, $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are $\alpha/2^{th}$ and $1-\alpha/2^{th}$ quintiles of the chi-squared distribution with *n*-1 df.

2.8 CI based on the large sample theory (LS)

If the normality assumption is invalid, then one can use the large sample theory, where $S^2 \sim N(\sigma^2, \frac{\sigma^4}{n}(\kappa_e + \frac{2n}{n-1}))$, κ_e is the excess kurtosis and defined as κ-3.

A (1-α)100% CI for the *Cp* based on the large sample confidence interval for variance is

$$LCL = \frac{\widehat{cp}}{\sqrt{\exp\left(Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)}} \text{ and } UCL = \frac{\widehat{cp}}{\sqrt{\exp\left(-Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)}} \quad (2.13)$$

where $A = \frac{G_{2+2n/(n-1)}}{n}$ and $G_2 = \frac{n-1}{(n-2)(n-3)}[(n-1)g_2 + 6]$, $g_2 = \frac{m_4}{m_2^2} - 3$, $m_4 = n^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^4$ and $m_2 = n^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$. For critical value of the test statistic, see the standard Z table.

2.9 CI based on the augmented large sample theory (ALS)

Burch (2014) considered a modification to the approximate distribution of log(S) by using a three-term Taylor's series expansion. Employing the large sample properties of S^2, and following Burch (2014), a (1-α)100% confidence interval for the *Cp* based on the augmented large sample confidence interval for variance is given by

$$LCL = \frac{\widehat{cp}}{\sqrt{\exp\left(Z_{1-\frac{\alpha}{2}}\sqrt{B}+C\right)}}$$

$$\text{and } UCL = \frac{\widehat{cp}}{\sqrt{\exp\left(-Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)+C}} \quad (2,14)$$

where

$B = \widehat{var}\log(S^2) \approx \frac{1}{n}\left(\kappa_e + \frac{2n}{n-1}\right)\left(1 + \frac{1}{2n}\left(\kappa_e + \frac{2n}{n-1}\right)\right)$, $C = \frac{\hat{\kappa}_{e,5}+2n/(n-1)}{2n}$,

$\hat{\kappa}_{e,5} = \left(\frac{n+1}{n-1}\right)G_2(1+\frac{5G_2}{n})$. For the critical value of the test statistic, see the standard *Z* table.

2.10 CI based on Trimmed Standard Deviation (ST)

The sample mean and the sample variation can be influenced by the outliers or extreme values of the distribution. To overcome the extreme value problem, the trimmed technique is very useful (Burch (2014), Tukey (1948), and Dixon and Yuen (1974) among others). To modify the variance of the trimmed mean, Sindhumol et al. (2016) recommended an amendment, which is multiplying the variance of the trimmed mean with a fine-tuning constant. This technique can be described as follows: Consider $X_i \sim N(\mu,\sigma^2)$, i=1,2,…,n. Assume that the order statistics of the above random samples is denoted by $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$. Then the r-times symmetrically trimmed sample is obtained by reducing both bottom-most and uppermost *r* values. Then the trimmed sample mean and the trimmed sample standard deviation is defined respectively as follows:

$\bar{X}_T = \frac{1}{n-2r}\sum_{i=r+1}^{n-r}X_{(i)}$ and

$S_T = \sqrt{\frac{1}{n-2r-1}\sum_{i=r+1}^{n-r}(X_{(i)} - \bar{X}_T)^2}$,

where $r = [\alpha n]$, trimming is done for α% ($0 \le \alpha \le 0.5$) of n. The modified trimmed standard deviation, suggested by Sindhumol et al. (2016)) and is defined as follows: $S^*_T = 1.4826 S_T$ (For details, see Abu-Shawiesh et al. (2020a, b)).

$$LCL = C_p^*\sqrt{\frac{\chi^2_{\left(\frac{\alpha}{2},n-1\right)}}{(n-1)}} \text{ and}$$

$$USL = C_p^*\sqrt{\frac{\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}}{(n-1)}} \quad (2,15)$$

where $\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ and $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$ are the $\alpha/2$ and $1-\alpha/2$ quantiles of the central chi-squared distribution with degrees of freedom respectively. The CI for *Cp* shown above is to be used where $\hat{C}_p^* = \frac{\text{USL} - \text{LSL}}{6\,S_T^*}$ is the sample estimate of population Cp.

2.11 CI based on IQR

measures based on IQR is given by

$$\text{LCL} = C_p^{IQR}\sqrt{\frac{\chi^2_{\alpha/2,(n-1)}}{n-1}} \text{ and}$$

$$\text{UCL} = C_p^{1QR}\sqrt{\frac{\chi^2_{1-\alpha/2,(n-1)}}{n-1}}, \quad (2.16)$$

where $C_p^{IQR} = \frac{USL-LSL}{6S_{SIQR}}$ is the modified sample estimate of population Cp and SIQR = IQR/1.349.

2.12 Modified Classical Intervals (MDF)

The (1-α)100% confidence interval for the Cp measures based on S_M is given by

$$\text{LCL} = C_p^M\sqrt{\frac{\chi^2_{\alpha/2,(n-1)}}{n-1}} \text{ and}$$

$$\text{UCL} = C_p^M\sqrt{\frac{\chi^2_{1-\alpha/2,(n-1)}}{n-1}} \quad (2.17)$$

where $C_p^M = \frac{USL-LSL}{6S_M}$ and S_M is defined as follows

$$S_M = \sqrt{\frac{\sum_{i=1}^{n}(X_i - Md)^2}{n-1}} \quad (2.18)$$

where Md is the median of the observations of X_1, $X_2, \ldots, X_n$.

2.13 Modified interval based on adjusted degrees of freedom (MDF)

The (1-α)100% confidence interval for the Cp measures based on S_M is given by

$$\text{LCL} = C_p^M\sqrt{\frac{\chi^2_{\alpha/2,(r-1)}}{\hat{r}}} \text{ and}$$

$$\text{UCL} = C_p^M\sqrt{\frac{\chi^2_{1-\alpha/2,(r-1)}}{\hat{r}}}, \quad (2.19)$$

where $C_p^M = \frac{USL-LSL}{6S_M}$ (see Kibria and Chen, 2021).

2.14 Modified interval based on the large sample theory (MLS)

The (1-α)100% confidence interval for the Cp measures based on S_M is given by

$$\text{LCL} = \frac{C_p^M}{\sqrt{\exp\left(Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)}} \text{ and}$$

$$\text{UCL} = \frac{C_p^M}{\sqrt{\exp\left(-Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)}} \quad (2.20)$$

where $C_p^M = \frac{USL-LSL}{6S_M}$. and $Z_{1-\frac{\alpha}{2}}$ is the $(1-\frac{\alpha}{2})$ percentile of a standard normal distribution (see Kibria and Chen, 2021).

2.15 Modified interval based on the large sample theory (MLS)

The (1-α)100% confidence interval for the Cp measures based on S_M is given by

$$\text{LCL} = \frac{C_p^M}{\sqrt{\exp\left(Z_{1-\frac{\alpha}{2}}\sqrt{B}+C\right)}} \text{ and}$$

$$\text{UCL} = \frac{C_p^M}{\sqrt{\exp\left(-Z_{1-\frac{\alpha}{2}}\sqrt{A}\right)+C}} \quad (2.21)$$

where $C_p^M = \frac{USL-LSL}{6S_M}$ (see Kibria and Chen, 2021).

2.16 Bootstrap approach (Boot)

Bootstrap is a frequently used non-parametric approach (Efron (1979)), which involves no assumptions about the primary population and can be applied to a range of situations. The accuracy of the bootstrap statistic relies on the number of bootstrap samples. If the number of bootstrap samples is large enough, the estimate may be precise. A bootstrap method is summarized as follows: Let $X^{(*)} = X_1^{(*)}, X_2^{(*)}, \ldots, X_n^{(*)}$, where the i^{th} sample is denoted $X^{(i)}$ for i=1,2 ,…, *B*, and *B* are the number of bootstrap samples. The number of bootstrap samples is naturally between 1000 and 2000. The (1-α)100% bootstrap version confidence interval for the *Cp* is given by

$$\text{LCL} = cp\sqrt{\frac{S^2\sqrt{2(n-1)}}{2t^*_{\frac{\alpha}{2}}+\sqrt{2(n-1)}}} \text{ and}$$

$$\text{UCL} = cp\sqrt{\frac{S^2\sqrt{2(n-1)}}{2t^*_{1-\frac{\alpha}{2}}+\sqrt{2(n-1)}}} \quad (2.22)$$

where $t^*_{\frac{\alpha}{2}}$ and $t^*_{1-\frac{\alpha}{2}}$ are the (α/2)100th and 1- (α/2)100th percentiles of the following statistic, $T^* = \frac{{s^*}^2 - s^2}{\sqrt{\widehat{var}\left({s^*}^2\right)}}$, where ${s^*}^2$ is a bootstrap replication of the statistic s^2, $\widehat{var}\left({s^*}^2\right) = \frac{1}{n}\left(\hat{\mu}_4^* - \frac{n-3}{n-1}{s^*}^4\right)$ and $\hat{\mu}_4^* = \frac{1}{m}\sum_{i=1}^{m}(X_i^* - \bar{X}^*)^4$. For details, see Panichkitkosolkul (2014).

3. SIMULATION STUDY

3.1. Simulation Design

The main objective of this paper is to find some good confidence interval estimators for estimating the population *Cp*. Since a theoretical comparison among the interval estimators is not possible, a simulation study has been made to compare the performances of the interval estimators under both symmetric and non-symmetric distributions, which are listed below.

- Standard normal distribution, N(50,1) with skewness 0
- Chi-Square distribution with 1 df with skewness 2.828, right skewed
- Chi-Square distribution with 3 df with skewness 1.633, right skewed
- t distribution with 5 df, t(5) with skewness 0
- Beta (4,1) distribution with skewness = - 1.05, left skewed
- Beta (10,1) distribution, with skewness = - 1.52, left skewed

The following LSL and USL were used to computer Cp.

- N(50,1): LSL = 47 and USL = 53
- Chi(1): LSL = -3.2426 and USL = 5.2426
- Chi(3): LSL = -4.3484 and USL = 10.348
- t(5): LSL = -3.8729 and USL = 3.8729
- B(4,1): LSL = 0.32 and USL = 1.28
- B(10,1): LSL = 0.654 and USL = 1.146

MATLAB R2018a programming language is used for all types of calculations. The number of simulation replications was 50000 for each case. Random samples were generated from each of the above-mentioned distributions for sample sizes, *n*=20, 30, 50, 70, 100 and 150 and *B*=5000, bootstrap samples. The most common 95% confidence interval is used to measure the confidence level. Results are tabulated in Tables 3.1 a,b to Tables 3.6 a,b for N(50,1), Chi-square with 1 DF, Chi-square with 3 DF, t with 5 DF, Beta (4,1) and Beta (10,1) distributions respectively.

3.2. Results and Discussions

In Tables 1 and 2, we reported coverage probabilities and average widths for all confidence intervals when data are generated from the N(50,1) distribution. For a better understanding, we presented them in Figures 1 and 2. For the brevity, we will discuss only two extreme cases (n=20 and 150) in this section. From Table 1 and Figure 1, it appear that for small sample size (20), the classical interval, AADM, ALS. Mchi, and MALS have high coverage probabilities compared to other CIs. However, Table 2 and Figure 2 indicate that ALS, and MALS have smaller average width compared to other CIs. We found that the ALS performed the best in terms of coverage probability and average width, followed by MALS, DF, LS, and MDF. For sample size (150), the classical interval, AADM, ALS. Mchi, and MALS have high coverage probabilities compared to the rest of the intervals. However, Table 2 and Figure 2 show that ALS and MALS have smaller average width compared to other CIs. We found that the ALS performed the best in terms of coverage probability and average width, followed by the MALS interval. The classical, AAMD and Mchi performed equivalently well. When data are from a normal population, both ALS and MALS are highly recommended to estimate the population Cp by the confidence interval.

Table 1. Coverage probability when data are generated from the N(50,1) distribution with skewness 0

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.9479	0.9492	0.9505	0.9498	0.9493	0.9499
Sps	0.7668	0.7786	0.7721	0.7719	0.7733	0.8059
AADM	0.9359	0.9355	0.9351	0.9350	0.9328	0.9332
MAD	0.7799	0.7784	0.7714	0.7717	0.7683	0.7679
GMD	0.4174	0.5560	0.7250	0.6547	0.7976	0.8374
Sn	0.8454	0.8551	0.8618	0.8642	0.8617	0.8676
DF	0.8715	0.8784	0.8863	0.8888	0.8617	0.8950
LS	0.8712	0.8769	0.8841	0.8878	0.8883	0.8938
ALS	0.9862	0.9880	0.9890	0.9892	0.9887	0.9900
ST	0.7033	0.7708	0.7985	0.8551	0.8433	0.8687
IQR	0.7890	0.7660	0.7702	0.7703	0.7684	0.7659
Mchi	0.9443	0.9466	0.9489	0.9490	0.9486	0.9489
MDF	0.8657	0.8747	0.8832	0.8866	0.8904	0.8936
MLS	0.8718	0.8774	0.8849	0.8878	0.8883	0.8942
MALS	0.9861	0.9886	0.9891	0.9894	0.9889	0.9898
Boot	0.8167	0.7953	0.9151	0.9503	0.8339	0.8324

Notes: C - Classical approach, Sps - Sps approach, AADM - AADM approach, MAD - MAD approach, GMD - GMD approach, Sn – Sn approach, DF - DF approach, LS - LS approach, ALS – ALS approach, ST – ST approach, IQR – IQR approach, Mchi – Mchi approach, MDF- MDF approach, MLS – MLS approach, MALS – MALS approach and Boot - Bootstrap -approach

Table 2. Average widths when data are generated from the N(50,1) distribution with skewness 0

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.6565	0.5257	0.4008	0.3365	0.2804	0.2280
Sps	0.6797	0.5410	0.4079	0.3404	0.2827	0.2293
AADM	0.6762	0.5361	0.4056	0.3393	0.2821	0.2290
MAD	0.7083	0.5513	0.4125	0.3432	0.2844	0.2301
GMD	0.5746	0.4620	0.3535	0.2972	0.2479	0.2018
Sn	0.6488	0.5195	0.3977	0.3346	0.2793	0.2275
DF	0.5387	0.4336	0.3328	0.2799	0.2793	0.1907
LS	0.5576	0.4421	0.3361	0.2817	0.2349	0.1912
ALS	0.4534	0.2926	0.1711	0.1208	0.0839	0.0556
ST	0.5421	0.4652	0.3496	0.2917	0.2376	0.1962
IQR	0.6238	0.5074	0.3925	0.3320	0.2773	0.2264
Mchi	0.6488	0.5213	0.3987	0.3352	0.2796	0.2276
MDF	0.5325	0.4301	0.3311	0.2788	0.2333	0.1903
MLS	0.5512	0.4385	0.3343	0.2806	0.2338	0.1908
MALS	0.4481	0.2902	0.1702	0.1203	0.0837	0.0555
Boot	0.5378	0.3727	0.3874	0.3634	0.2055	0.1640

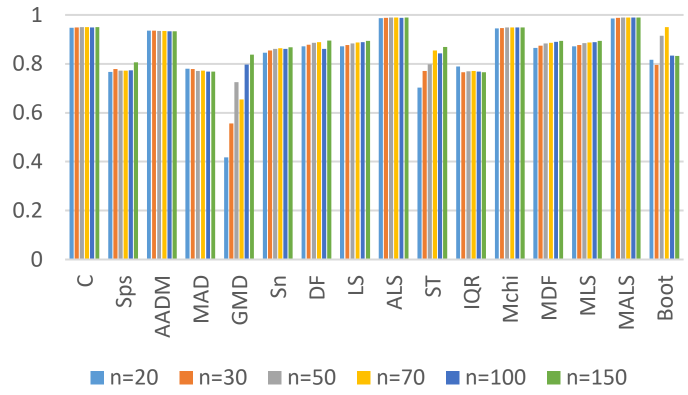

Figure 1. Coverage Probability for all selected tests when data generated from the standard normal distribution N(50,1)

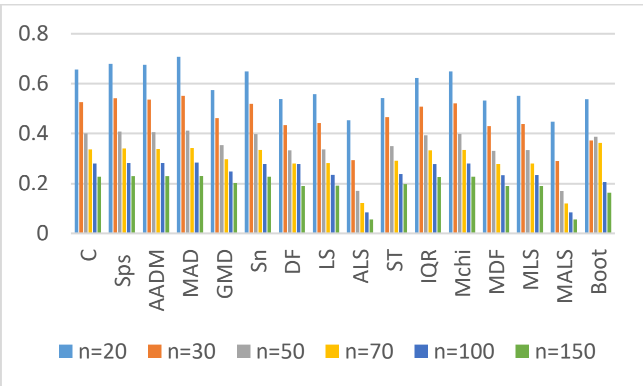

Figure 2. Average width for all selected tests when data generated from the standard normal distribution N(50,1)

In Tables 3 and 4, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from the Chi-square distribution with 1 DF. From Table 3 and Figure 3, it appear that for small sample size (20), DF, ALS, MDF, MLS and MALS have higher coverage probabilities compared to the rest of the intervals. However, Table 4 and Figure 4 indicates that ALS and MALS have smaller average widths compared to other CIs. We found that the ALS performed the best in terms of coverage probability and average width, followed by MALS, MDF and MLS. For large sample size (150), the classical interval, DF, LS, ALS, MDF, MLS, MALS and Boot have high coverage probabilities compared to the rest of the intervals. However, Table 4 and Figure 4 indicate that ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS and Boot intervals. When data are from a highly positive skewed distributions both ALS and MALS are highly recommended to estimate the population Cp by confidence interval.

Table 3. Coverage probability of selected confidence interval when data are generated from the Chi-square distribution with df 1 and skewness 2.828

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.6082	0.5867	0.5742	0.5644	0.5582	0.5564
Sps	0.5808	0.5811	0.5914	0.6029	0.6184	0.6232
AADM	0.6220	0.6357	0.6585	0.6736	0.6904	0.6925
MAD	0.6218	0.6317	0.6413	0.6424	0.6502	0.6665
GMD	0.6519	0.6657	0.6759	0.6803	0.6960	0.6958
Sn	0.5212	0.5543	0.5566	0.5808	0.5900	0.6011
DF	0.7141	0.7434	0.7566	0.7912	0.8112	0.8305
LS	0.6887	0.7215	0.7749	0.7813	0.8034	0.8237
ALS	0.7652	0.7974	0.7608	0.8477	0.8660	0.8818
ST	0.6263	0.6893	0.7010	0.7168	0.7335	0.7564
IQR	0.5404	0.5533	0.5803	0.5849	0.6373	0.6580
Mchi	0.6093	0.5877	0.5932	0.6085	0.6123	0.6386
MDF	0.7273	0.7556	0.7692	0.8006	0.8290	0.8411
MLS	0.7028	0.7388	0.7641	0.7985	0.8132	0.8269
MALS	0.7698	0.7984	0.8261	0.8477	0.8754	0.8980
Boot	0.5344	0.5148	0.7768	0.6963	0.8384	0.9659

Table 4. Average width of selected confidence interval when data are generated from the Chi-square distribution with df 1 and skewness 2.828

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.7816	0..5946	0.4331	0.3566	0.2921	0.2348
Sps	1.1389	0.8916	0.6573	0.5439	0.4480	0.3618
AADM	0.9582	1.3613	0.5492	0.4560	0.3765	0.3044
MAD	1.8159	0.6149	0.9860	0.8086	0.6624	0.5326
GMD	0.7877	1.4567	0.4589	0.3821	0.3161	0.2560
Sn	1.8492	1.1134	1.0724	0.8919	0.7392	0.5997
DF	0.9870	0.8317	1.0724	0.5912	0.5135	0.4347
LS	1.0264	0.8556	0.6761	0.6008	0.5201	0.4388
ALS	0.7793	0.5624	0.6900	0.2877	0.2160	0.1532
ST	0.7364	0.6331	0.3762	0.3794	0.3028	0.2510
IQR	0.5864	0.4694	0.4605	0.3035	0.2532	0.2060
Mchi	0.7286	0.5539	0.3607	0.3321	0.2721	0.2188
MDF	0.9214	0.7762	0.4033	0.5517	0.4793	0.4058
MLS	0.9584	0.7986	0.6310	0.5608	0.4856	0.4096
MALS	0.7273	0.5251	0.3514	0.2688	0.2019	0.1431
Boot	0.7613	0.4965	0.6344	0.4971	0.3966	0.3379

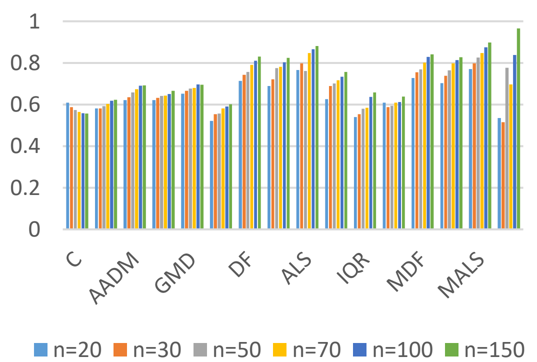

Figure 3. Coverage Probability for all selected tests when data are generated from the Chi-square distribution with 1 df.

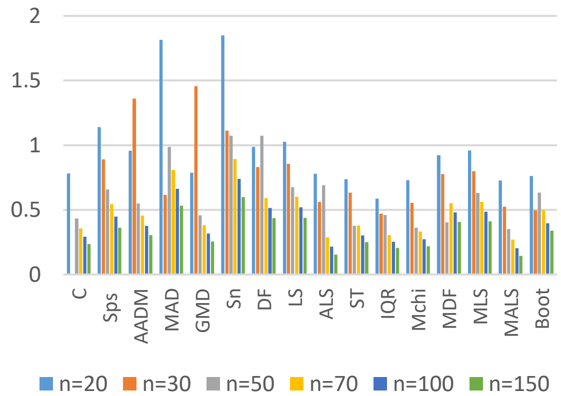

Figure 4. Average width for all selected tests when data are generated from the Chi-square distribution with 1 df.

In Tables 5 and 6, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from the Chi-square distribution with 3 DF. From Table 5 and Figure 5, it appear that for small sample size (20), AAMD, GMD, ALS, and MALS have higher coverage probabilities compared to the rest of the intervals. However, Table 6 and Figure 6 indicate that ALS and MALS have smaller average widths compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, GMD and AAMD intervals. For large sample size (150), the Sps, AAMD, ALS, MLS, MALS and Boot intervals have high coverage probabilities compared to the rest of the intervals. However, Table 6 and Figure 6 indicate that ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Boot, AAMd and Sps intervals. When data are from a moderate positive skewed distributions both ALS and MALS are highly recommended to estimate the population Cp by confidence interval.

Table 5. Coverage probability of selected confidence interval when data are generated from the Chi-square distribution with df 3 and skewness 1.6333

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.7521	0.7803	0.7977	0.8003	0.8197	0.8205
Sps	0.7017	0.7525	0.8103	0.8652	0.8711	0.8909
AADM	0.8057	0.8119	0.8479	0.8687	0.8704	0.8960
MAD	0.6044	0.6496	0.6596	0.6756	0.6876	0.7261
GMD	0.8165	0.8185	0.8195	0.8364	0.8476	0.8691
Sn	0.6741	0.6929	0.7316	0.7508	0.7836	0.8381
DF	0.7670	0.7901	0.8169	0.8294	0.8459	0.8539
LS	0.7577	0.7804	0.8117	0.8233	0.8414	0.8514
ALS	0.8963	0.9108	0.9239	0.9294	0.9366	0.9411
ST	0.7326	0.7774	0.7975	0.8040	0.8438	0.8505
IQR	0.6285	0.6515	0.6657	0.6706	0.6700	0.6812
Mchi	0.7727	0.7921	0.8042	0.8119	0.8268	0.8618
MDF	0.7705	0.8017	0.8263	0.8480	0.8503	0.8686
MLS	0.7580	0.7983	0.8255	0.8386	0.8636	0.8907
MALS	0.8986	0.9222	0.9351	0.9414	0.9470	0.9482
Boot	0.6027	0.7246	0.8596	0.8529	0.8633	0.9490

Table 6. Average width of selected confidence interval when data are generated from the Chi-square distribution with df 3 and skewness 1.6333

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.6970	0.5482	0.4112	0.3432	0.2842	0.2302
Sps	0.7895	0.6255	0.4682	0.3911	0.3235	0.2623
AADM	0.7585	0.5970	0.4488	0.3751	0.3109	0.2522
MAD	0.8902	0.6895	0.5114	0.4259	0.3516	0.2845
GMD	0.6403	0.5103	0.3873	0.3251	0.2703	0.2198
Sn	0.8386	0.6644	0.5019	0.4208	0.3494	0.2837
DF	0.7048	0.5951	0.4818	0.4198	0.3605	0.3023
LS	0.7282	0.6063	0.4868	0.4228	0.3623	0.3033
ALS	0.5747	0.3969	0.2506	0.1852	0.1333	0.0915
ST	0.5995	0.5140	0.3826	0.3186	0.2577	0.2131
IQR	1.3479	1.0949	0.8465	0.7128	0.5966	0.4865
Mchi	0.6739	0.5303	0.3979	0.3321	0.2751	0.2228
MDF	0.6810	0.5754	0.4662	0.4062	0.3489	0.2927
MLS	0.7037	0.5863	0.4710	0.4091	0.3506	0.2936
MALS	0.5555	0.3838	0.2425	0.1792	0.1291	0.0885
Boot	0.2509	0.2442	0.2403	0.1514	0.1273	0.1681

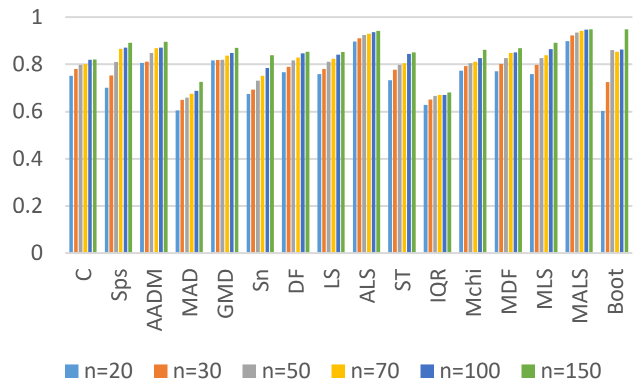

Figure 5. Coverage Probability for all selected tests when data are generated from the Chi-square distribution with 3 df.

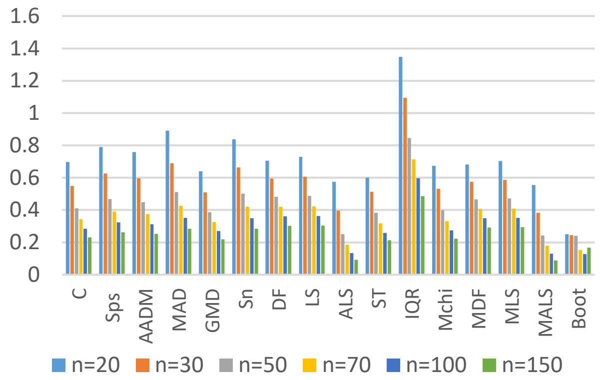

Figure 6. Average width for all selected tests when data are generated from the Chi-square distribution with 3 df.

In Table 7 and Table 8, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from t-distribution with 5 DF. From Table 7 and Figure 7, it appears that for small sample size (20), LS, ALS, MDF, MLS and MALS have higher coverage probabilities compared to the rest of the intervals. However, Table 8 and Figure 8 indicates that among these five intervals, ALS and MALS have smaller average widths. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, MLS, MDF and LS. For large sample size (150), the DF, LS, ALS, MDF, MLS, MALS and Boot intervals have high coverage probabilities compared to the rest of the intervals. However, Table 4.3b and Figure 4.3b indicates that among these 7 intervals, ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Boot, MLS, LS, and DF intervals. When data are from a symmetric distribution both ALS and MALS are highly recommended to estimate the population Cp by a confidence interval.

Table 7. Coverage probability of selected confidence intervals when data are generated from the t distribution with df 5 and skewness 0

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.4595	0.4919	0.5262	0.5600	0.6190	0.6190
Sps	0.5011	0.5111	0.5455	0.5744	0.6062	0.6762
AADM	0.5115	0.5575	0.5736	0.5975	0.6189	0.6989
MAD	0.4911	0.5100	0.5401	0.5932	0.6103	0.6903
GMD	0.6637	0.6900	0.7062	0.7294	0.7562	0.7562
Sn	0.5025	0.5236	0.5795	0.6080	0.6787	0.7387
DF	0.6961	0.6993	0.7090	0.7267	0.7841	0.8041
LS	0.7046	0.7148	0.7363	0.7603	0.7870	0.8070
ALS	0.7634	0.7660	0.7695	0.7767	0.7859	0.8259
ST	0.5507	0.5779	0.5897	0.6020	0.6524	0.7524
IQR	0.4672	0.5559	0.6138	0.6600	0.7018	0.7018
Mchi	0.4629	0.4949	0.5306	0.5669	0.6264	0.6264
MDF	0.6984	0.7049	0.7337	0.7902	0.8296	0.8296
MLS	0.7382	0.7391	0.7412	0.7859	0.8046	0.8546
MALS	0.7663	0.7693	0.7741	0.7812	0.7910	0.8910
Boot	0.6353	0.7392	0.7754	0.8144	0.8512	0.9512

Table 8. Average width of selected confidence interval when data are generated from the t distribution with df 5 and skewness 0

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.7767	0.6049	0.4481	0.3708	0.3037	0.2436
Sps	0.7199	0.7178	0.5193	0.4184	0.3004	0.2403
AADM	0.7705	0.6849	0.5147	0.4098	0.2557	0.2382
MAD	1.0658	0.8339	0.6261	0.5225	0.4329	0.3508
GMD	0.7127	0.5652	0.4273	0.3577	0.2964	0.2403
Sn	0.9416	0.7536	0.5761	0.4847	0.4039	0.3288
DF	0.8368	0.7139	0.5957	0.5328	0.4749	0.4182
LS	0.8640	0.7300	0.6061	0.5411	0.4818	0.4237
ALS	0.6717	0.4756	0.3204	0.2514	0.1971	0.1506
ST	0.7354	0.6476	0.4852	0.4050	0.3267	0.2718
IQR	0.7289	0.5867	0.4500	0.3890	0.2963	0.2277
Mchi	0.7674	0.5998	0.4457	0.3693	0.3029	0.2432
MDF	0.8267	0.7078	0.5925	0.5307	0.4735	0.4174
MLS	0.8534	0.7237	0.6028	0.5390	0.4805	0.4229
MALS	0.6636	0.4715	0.3187	0.2504	0.1965	0.1503
Boot	0.6241	0.5813	0.2558	0.3768	0.5731	0.2451

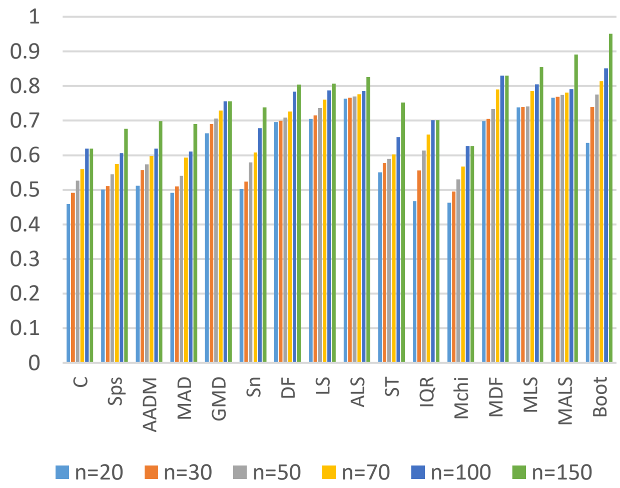

Figure 7. Coverage Probability for all selected tests when data are generated from the t distribution with 5 df.

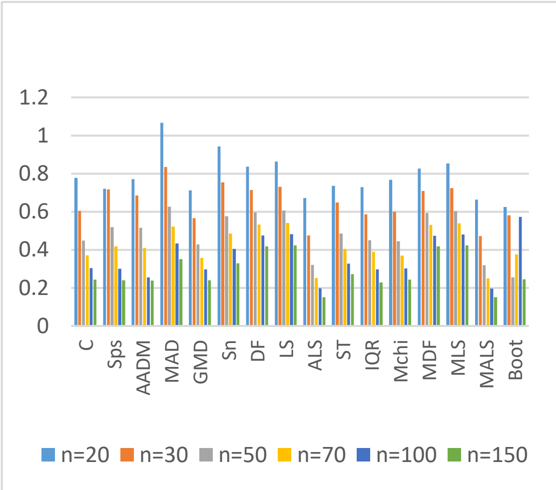

Figure 8. Average width for all selected tests when data are generated from the t distribution with 5 df.

In Table 9 and Table 10, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from a Beta (4,1) distribution with skewness -1.05. From Table 9 and Figure 8, it appears that for small sample size (20), Classical, AAMD, ALS, Mchi and MALS have higher coverage probabilities compared to the rest of the intervals. However, Table 10 and Figure 9 indicates that among these five intervals, ALS and MALS have smaller average widths. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Classical, Mchi and AADM. For large sample size (150), the Classical, AAMD, ALS, Mchi, MALS and Boot intervals have high coverage probabilities compared to the rest of the intervals. However, Table 4.3b and Figure 4.3b indicates that among these six intervals, ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Boot, Classical, Mchi and AADM intervals. When data are from a left symmetric distribution both ALS and MALS are highly recommended to estimate the population Cp by a confidence interval. In Tables 9 and 10, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from a Beta (4,1) distribution with skewness -1.05. From Table 9 and Figure 9, it appear that for small sample size (20), Classical, AAMD, ALS, Mchi and MALS have higher coverage probabilities compared to the rest of the intervals. However, Table 10 and Figure 10 indicate that among these five intervals, ALS and MALS have smaller average widths. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Classical, Mchi and AADM. For large sample size (150), the Classical, AAMD, ALS, Mchi, MALS and Boot intervals have high coverage probabilities compared to the rest of the intervals. However, Table 10 and Figure 10 indicate that among these six intervals, ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Boot, Classical, Mchi and AADM intervals. When data are from a left symmetric distribution both ALS and MALS are highly recommended to estimate the population Cp by a confidence interval.

Table 9. Coverage probability of selected confidence interval when data are generated from the Beta (4,1) distribution with skewness - 1.05

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.9543	0.9535	0.9532	0.9541	0.9535	0.9504
Sps	0.6422	0.6876	0.7024	0.7215	0.7438	0.7727
AADM	0.9182	0.9278	0.9298	0.9329	0.9365	0.9380
MAD	0.7769	0.7699	0.7819	0.7920	0.8285	0.8413
GMD	0.5149	0.5316	0.6309	0.6826	0.7719	0.8304
Sn	0.8301	0.8363	0.8372	0.8374	0.8396	0.8409
DF	0.8564	0.8635	0.8743	0.8785	0.8838	0.8819
LS	0.8671	0.8690	0.8797	0.8834	0.8880	0.8864
ALS	0.9874	0.9880	0.9895	0.9900	0.9915	0.9905
ST	0.5980	0.6140	0.6446	0.6942	0.7038	0.7193
IQR	0.3549	0.5116	0.6222	0.7021	0.7910	0.8299
Mchi	0.9376	0.9377	0.9374	0.9367	0.9337	0.9466
MDF	0.8352	0.8431	0.8511	0.8538	0.8547	0.8486
MLS	0.8546	0.8562	0.8618	0.8645	0.8642	0.8880
MALS	0.9855	0.9857	0.9856	0.9858	0.9867	0.9837
Boot	0.7801	0.8032	0.8595	0.8791	0.8202	0.9044

Table 10. Average width of selected confidence interval when data are generated from the Beta (4,1) distribution with skewness -1.05

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.6483	0.5201	0.3965	0.3330	0.2774	0.2255
Sps	0.6405	0.5089	0.3828	0.3190	0.2645	0.2143
AADM	0.6613	0.5249	0.3969	0.3319	0.2758	0.2237
MAD	0.6806	0.5289	0.3942	0.3271	0.2709	0.2188
GMD	0.5685	0.4581	0.3505	0.2947	0.2458	0.2000
Sn	0.6388	0.5114	0.3908	0.3284	0.2741	0.2230
DF	0.5152	0.4161	0.3197	0.2691	0.2247	0.1829
LS	0.5355	0.4253	0.3232	0.2710	0.2258	0.1835
ALS	0.4397	0.2843	0.1660	0.1171	0.0813	0.0537
ST	0.5217	0.4451	0.3342	0.2788	0.2273	0.1872
IQR	0.1048	0.0853	0.0662	0.0560	0.0469	0.0383
Mchi	0.6365	0.5121	0.3914	0.3291	0.2743	0.2232
MDF	0.5058	0.4097	0.3155	0.2659	0.2222	0.1810
MLS	0.5257	0.4187	0.3190	0.2678	0.2233	0.1816
MALS	0.4317	0.2799	0.1639	0.1157	0.0804	0.0532
Boot	0.7395	0.6273	0.6606	0.6472	0.1601	0.1394

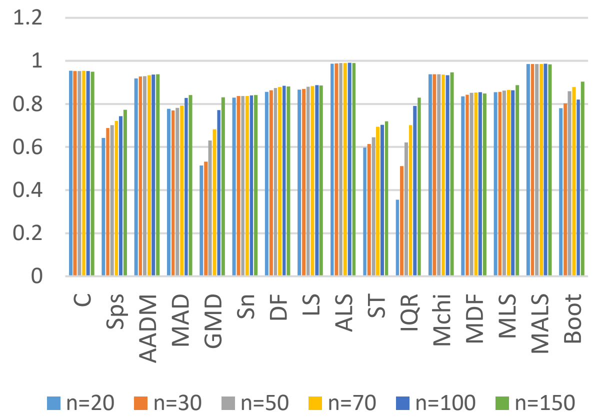

Figure 9. Coverage Probability when data are generated from the B(4,1) distribution

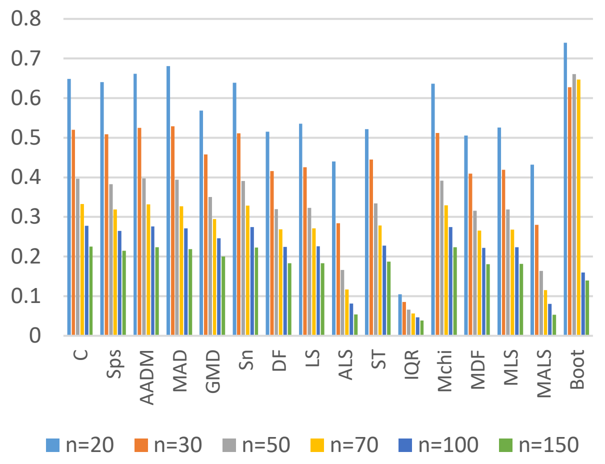

Figure 10. Average width when data are generated from the B(4,1) distribution

In Tables 11 and 12, we have reported the coverage probabilities and average widths for all confidence intervals when data are generated from a Beta (10,1) distribution with skewness -1.53. From Table 11 and Figure 11, it appear that for small sample size (20), ALS, Mchi, MALS and Boot have higher coverage probabilities compared to the rest of the intervals. However, Table 12 and Figure 12 indicate that among these three intervals, ALS and MALS have smaller average widths. For large sample size (150), the Classical, AAMD, ALS, ST, Mchi, MLS, MALS and Boot intervals have high coverage probabilities compared to the rest of the intervals. However, Table 12 and Figure 12 indicate that among these eight intervals, ALS and MALS have smaller average width compared to other CIs. We found that the MALS performed the best in terms of coverage probability and average width, followed by ALS, Boot, MLS, Mchi, AADM and Classical intervals. When data are from a highly left symmetric distributions both ALS and MALS are highly recommended to estimate the population Cp by confidence interval.

Table 11. Coverage probability of selected confidence interval when data are generated from the Beta (10, 1) distribution with skewness -1.53

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.4124	0.5657	0.6693	0.7432	0.8244	0.8609
Sps	0.2405	0.3548	0.4428	0.5199	0.6069	0.6637
AADM	0.4679	0.5286	0.6410	0.7266	0.8211	0.8627
MAD	0.3094	0.4282	0.5112	0.5808	0.6622	0.7053
GMD	0.3330	0.3654	0.4589	0.4805	0.6699	0.7244
Sn	0.3742	0.4882	0.5627	0.6197	0.6867	0.7147
DF	0.2870	0.4267	0.5329	0.6083	0.6989	0.7350
LS	0.3161	0.4679	0.5779	0.6600	0.7529	0.7939
ALS	0.6766	0.8102	0.8804	0.9230	0.9594	0.9742
ST	0.5623	0.6131	0.6805	0.7244	0.7915	0.8307
IQR	0.3594	0.4397	0.5376	0.5998	0.6829	0.7287
Mchi	0.4498	0.5735	0.6713	0.7907	0.8402	0.8816
MDF	0.2980	0.4735	0.5809	0.6583	0.7027	0.7938
MLS	0.3644	0.4810	0.5944	0.6894	0.7610	0.8382
MALS	0.6860	0.8546	0.9072	0.9208	0.9473	0.9578
Boot	0.7583	0.7655	0.7872	0.8530	0.7958	0.8876

Table 12. Average width of selected confidence interval when data are generated from the Beta (5,2) distribution with skewness -1.53

			Sample sizes			
CIs	**n=20**	**n=30**	**n=50**	**n=70**	**n=100**	**n=150**
C	0.5799	0.4652	0.3547	0.2979	0.2481	0.2018
Sps	0.5730	0.4552	0.3425	0.2853	0.2367	0.1917
AADM	0.5916	0.4696	0.3550	0.2969	0.2467	0.2001
MAD	0.5053	0.4732	0.3526	0.2926	0.2423	0.1958
GMD	0.5086	0.4098	0.3136	0.2636	0.2199	0.1789
Sn	0.5714	0.4575	0.3496	0.2938	0.2452	0.1995
DF	0.4609	0.3723	0.2860	0.2407	0.2810	0.1636
LS	0.4790	0.3805	0.2892	0.2425	0.2020	0.1641
ALS	0.3934	0.2543	0.1485	0.1048	0.0727	0.0481
ST	0.4667	0.3982	0.2990	0.2494	0.2033	0.1675
IQR	0.1048	0.0853	0.0662	0.0560	0.0469	0.0383
Mchi	0.5694	0.4581	0.3501	0.2944	0.2454	0.1997
MDF	0.4525	0.3665	0.2822	0.2378	0.1988	0.1619
MLS	0.4703	0.3746	0.2854	0.2396	0.1997	0.1624
MALS	0.3862	0.2504	0.1466	0.1035	0.0719	0.0476
Boot	0.4507	0.3503	0.4856	0.4735	0.3878	0.0551

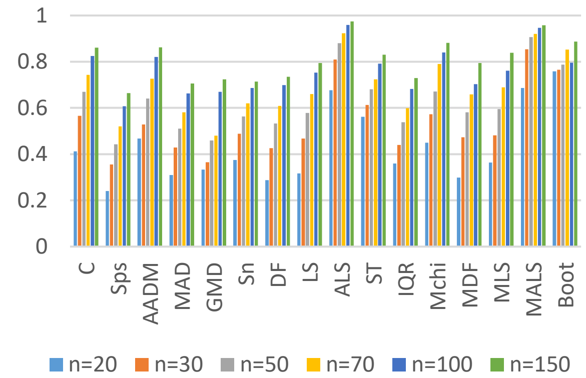

Figure 11. Coverage Probability for all selected tests when data generated from the B(5,2) distribution

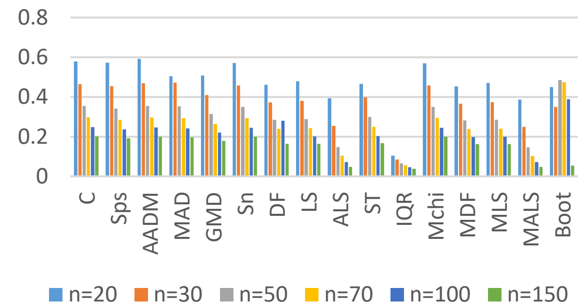

Figure 12. Average width for all selected tests when data generated from the B(5,2) distribution

Overall, ALS and MALS performed better in all simulation conditions and highly recommended for the practitioners.

4. SOME CONCLUDING REMARKS

This paper considers sixteen available different confidence intervals for estimating the population process capability index, Cp. We compared their performances under the same simulation condition but with different kinds of distributions, such as symmetric, right and left-skewed distributions. Both coverage probability and average width were considered as performance criteria. Our simulation study indicates that both ALS and MALS confidence intervals outperformed in all simulation conditions in the sense of high coverage probability and average width and can be recommended for practitioners. We sincerely believe that this paper will contribute to process capability literature, and it will be helpful to choose an interval estimate formula when researchers are interested in estimating the population process capability index. The findings of this paper are restricted to the simulation conditions of this paper. However, for a definite statement about the interval estimators, one may need more analysis.

Acknowledgements: Authors are thankful to reviewers and Editor for their comments and suggestions towards improving the presentation of the paper.

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