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TASK DESIGN FOR TEACHING CARDIOID CURVE WITH DYNAMIC GEOMETRY SOFTWARE AND EDUCATIONAL ROBOTICS IN UNIVERSITY PRACTICE

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Abstract

Considering Gen Z's learning needs, primarily focusing on bachelor IT students, a STEAM-based methodology was developed and tested for teaching and learning the principal properties of the cardioid curve. The four-component methodology is based on visuality and combines frontal teaching techniques with educational robotics, dynamic geometry software and project-based learning elements. Interactive learning materials were set up to support independent learning, including tasks which follow the guidelines of recent research on task design. Moreover, a cardioid drawing LEGO robot model was designed and built to visualise the generation process of the curve and to provide first-hand experiences for students. The participants involved in testing the methodology had to solve two homework problems using dynamic geometry software. An assessment system was set up to evaluate students' solutions. After a statistical analysis of the data obtained, it could be concluded that the four-component methodology is an effective didactic technique to facilitate the learning of the cardioid curve.
Keywords: *cardioid curve, dynamic geometry software, educational robotics, project-based learning*

Introduction

Visualisation and representation of mathematical laws are essential in realising mathematics teaching. Many famous mathematicians explicitly advocate visualisation because it is a powerful tool for exploring mathematical problems and interpreting mathematical concepts and their relationships (Rösken & Rolka, 2006). Numerous studies have shown that visual representation plays an essential role in learning and teaching mathematics because it reduces the complexity of the task. The researchers point out the importance of visual thinking for the learning process (Arcavi, 2003; Raiyn, 2016; Sedig & Sumner, 2006; Tiwari et al., 2021) and present convincing experiments and arguments for the central role of visualisation in the teaching process of mathematics at all levels of the education from elementary school to university (Bråting & Peljare, 2008; Lukáč & Gavala, 2019; Presmeg, 1986; Zorzos & Avgerinos, 2023). Visualisations and sketches are an integral part of teaching in many areas of mathematics. Visual techniques that rely on non-procedurally safe routines are more cognitively demanding than analytical methods (Arcavi, 2003). An example is the theory of plane curves, where the derivation of parametric equations for different types of curves is usually a complex, multi-step process, for which the preparation of an illustrative figure during a frontal teaching activity is a lengthy procedure requiring a lot of attention and concentration relying heavily on the student's prior knowledge. For this reason, static tools alone - formulas written on a blackboard and schematic diagrams - are not a good way of explaining this subject

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because it is difficult to visualise the curves correctly in lectures. With few exceptions, accurate representation of curves is rarely used in higher education.

The cardioid curve has fascinated mathematicians for centuries because of its properties, its graph's beauty, and practical applications (Pickover, 2012). The cardioid as an epicycloid was studied by Jacob Ozaniaill in 1691, and afterwards, it appeared in the works of many famous mathematicians (Archibald, 1903; Stillwell, 2010). Because of its practical importance, the cardioid curve's properties and possible equations are worth discussing in detail in the Calculus II course for computer science students. The tight curriculum leaves little opportunity to experiment with new methodological techniques. However, the changing generational needs should be considered if the aim is for students to complete their foundation semesters with solid mathematical knowledge they can use in the future. Project-based learning (PBL) allows students to work on mathematical tasks independently or in a team, creatively implementing their ideas. Zhou's (2023) summary results have shown that PBL positively affects students compared to traditional education. Also, it can improve students' active thinking, hands-on, and teamwork cooperative abilities. Successful project work requires the right basic skills and motivation. There is a significant relationship between PBL and collaborative learning, disciplinary subject learning, iterative learning, and authentic learning. The experience results collected by Almulla (2020) have shown that PBL techniques improve student engagement by enabling the sharing and discussing of knowledge and information.

Using new technologies, mathematical representations and visualisation are closely related to research issues in mathematics education. In the research of Ibili (2019), the literature on DGS (dynamic geometry software) was reviewed, and it was stated that DGS provides an effective pedagogical environment, allowing for interaction with digital materials and for geometric objects to be seen from all sides for users. In addition, interacting with geometric materials through DGS helps students understand geometric concepts more efficiently and explore the relationships between geometric concepts (Dogruer & Akyuz, 2020; Jones, 2002; Owens, 2014). Students belonging to Generation Z respond well to visual and interactive learning content, such as videos, infographics, simulations, and interactive quizzes. They value hands-on experiences and real-world applications of their learning (Seemiller & Grace, 2016). Computer science students enjoy working with dynamic geometry software and creating animations is a powerful motivator for them (Körei & Szilágyi, 2022).

Task Design

Task design constitutes a growing field of research in mathematics education. In particular, task design in dynamic and interactive mathematics learning environments has become very popular in the last decade. Cevikbas and Kaiser (2021) have systematically analysed the state-of-the-art research on mathematical task design in dynamic and interactive environments. The results show that most tasks were created in geometry, followed by algebra and calculus. It was further found that different frameworks were used in the task design process and that the authors of the studies placed a strong emphasis on developing and testing design principles. The objectives to be achieved in task design included the implementation of problem-based learning. The study also showed that task design research contributed to the literature by extending existing frameworks. However, in addition to the positive results, challenges related to task design in dynamic and interactive environments were also reported by the study's authors, such as negative attitudes of students, poor mathematical background of participants, problems with assessment, and the time-consuming nature of task design. Overall, the results indicate that further studies on the design of dynamic and interactive tasks are needed.

According to Olive (2013), educators need to understand how Generation Z young people use the modern technological environment to consider the generation's needs in the teaching process. Mathematics educators need to understand how to use this technological environment to improve the teaching and learning of mathematics - both in and out of school. In addition, due to differences in students' learning styles, students need to use not only one representation to create their learning environment (Özdemir & Ayvaz Reis, 2013). The study of Ondes (2021) provided a comprehensive overview of current research on DGS, analysing 210 articles between 2005 and 2021. The researchers agree that carefully designed practices are needed to exploit the potential of dynamic and interactive software. The creation of these exercises and the evaluation of existing exercises also pose difficulties for teachers. Researchers have proposed models and principles for designing tasks to address this problem that take advantage of DGSs, focusing on discovery learning that can lead to assumptions, explanations, and proofs (Fahlgren & Brunström, 2014; Leung, 2011). Researchers also agree that the quality and quantity of instructions attached to tasks significantly impact the successful completion of tasks. Trocki and Hollenbrabds (2018) have compiled a general framework that serves as a guide for defining the quality of DGS tasks and writing them. They describe three levels (low, medium, and high) for defining task quality. High-quality tasks are instructions that coordinate mathematical knowledge and technological operations. Moreover, the learner must draw general conclusions based on knowledge and context beyond a static task representation.

The Desmos graphing calculator is a user-friendly and easy-to-use tool DGS. The findings from the papers of Chorney (2022), Körei and Szilágyi (2022) and Chechan et al. (2023) also support that using Desmos as a technological tool for learning mathematics improves students' performance and helps them develop conceptual understanding through visualisation.

Educational Robotics in Teaching of Mathematics

In recent decades, education has transformed and transitioned beyond traditional learning methods. It is now enriched with technological procedures, mainly Information Communication Technology (ICT) related tools. Many researchers have studied the convergence of ICT in education while highlighting the growing and successful incorporation between of ICT applications and teaching (Sophokleous et al., 2021). Benitti (2021) reviewed recently published scientific literature on the use of robotics in schools to identify the potential contribution of the incorporation of robotics as an educational tool in schools, present a synthesis of the available empirical evidence on the educational effectiveness of robotics and define future research perspectives. The results suggested that educational robotics usually acts as an element that enhances learning. Several other authors also report the motivational effects of educational robotics (Francis et al., 2018; Ruiz et al., 2019).

Nowadays, STEAM (Science, Technology, Engineering, Art, Mathematics) education has been emphasised at all education levels, and using robotics has played an essential role in STEAM learning design. Darmawansah et al. (2023) systematically reviewed publications on robotics-based STEAM (R-STEAM) education by existing research in this area. This review examined the role of robotics and research trends in STEAM education. A total of 39 papers published between 2012 and 2021 were analysed. The review indicated that LEGO was the most used tool to accomplish the learning objectives, and PBL was the most frequently employed learning method. In addition, STEAM learning and transferable skills were the most popular educational goals when applying robotics.

STEAM-based and R-STEAM-based teaching methods have been proven effective in mathematics education, including higher education (Nerantzi & James, 2019; Sánchez et al., 2019). Zhong and Xia (2020) reviewed 20 empirical studies on how to teach and learn mathematical knowledge through robotics. Their results indicate that most studies used LEGO robots; the robots were primarily applied to teach and learn graphics, geometry, and algebra. The 20 papers suggest that robotics plays an active role in mathematics education. There are only a few sources in the literature where educational robotics were used to support teaching Calculus in the university practice. For example, the article of Zalvidar-Colado et al. (2017) presents a methodological proposal to enhance the learning environment through ludic practices using new technologies with holistic learning, using LEGO Mindstorms EV3 robot set for teaching functions. Another example is when a robot built from LEGO Education SPIKE Prime set is used to draw an ellipse and then analyse the characteristics of the curve drawn (Petrovič, 2022).

In summary, LEGO robotic sets are excellent tools for constructivist and constructionist learning tools and PBL (Petrovič, 2022; Zhong & Xia, 2020). The primary goal of R-STEAM education is to let the students collect actual practical experience, which is very important for the learning process to establish analogies and link the theoretical material to the personal practical experiences of students (Petrovič, 2022).

Research Problem

At the university level, mathematics is becoming both abstract and complex. The curriculum relies heavily on students' prior knowledge. The frontal teaching technique in university practice follows the traditional hierarchical model. This structure is logical and consistent from the point of view of instructors who are already familiar with mathematics and know the complete structure (Merenluoto & Lehtinen, 2002), but challenging to follow for first-year students entering a new educational environment. Students must learn mathematics with understanding, actively building new knowledge and connecting them to existing knowledge. When organising the teaching process, it is worthwhile to choose a methodological approach that allows for the construction and strengthening of conceptual knowledge in addition to procedural knowledge. According to Rittle-Johnson et al. (2001), the two types of knowledge (conceptual and procedural) in mathematics develop iteratively, and they are in a permanently renewing relationship: if one type of knowledge increases, then the other type of knowledge increases, too.

There are many good practices related to parametric curves, where the methodology used supports the development of procedural knowledge through different types of visualisation techniques. However, no methodological model was found on the topic of cardioid, where much is done to include components that focus on both types of knowledge in a consistent way. The need for this research was motivated by the fact that cardioid is one of the most important two-dimensional plane geometric shapes which is widely used in practice; however, alternative methods for teaching it in higher education were not available to provide practical experience.

This study addressed the question of how the cardioid curve can be taught using a new methodology that emphasises the practice-oriented parts of the learning process and supports the iterative development of procedural and conceptual knowledge. In addition to developing an innovative methodology, the research involved live testing with first-year computer science students.

Research Focus

The focus of the study was the teaching of the cardioid. After thoroughly mapping and studying the literature and considering the available information, a new learning structure was created to acquire knowledge related to the cardioid curve. The new methodological model, the four-component methodology, considers the needs of Generation Z while making optimal use of the available academic infrastructure. Following the development of this methodology, the ICT- and R-STEAM-based didactic framework was tested in practice at the University of Miskolc in Hungary. This experiment focused on the positive effects of the implementation of the four-component methodology among the participants.

Theoretical Framework: The Four-Component Methodological Model

Frontal lectures and practical lessons have a long history in university education. This classical way of teaching has come under attack regarding the quality of knowledge transfer over the last decade (Bosio & Origo, 2019; Hashim, 2018; Schwerdt & Wuppermann, 2011). However, today, there is no better alternative for extensive courses with many members. From the point of view of student success, one does not necessarily have to give it up if placed in a complex unit and accompanied by teaching-learning techniques that promote the success of frontal teaching. Frontal teaching is the cornerstone of the new methodology. How modern didactic techniques can complement this traditional form of teaching is described below.

I) The first component of the methodology is the classical classroom lecture and practical lessons on the cardioid curve.

II) The practical application of the theory is supported by interactive online learning material, with exercises requiring dynamic geometry software. The online tasks can be solved independently or in groups so that students can learn at their own pace and according to their needs. An essential feature of the online course material is that it includes a possible solution for every problem using the Desmos graphing calculator. It is up to the student to decide at which stage of the learning process to look at the solution.

III) As a third element, educational robotics was chosen to provide reinforcement and positive feedback to build on the advantages of R-STEAM pedagogy. A LEGO SPIKE Prime robot draws a cardioid curve, engages students' attention and allows for active knowledge transfer. Explaining and understanding how the robot works is essential; therefore, efforts have been made to make it easy for students to understand the theoretical background. Working with robots can be implemented in one of the following alternatives:

i) Using a step-by-step guide, students can build and test the cardioid drawing robot in groups of 2-3.

ii) If there are few robots available, larger groups are formed, and each group works with a pre-built robot. Although the construction phase of R-STEAM-based learning does not take place, the participants gain valuable first-hand experience by being able to study and test the robot model.

iii) If there is no chance for hands-on activities (which are essential in R-STEAM-based learning), educational robots can be used as a model for observation. In this case, the robotic demonstration can be recorded and integrated into the interactive learning material.

In the methodological experiment, the drawing robot was used in a 90-minute seminar in groups of 5-7 people according to option ii).

IV) Implementing project-based learning (PBL) is the final step of the methodology. For this reason, it is advisable to create DGS project tasks that are interesting and motivating for the participants and adapted to Gen Z students' needs. These projects allow students to work cooperatively. However, the solutions should also have unique features, given that the ultimate goal is the development of individuals and the complete mastery of the course material by all participants. In the experiment, two online homework assignments were set.

Creating the components went hand in hand with fine-tuning the model. The development method for each component was the widely known and used PDSA (Plan - Do - Study - Act) cycle (Deming, 2018). This framework ensured that errors were eliminated because at least three complete cycles were performed for each component. The use of PDSA cycles has given a structured framework for the development of the new teaching methodology model. The changes were tested in small steps, and used the lessons learned from the testing cycles were used to optimise the four-component methodology model before it was rolled out to students.

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The complete scheme of the four-component methodology can be seen in Figure 1, and Table 1 shows the in-class and post-class activities of the methodology. As shown in Figure 1, the second and third components can be implemented in parallel.

Figure 1
The Structure of the Four-Component Methodology

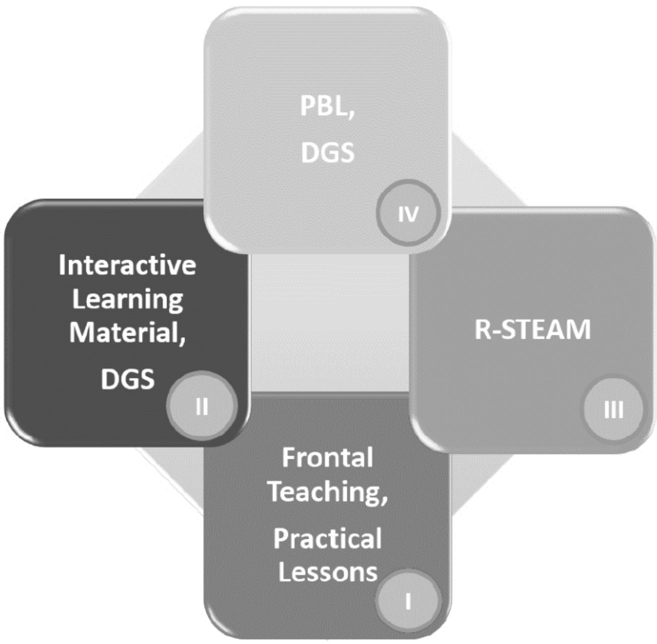

Table 1
Timeline of the Weekly Learning Activities on the Topic of the Cardioid During the Semester

Week	Week 1 - 4	Week 5	Week 6 - 7	Week 8
In-class activity	Frontal teaching lectures and practical lessons	R-STEAM seminar		Project presentation
Post-class activity	Getting started with Desmos		Interactive learning material with ten tasks Online homework projects	

Presenting mathematical concepts using concrete models can help in the abstraction of concepts; therefore, an educational robot was used for drawing the cardioid curve. R-STEAM-based education is an innovative and unique part of the four-component methodology model; however, it would not effectively support the acquisition of conceptual knowledge on its own. The problems of the task chain in the online learning material support the strengthening of both types of knowledge in an iterative way through problem-based and project-based approaches. Successive problems require the use of more and more new knowledge elements, actively using previously acquired knowledge to produce solutions. The effectiveness of the methodology can be measured by the homework tasks, given that solving the tasks requires a high level of use of both procedural and conceptual knowledge.

Task Design for Interactive Learning Materials

The interactive online course materials were uploaded weekly, and all students in the Calculus II course had access to them. The cardioid was covered in chapter six. Every chapter included tasks related to theoretical knowledge. In each case, in addition to the tasks' text, at least a colour illustration was included in the learning material to provide visual stimulation. The instructions for the solution were given step-by-step and placed in the course material after the task description but were only visible if the student wanted to display them. In addition, a link to a solution using the Desmos graphing calculator was included in the tutorial similarly.

Table 2
List of Tasks in the Interactive Learning Material

Task	Difficulty level
Cardioid in different positions	Easy
Horizontal and vertical tangent lines of a cardioid	Easy
Envelope of a pencil of circles	Easy
Envelope of a pencil of lines	Medium
Chords through the cusp	Medium
Tangent lines to a cardioid with any particular gradient	Medium
The evolute of a cardioid	Medium
The involute of a cardioid	Medium
Cardioid as the pedal curve of a circle	Hard
Cardioid as caustic of a circle	Hard

Ten tasks can be found in the interactive learning material on the topic of the cardioid curve; these are solved using the Desmos graphing calculator. The participants processed online learning materials in their free time, spending as much time as they wanted to solve the problems and study the solutions. Table 2 collects the tasks with different difficulty levels. The classic books by Lawrence (2014), Lockwood (1961), Pedoe (1979) and Yates (2012), were used to compile the interactive learning material. A complete description of the task "The Evolute of a Cardioid" is given in Appendix 1 to illustrate the structure of tasks.

Cardioid Drawing Robot

The literature on the cardioid curve is abundant, and the mathematics of this curve is varied and diverse. The cardioid curve can be derived in a surprisingly wide variety of ways. It is most often defined as an epicycloid: a cardioid is a curve in a plane followed by a fixed point on the circumference of a circle if the circle rolls without sliding along a fixed circle of the same radius (Lawrence, 2014; Yates, 2012).

The cardioid-drawing robot model was built from LEGO elements. The main components (motor, hub) are from the LEGO Education SPIKE Prime Set (45678), while the gears and some of the connecting elements are LEGO Technic parts (see Figure 2).

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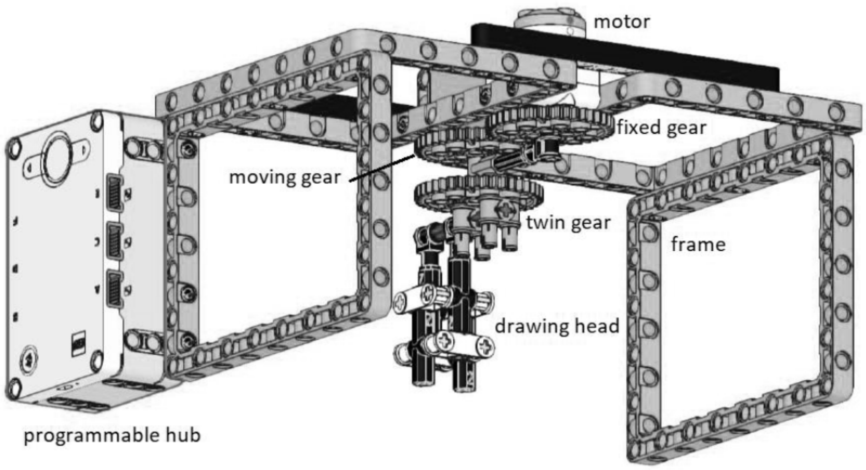
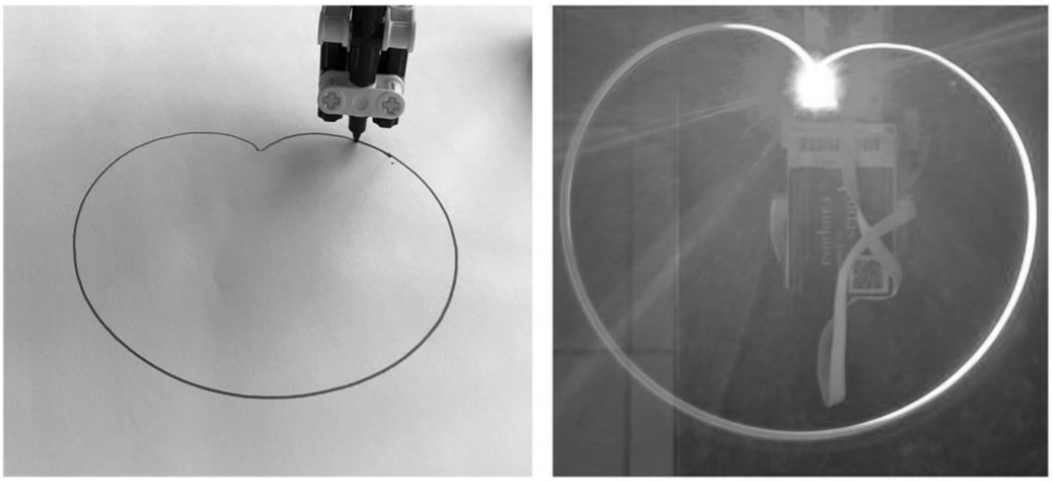
Figure 2
Main Parts of the Cardioid Drawing Robot Model



Using gears is an excellent way to model the slip-free rolling of circles on each other. From this point of view, implementing the cardioid as a hypocycloid is much more complicated since it would require a unique part, a ring toothed from the inside. In contrast, to represent the cardioid as a special epicycloid, only two gears of the same size are needed, one of which is fixed, and the other is rolled around it by a motor. The aim was to follow the path of a point on the circumference of the moving gear, so it was necessary to mount a pen holder on the gear so that the distance of the pen tip from the centre was equal to the radius. A relatively larger gear with many connection options was needed to attach a drawing head that held the writing instrument easily. These criteria were best met by LEGO Technic gears with a diameter of 40 mm. Three of these types of gears are built into the drawing robot. One of them is fixed to the frame, corresponding to the fixed circle in the definition of a cardioid. An axle is passed through the centre of the fixed gear, which is rotated by the motor. A lever was attached to the axle and another gear to the end of the lever so that the teeth fit. When the motor starts, this gear rolls along the fixed one. The third gear is also mounted on the same axle as the rolling gear so that their movements are synchronised. This twin gear is needed to have a place to attach the drawing head, which must be sturdy enough to hold the writing instrument securely in place.

The drawing was usually done with felt-tip pens cut to size. The position of the drawing head is adjustable because the entire support can be slid on an axis. With the setting shown in Figure 2, the tip of the pen inserted into the drawing head corresponds to a circumference point on the moving circle; as the motor makes one revolution, the pen returns to the starting point and draws the entire cardioid (see Figure 3). Another, even more spectacular way to display a cardioid with a robot is to place a lighted LED on the edge of the moving gear instead of a pen. The experiment is carried out in a dark room. Turn the robot on its side, turn on the LED and start the motor. Then, take long exposure photos of the movement. The best results are achieved using a camera and tripod (see Figure 3), but with some practice and the correct settings, one can take spectacular pictures of the curve even with a smartphone.

Figure 3
Cardioids Drawn by LEGO Robots with Pen and Light



During the robotics event, the students were first shown in detail how the robot works, and then, after testing the drawing with a felt-tip pen, the robot was rebuilt together to be able to draw with light. The students were particularly interested in drawing with light and tried out different settings. A significant advantage of this experiment was that it was quick to perform, required minimal equipment and was truly spectacular. Many of the robotics seminar participants took pictures with their smartphones. The picture on the right in Figure 3 is one of the pictures the students took. The best quality photo taken with a phone can be dragged into the Desmos graphing calculator to check that the shape drawn was a cardioid. After writing the parametric equations, it was possible to answer the relevant calculus questions for this curve, such as the length of the curve and the area enclosed by the cardioid.

Task Design of Homework Problems

Animations produced using DGS differ significantly from traditional task-solving in that they can be provided with many unique features. The complexity of the animation's appearance is a good indication of the creator's mathematical skills. Given that in the four-component methodology, procedural and conceptual knowledge develop in a mutually supportive and continuous way as animations are created, tasks requiring animation creation were needed to measure the success of the methodology. Two homework problems were created on the topic of cardioid. Designing the tasks aimed to ensure that the interactive online learning material was usable for the solution but also went beyond it. Complex tasks were provided with several parts to reinforce the project work. A detailed description of the homework tasks is given in Appendix 2. The first problem was an interesting wheel-road problem inspired by Hall and Wagon (1992). The task was to animate the moving of a vehicle with cardioid wheels on an appropriate road. The second problem was related to fractal geometry since the main shape of the Mandelbrot fractal is a cardioid. Pickover (2012) found that fractals are among the most fascinating objects in mathematics; therefore, they are very well suited to capturing students' interest. The wheel-road problem aimed to strengthen cross-curricular links, while the Mandelbrot fractal task increased working efficiency with the drawing robot.

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Research Aim and Research Question

This study aimed to create, demonstrate, and test a new innovative methodology that can be effectively applied to teach and learn the relevant properties of cardioid for first-year computer science undergraduate students. The new methodology combines frontal teaching, educational robotics and project work supported by DGS and focuses on visualisation. This study gives a complete description of the four-component methodology and shows the results of the pilot experience. High-quality DGS tasks were created with step-by-step instructions for the online learning material, supporting the successful learning process, and the Desmos graphing calculator was used as DGS. In the last component of the methodology, students had to solve two complex homework problems using Desmos. Students were expected to submit creative, self-designed solutions but were encouraged to work in groups to have the opportunity to share their experiences. Homework tasks were marked according to a fixed set of criteria. Through the data collected, this study explores the effectiveness and efficiency of the new methodology for university computer science students.

This research aimed to analyse the positive effects of the four-component methodology, so the research question was the following:

(RQ) Can the four-component methodology be used as an effective didactic technique to facilitate learning the cardioid curve?

Research Methodology

General Background

The research started in June 2021 and lasted two years. The first step was the development of a new methodological model. An important aspect was implementing practice-oriented teaching and learning in higher education for a maths topic for which first-hand experience learning was yet to be available. The entire development of the four-component methodological model for teaching cardioid took eighteen months, after which an experiment was conducted with students ($N = 27$) to test the effectiveness of the new methodology. The experiment was the second step of the research. The results obtained from the experiment were used to explore whether applying the four-component methodology improved conceptual and procedural understanding.

Research Design

In this study, exploratory research was used with quantitative observation and a quasi-experimental research design to assess the impact of the four-component methodology on the development of first-year computer science students' procedural and conceptual knowledge of the cardioid curve. The effectiveness of the methodology was assessed through descriptive analysis of the homework problems completed by the participants. For the evaluation of the homework tasks and obtaining quantitative data, a set of criteria was developed in the design of the experiment that not only assesses the mathematical correctness of the homework problems but also the additional features related to the creation of the animations. Moreover, a correlational research design format was used to identify relationships between collected data. The experiment took six months to design, conduct, and summarise the results.

Participants

The research was conducted among first-year undergraduate computer science students in the spring semester of the academic year 2022-2023 in the Calculus II course at the University of Miskolc. The participants had no relevant educational robotics experience, but they all had basic calculus skills because they had taken the Calculus I course. A project-based learning approach provided a blended learning environment for participants to complement the face-to-face teaching format. In addition to the lectures and exercises, learning was supported by the Desmos graphing calculator with online interactive learning materials to be completed independently or in a team during the first six weeks of the semester. These learning materials were freely downloadable. The online teaching materials covered chapters related to parametric graphs but were beyond the scope of the Calculus II course. After the theoretical knowledge, the students were allowed to test whether they had successfully mastered the knowledge needed to carry out the tasks by implementing Desmos projects. Detailed solutions to the complex tasks were also included in the online learning materials, thus ensuring optimal learning. The Calculus II course was attended by 79 computer science students, 27 of whom volunteered to participate in the experiment and to work on homework projects on the topic of the cardioid. The participants comprised 25 males and 2 females, with an average age of 20.04. Students were informed about the objectives of this research at the beginning of the semester, and all students agreed to participate. This study followed the ethical standards laid down in the Ethics Code of the University of Miskolc.

Procedures

The scoring criteria, the scoring system, and the way the data was processed were all developed during the research design. Homework tasks were assessed according to the following criteria: mathematical correctness, completeness, quality of Desmos use, spectacularity, creativity, and quality of documentation. All aspects were scored from 0 to 5. Table 3 shows the scoring technique used in the evaluation. The assessment was carried out jointly by three experienced teachers.

Table 3
The Scoring System Used in the Evaluation of Homework Projects

Score	Mathematical correctness	Other criteria
0	There is no content of value.	Nothing to evaluate.
1	It has a minimum of valuable content.	Very poor.
2	More than the minimum, but it is far from an acceptable solution.	Poor.
3	Acceptable, several errors.	Average.
4	Almost perfect, with some minor errors.	Good.
5	Perfect, free of all mistakes.	Excellent.

Data were collected after all students (N = 27) sent solutions for homework tasks. Students had two weeks to prepare their solutions. All participants who reported for homework solved the wheel-road task, but only 16 solved the Mandelbrot fractal problem.

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Data Analysis

During the homework assessment, the results were recorded in .xlsx spreadsheet format, and the data were analysed using the built-in functions and chart generator of Microsoft Excel spreadsheet software. Descriptive and correlational analyses were used to analyse the quantitative data. The mean (*M*), median (*Mdn*) and mode (*Mo*) are the three measures of central tendency and were calculated for both homework tasks. In addition, the value of the standard deviation (*SD*) was also given. Kendall's correlation coefficients were calculated to determine the correlation between criteria since the data did not have a normal distribution. Conover (1980) detailed how to calculate Kendall's tau; this method was used. Regarding the strength of the relationship between two variables and the direction of the relationship, the value of Kendall's tau correlation coefficient varies between -1 and +1, similar to other correlation measures.

Research Results

The basic statistical data of the wheel-road problem are shown in Table 4. The mean score of total points is 24.93 (83.09%). The low standard deviation (SD) values mean data are clustered around the mean, indicating that the mean scores are a good sample characterisation in all criteria.

Table 4
Main Descriptive Statistical Data of the Wheel-Road Problem (N = 27)

Criteria	*M*	*SD*	*Mdn*	*Mo*
Mathematical correctness	4.81	0.40	5	5
Completeness	3.56	0.75	3	3
Quality of Desmos using	4.74	0.59	5	5
Spectacularity	4.37	0.74	5	5
Creativity	4.30	0.78	4	4
Quality of documentation	3.15	1.51	3	3
Total	24.93	2.91	26	26

The high mean score of mathematical correctness shows that every student has done an excellent job. The similarly high average scores in the other test criteria also support the success of active learning. For the three criteria from the six, both the mode and the median are the maximum values that can be given (5), showing that most students achieved the greatest scores of these criteria. All students solved part (a) of the problem. The mean score for completeness is low because part (b) was missing or contained errors in several cases. Interestingly, while the work done in Desmos results from precise and well-thought-out steps, students did not pay as much attention to the preparation of the documentation. Only one student achieved the maximum score (30), but all participants performed at least 60%.

Kendall's tau was calculated pairwise to examine whether there is a correlation between mathematical correctness and other criteria. Table 5 shows the results. The greatest Kendall's tau value (.93) is between mathematical correctness and project spectacularity. The association between them can be considered statistically significant. In examining the data, Kendall's tau was calculated for all criteria pairs. There is also a relatively strong correlation between the project's spectacularity and the quality of Desmos use because Kendall's tau is .76. The project's spectacularity and the quality of Desmos use by the students in the project have a strong positive correlation with mathematical knowledge.

Table 5
Kendall's Tau Correlation Coefficient for the Wheel-Road Problem Respecting to Mathematical Correctness and Other Criteria (N = 27)

Criteria	Mathematical correctness
Completeness	.41
Quality of Desmos using	.42
Spectacularity	.93
Creativity	.74
Quality of documentation	.50

In the case of the Mandelbrot fractal problem, the mean score of total points is lower than the previous task; it is 23.00 (76.67%). Descriptive results are shown in Table 6. The standard deviation values are higher for all the criteria examined because the scores obtained for the solutions are, in all respects, in a broader range.

Table 6
Main Descriptive Statistical Data of the Cardioid in the Mandelbrot Fractal Problem Respecting to Mathematical Correctness and Other Criteria (N = 16)

Criteria	*M*	*SD*	*Mdn*	*Mo*
Mathematical correctness	4.06	1.39	5	5
Completeness	3.63	1.41	4	5
Quality of Desmos using	4.31	1.35	5	5
Spectacularity	4.25	1.57	5	5
Creativity	3.94	1.34	4	4
Quality of documentation	2.81	1.80	3	5
Total	23.00	7.76	26	27

Three students gave a perfect solution (100%), and five more gave 90% or above. Two students failed the task, and several participants had partially incorrect solutions. Nevertheless, for the five criteria from the six, the mode is the maximum value that can be given (5). In general, the students solved the first three parts of the problem, but part (d) was wrong or missed. Creativity scores are lower than in the previous exercise. The solutions generally met the criteria in the task specification, but with few exceptions, there was no added extra. There were several problems with the documentation preparation, as shown by the low mean score (2.81).

Kendall's tau was calculated pairwise for this task to discover the correlation between mathematical correctness and other criteria. As seen in Table 7, mathematical correctness correlates strongly with completeness. The other criteria also show a relatively strong correlation with mathematical correctness, but surprisingly, the weakest correlation is for spectacularity.

Table 7
Kendall's Tau Correlation Coefficient for the Cardioid in the Mandelbrot Fractal Problem (N = 16)

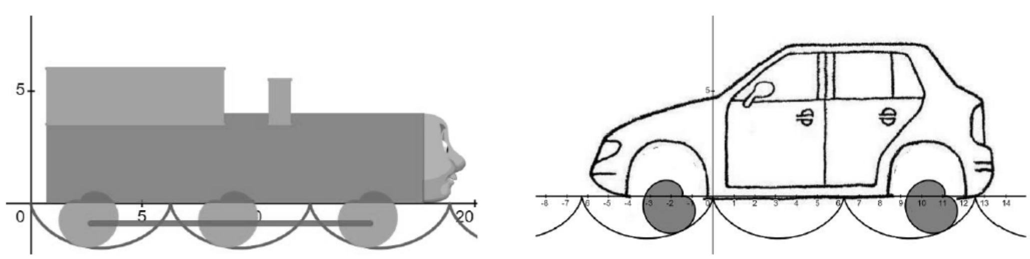
Criteria	*Mathematical correctness*
Completeness	.87
Quality of Desmos using	.68
Spectacularity	.55
Creativity	.71
Quality of documentation	.69

Discussion

The difficulty level of both homework tasks was hard. Nevertheless, the students successfully solved the problems, especially the road-wheel problem, which had a consistently high total score value. Statistics showed that the number of mathematically correct solutions is high, mainly because students worked in groups using the PBL method. When assessing the problems, it was clear that the students worked in groups of 5-6, yet no two solutions were perfectly identical. The denotations used, and the solution steps allowed us to deduce how the groups of students were formed. In addition to the mathematical correctness of the solution, creativity and visual appeal also played an important role. Advanced conceptual knowledge was needed to implement creative ideas.

In the animations, cars, buses, tanks, and trains rolled on cardioid wheels on the inverted cycloidal road, as shown in Figure 3. The number of wheels varied from two to five.

Figure 3
Examples of Students' Solutions for the Wheel-Road Problem

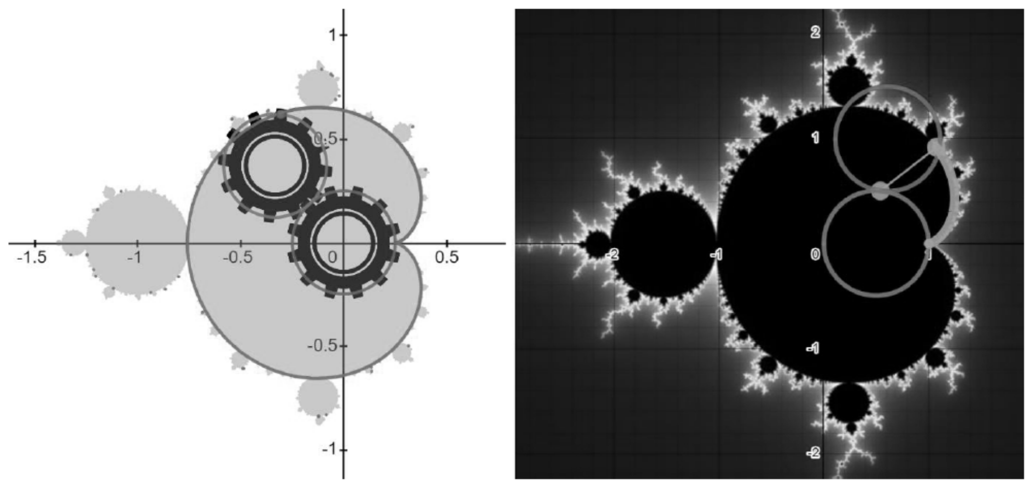


There were three ways to create the vehicle: an imported image of the vehicle without wheels (on the right side of Figure 3), the complete vehicle drawn in Desmos, or a combination of these two. This last was the most complex and gave the best spectacularity, for example, the locomotive on the left side of Figure 3. The first option was the simplest but synchronised moving of the imported picture and the wheels had to be solved in this case, too. It was a big challenge to coordinate the wheels' movement with the vehicle's body. This problem required a confident knowledge of mathematics and routine with using Desmos. The monitoring of cross-curricular links was also one of the objectives. For example, correctly positioning the previously learnt cycloid in the Cartesian coordinate system was crucial in solving the problem. The inverse cycloid did not cause any problems for students, and the cardioid that can be fitted and scrolled on it was also correct in all solutions.

The second exercise measured the success of the R-STEAM event. Based on the text of the task, all but two students found the circles that generated the cardioid and made working animations. There were also some valuable moments in the incorrect solutions, e.g., using the correct parametric equation for a circle, which is a basic procedural knowledge item. In all solutions, the generation of the cardioid was solved as an epicycloid, similar to the working principle of the cardioid drawing robot. One student's creative solution exactly mimics the robot by fitting gears to the generating circles (Figure 4). The gears of the drawing robot are not hidden (see Figure 2). The design of the robot is specifically intended to allow the movement that produces the cardioid to be clearly tracked during operation. During R-STEAM-based learning, the students tested the robot several times. They could observe the drawing of several cardioids and see the movement of the gears, which contributed significantly to the growth of advanced conceptual knowledge.

The different student groups working on the solutions were well separated in this exercise. The success of PBL is also shown by the fact that students who submitted the wrong solution worked independently and did not consult their peers. In the problem specification, two possibilities for representing the Mandelbrot set were pointed out. Both appear in the solutions in approximately equal proportions. On the left side of Figure 4, the Mandelbrot set is drawn iteratively based on the defining equation, while an imported image provides the background on the right side. Those who created a solution where the Mandelbrot set is generated iteratively had to fit the generating circles to the resulting cardioid. In the other solution, circles of the same radius touching each other could be placed anywhere in the coordinate system, and the animation could be made for drawing cardioid. Finally, the solution is completed by positioning and scaling the background image.

Figure 4
Examples of Students' Solutions for the Cardioid in the Mandelbrot Fractal Problem



The development of procedural knowledge was mainly supported by frontal teaching and interactive online learning materials. Basic procedural skills include, for example, managing parametric equations, specifying the domain of the curve using relations, creating lists, and defining different colours in Desmos. Advanced procedural skills are considered to include operations in the field of calculus: defining a derivative, defining the tangent and normal of curves and calculating arc length and area using integration. The evaluation of the homework assignments clearly showed that the participants had basic procedural knowledge, but there were gaps in the advanced procedural knowledge. For several students, the mathematical

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background of the animation was correct, but the calculation of the area and arc length requested as a sub-task contained errors. In the online material, the operation of derivation was used in many cases; however, the integral calculus did not occur in the exercises, so students did not uniformly develop advanced procedural knowledge in this field.

In addition to the creative use of procedural knowledge elements, conceptual understanding was required to produce a high-quality homework assignment. Basic conceptual skills include, for example, the correct use of parametric constants and the development of a suitable set of conditions to represent a given part of a curve, while advanced conceptual skills include the ability to move different geometric elements in a coordinated way, to use a novel approach to solving, to produce multiple possible solution variations.

One of the novelties of the four-component methodology was implementing R-STEAM-based learning using a robot model drawing a cardioid. However, the application's bottleneck was working with robots because the number of available robots limits the implementation in the case of many students. Although three alternatives were outlined in the methodology for implementing R-STEAM-based learning, the most efficient way to work is when students build the robot. This option was not available to the students ($N = 27$) participating in the experiment, so they studied and tested pre-built robots in small groups. The results showed that this alternative also significantly contributed to the development of conceptual knowledge. Another novelty of the methodology was that mathematical knowledge was tested by creating animations. The high-quality tasks in the online learning material actively supported independent learning with step-by-step instructions and developed mathematical skills needed to use Desmos. The measurement results showed that reinforcing the knowledge of integral calculus needs more attention in the future.

Conclusions and Implications

The four-component methodology for teaching the cardioid curve in university education is a tiny part of the reform of practice-oriented mathematics education. Nowadays, due to changing expectations and beliefs about the teaching and learning of mathematics, new didactic methods are emerging in science education. This work presents a compelling alternative learning method for teaching the cardioid curve and provides evidence of its effectiveness.

The quantitative results of the experiment confirmed the need to exploit the potential of R-STEAM and to provide students with tasks that capture their attention. Educational robotics and Desmos were successful pairings for computer science students, given that both were close to their areas of interest. The four-component methodology relies heavily on visualisation and visual stimuli to encourage students to learn and solve tasks. In essence, they learnt while having fun because the result of animation was usually spectacular, with many unique features. They were happy to show these animations to each other, and there was an active transfer of knowledge because they exchanged ideas about implementation options during PBL. It was essential because creating Desmos animations required much mathematical knowledge. The missing knowledge was often made up for without effort because they were motivated in this process. At the same time, the research has highlighted the shortcomings of the methodological model in that it does not sufficiently support the practice of different applications of integral calculus.

The results from the data analysis showed that the four-component methodology performed well in the iterative development of procedural and conceptual knowledge. In summary, a positive answer to the research question can be given, as the four-component methodological model can be considered an effective didactic technique for teaching and learning the cardioid curve. However, the study had a limited number of participants, and a more extensive study with more participants is needed to give conclusive results. Thus, the results cannot be generalised to a broader number of students.

Preparing and developing interactive learning materials and educational robots were time-consuming tasks but helped achieve effective learning. The researchers plan to apply the four-component methodology to other chapters of the Calculus II course. Further possibilities for integrating R-STEAM education in other areas of mathematics could be explored in university practice.

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Declaration of Interest

The authors declare no competing interest.

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Appendix 1

The Evolute of a Cardioid

Task:

The meaning of evolute is the locus of the centre of curvature or the envelope of the normals of a curve. To solve this task, you need to know the parametric equations of the cardioid in vector form, the derivative of the parametrically defined curve, and equations of tangent and normal of a curve. We note that the evolute of the cardioid is a mirror-image cardioid, but not in the same size. Let's create an evolute.

a) Give a set of tangent lines for the cardioid given by parametric equations.

where

b) Find the evolute of the cardioid.

c) Give the parametric equations of the evolute.

d) Make an animation to generate the evolute.

Steps to create the evolute with Desmos graphing calculator:

1) Give the parametric equations of the cardioid and the interval of the parameter:

2) Divide interval into 60 parts. Create a list that takes 60 points from interval :

3) Give the coordinates of the points in list L:

4) Enter the slope of the tangent at points corresponding to the parameters in list *L*. The slope of the tangent line in the point at which the parameter is :

so, it needs to generate a new list with slopes:

5) Give the equation of the tangent at the points in list *L*:

6) Give the equation of the normal at the points in list *L*:

+

The envelope of the normals is the evolute.

7) To create an animation, change the endpoint of list *L* to and set the slider *a* to the interval

8) Check parametric equations of the evolute:

where

Appendix 2

Homework task 1 - An interesting wheel-road problem

Two curves g_1 and g_2 form a wheel-road couple if g_2 can roll without slipping on g_1 so that a fixed point on the plane of g_1 (the wheel hub) has a linear trajectory in the fixed plane. The cardioid rolls on an inverted cycloid, and the locus of the point opposite the cardioid's cusp is a vertically scaled cycloid.

a) Create an animation with the Desmos graphing calculator where a vehicle with cardioid wheels drives on an inverted cycloid road.

b) Determine the circumference and area of the cardioid used as the wheel.

c) Document your solution.

Homework task 2 - Cardioid in the Mandelbrot Fractal

The Mandelbrot set is a very famous set named after mathematician and computer scientist Bernoit Mandelbrot. Its popularity is partly due to its aesthetic appeal once graphed upon the complex plane. The main cardioid of the Mandelbrot set is the most visually apparent feature of the fractal.

a) Import an image of a Mandelbrot set into the Desmos graphing calculator or use one of the links available on the web to display the Mandelbrot set in Desmos.

b) Find the circles that can be used to generate the central cardioid point by point. Give the parametric equations for these circles.

c) Create an animation demonstrating how to draw the cardioid by rolling the circles on each other. The animation requires knowledge of cardioid production. Both the epi- and hypocycloid approaches are possible.

d) Calculate the area of the cardioid by a double integral.

e) Document your solution.

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