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Article



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## LONGITUDINAL-RADIAL VIBRATIONS OF A CYLINDRICAL SHELL

**Abstract:** The article solves the problem longitudinal-radial vibrations of a cylindrical shell. To solve the problem, we used the refined equations of oscillation of such a shell, derived earlier from the exact three-dimensional formulation of the problem and its solution in transformations. An extensive review of works devoted to the study of harmonic and nonstationary processes in elastic bodies on the basis of classical and refined Timoshenko type theories is given. Four frequency equations are obtained for the main parts of the longitudinal and radial displacements of the cylindrical shell. These frequency equations admit, as special cases, frequency equations and a thin-walled shell. Based on the solution of the obtained frequency equations, the frequencies of natural vibrations of the shell, including the thin-walled one, are determined. On the basis of the results obtained, conclusions were drawn regarding the applicability of the studied oscillation equations, depending on the waveform and shell length. In particular, it was found that all the considered equations are unsuitable for describing wave processes in short shells, the lengths of which are commensurate with the transverse dimensions of the shells.

**Key words:** shell, solutions, equations, longitudinal-radial vibrations, theory, algorithm.

**Language:** English

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### Introduction

In various fields of science and technology, in particular physics and mechanics, researchers try to reduce the analysis of the behavior of waves in the general case to the analysis of the simplest harmonic waves [1]. In this case, the reverse transition, i.e. the transition from the characteristics of a harmonic process to estimates of the general wave motion in the body under consideration with the initial conditions is considerably difficult [2]. Despite this, much attention is paid to the study of harmonic processes in elastic bodies. This desire of researchers is due to the fact that already at the intermediate stage of solving the

problem, it is possible to obtain important data on such characteristics of oscillatory systems as phase and group velocities, natural frequencies and modes of oscillations [3]. Such studies are carried out on the basis of refined equations of the Timoshenko type, taking into account the transverse shear deformation and the inertia of rotation [4]. When constructing new theories of shell vibrations, they try to derive refined equations of vibrations, taking into account certain factors of a physical, mechanical or geometric nature [5].

Depending on the factors taken into account, the methods for deriving the equations of oscillation,

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based on the dynamic theory of elasticity, are divided into several directions. The analysis of scientific research devoted to the derivation of equations of vibration and the development of refined theories of deformable solids, in particular, circular cylindrical layers, shells and rods, as well as a detailed analysis of various directions of this problem are given in monographs. In works [6,7], on the basis of the three-dimensional formulation of problems of the linear theory of viscoelasticity, general equations of longitudinal and transverse vibrations of viscoelastic plates, round rods and cylindrical shells, as well as the equations of vibration taking into account the environment and friction forces, were derived. The anisotropic properties and temperature of the plates and rods were taken into account (related theory). On the basis of exact equations, approximate equations of the type of equations of S.P. Timoshenko and others, containing derivatives with respect to coordinates and time of a higher order, are obtained. On the basis of exact and refined approximate equations, particular problems of vibrations of rods, plates and shells are solved.

In monograph [9], this method was developed for a circular cylindrical layer interacting with a deformable solid medium and an ideal liquid, taking into account the viscoelastic properties of the layer material and for various modes of contact between the layers and the medium. In it, for the first time, an intermediate surface was introduced as the main surface that carries information about the oscillation of the layer and passes, in limiting cases, to the inner, outer or middle surface of the layer, depending on the values  $\chi$  of a certain parameter, which has a continuous spectrum of values, bounded from above and below.

It should be noted that in the works [5,8], a method was developed for deriving the equations of vibration, based on the use of general solutions in transformations of three-dimensional problems of elasticity theory. The method is based on the use of integral transformations in coordinate and time, and the use of general solutions in transformations of three-dimensional problems of elasticity theory with the subsequent expansion of these solutions in power series for the approximate satisfaction of the dynamic conditions specified on the boundary surfaces of the considered elastic system [10, 11].

An essential and successful application of this method to problems of dynamics was obtained in [12-13]. In them, the general equations of vibrations of circular cylindrical shells and rods are obtained taking into account the interacting viscous fluid and the rotation of the rod. The essence of the method is to study the constructed solutions under various types of external influences and to find out the conditions under which the displacements or their "main parts" satisfy simple oscillation equations, and to find an algorithm that allows calculating the approximate

field values from the field of these "main parts" displacements and stresses in any section for an arbitrary moment in time.

In the works of the authors [14,15], equations of oscillation of circular cylindrical viscoelastic shells and layers interacting with a liquid were developed. The developments were carried out without the use of additional hypotheses and prerequisites of a physical or mechanical nature, from which it is possible to obtain the known classical and refined equations of oscillation. An algorithm is proposed that makes it possible to unambiguously determine the stress-strain state of points of an arbitrary section of the system under consideration from the values of the sought functions using the field of the sought functions.

The analysis of vibrations of elements of engineering structures, such as rods, plates and shells on the basis of both classical (Kirchhoff-Love) and refined (Timoshenko type) theories is carried out at the present time. At the same time, in most of such studies, the tendency to take into account the inertia of rotation, transverse shear deformation, as well as the multilayer structure prevails [16,17]. In addition, attention is paid to taking into account the rheological, in particular, the viscoelastic properties of the material [18, 19], as well as the interaction of structures with deformable media such as a viscous fluid [20] or dispersive waves [21]. The issues of studying natural frequencies and natural modes of vibrations of rods, plates and shells have also not lost their relevance. Proof of this statement can be found in publications where the problems of influence on the frequency characteristics of violations of the boundary forms [22] and the conduct of biharmonic [23] and frequency analyzes [24,25] are discussed.

Within the framework of this article, a circular cylindrical elastic shell is considered. The task is to study its harmonic longitudinal-radial oscillations on the basis of classical and refined theories. To carry out a comparative analysis of the numerical values of the frequencies of natural longitudinal-radial oscillations of an elastic cylindrical shell, obtained according to the equations proposed by the authors, according to the equations of the classical Kirchhoff-Love theory, on the basis of the refined theories of Hermann - Mirski (of the SP Timoshenko type) and Khudoinazarov Kh. As one of the equations of the refined theories, the refined oscillation equations developed by the authors [14, 15] are taken.

### Methods.

In a cylindrical coordinate system  $(r, \theta, z)$ , we investigate natural longitudinal-radial vibrations of a circular cylindrical elastic shell, freely supported on the ends. The shell with length  $l$  has an interior and exterior radii,  $r_1$ ,  $r_2$  respectively. The direction of the coordinate axes, radii and displacements are shown in Fig. 1. It is believed that the shell is not exposed to

external influences and its surfaces are also free from external forces.

In article [14], general equations of oscillation of a circular cylindrical shell were developed, and then they were generalized in [15] to the case of interaction of a circular cylindrical layer with a viscous fluid. To solve the problem, we assume that the terms responsible for the effect of the interacting fluid are equal to zero and the shell surfaces are free from external loads. Then, in the indicated equations, we pass to the dimensionless variables by the following formulas

$$U_{r,0} = U_{r,0}^*; \quad U_{r,1} = lU_{r,1}^*; \quad U_{z,0} = lU_{z,0}^*;$$

$$U_{z,1} = U_{z,1}^*; \quad z = lz^*; \quad r = lr^*; \quad t = \frac{l}{b}t^*.$$

And for ease of writing, omitting the asterisks above the values in what follows, we obtain

$$q_1 U_{r,0} + 2\nu q_1 \frac{\partial U_{z,0}}{\partial z} + \frac{1}{2} \partial_2 U_{r,1} + \frac{1}{2} \frac{\partial U_{z,1}}{\partial z} = 0,$$

$$\nu q_1 \frac{\partial U_{r,0}}{\partial z} + \partial_1 U_{z,0} - \frac{\partial U_{r,1}}{\partial z} - U_{z,1} = 0,$$

$$q_1 U_{r,0} + \nu q_1 \frac{\partial U_{z,0}}{\partial z} + \left[ \partial_2 q_2 \ln \frac{r_2}{r_1} + 2 \frac{r_1^2}{r_2^2} \right] U_{r,1} -$$

$$- \left[ \frac{q_2}{q_1} \ln \frac{r_2}{r_1} + \frac{1}{2} \right] \frac{\partial U_{z,1}}{\partial z} = 0, \quad (1)$$

$$\nu q_1 \frac{\partial U_{r,0}}{\partial z} + \partial_1 U_{z,0} - \nu q_2 \partial_2 \frac{\partial U_{r,1}}{\partial z} -$$

$$- \left[ \left( 2q_2 \frac{\partial^2}{\partial z^2} + \lambda_2 \right) \ln \frac{r_2}{r_1} + 2 \frac{r_1^2}{r_2^2} \right] U_{z,1} = 0,$$

where

$$\partial_1 = \frac{b^2}{a^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}; \quad \partial_2 = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2};$$

$$q_1 = -\frac{1}{1-2\nu}; \quad q_2 = -\frac{1}{2(1-\nu)};$$

$\nu$  - Poisson's ratio of the layer material;  $a$  - the speed of propagation of longitudinal waves in the shell material. The boundary conditions of the problem of natural vibrations of a cylindrical shell with free feathering of its ends at  $z=0$  and  $z=l$ , where  $l$  is the length of the shell, will have the form [9]

$$U_{z,0} = \frac{\partial^2 U_{z,0}}{\partial z^2} = 0; \quad U_{z,1} = \frac{\partial^2 U_{z,1}}{\partial z^2} = 0;$$

$$\frac{\partial U_{r,0}}{\partial z} = \frac{\partial^3 U_{r,0}}{\partial z^3} = 0; \quad \frac{\partial U_{r,1}}{\partial z} = \frac{\partial^3 U_{r,1}}{\partial z^3} = 0. \quad (2)$$

Note that conditions (2) are written on the basis of expressions for the displacement values determined by the formulas

$$U_r(r, z, t) = \frac{r}{2} U_{r,0}(z, t) - \frac{1}{2} U_{r,1}(z, t),$$

$$U_z(r, z, t) = U_{z,0} - \frac{r}{4} U_{z,1}. \quad (3)$$

Thus, the problem of natural longitudinal-radial vibrations of an elastic circular cylindrical shell is reduced to solving equations (1) under boundary conditions (2), which satisfy the series term by term.

$$U_{r,0} = \sum_m W_{0,m}(t) \cos(\gamma_m z)$$

$$U_{r,1} = \sum_m W_{1,m}(t) \cos(\gamma_m z)$$

$$U_{z,0} = \sum_m U_{0,m}(t) \sin(\gamma_m z)$$

$$U_{z,1} = \sum_m U_{1,m}(t) \sin(\gamma_m z)$$

where  $\gamma_m = \frac{m\pi}{l}$ ,  $l$  - the length of the shell

$m = 0, 1, 2, 3, \dots$ . Putting these series into the system of equations (1), we obtain

$$W_{0,m}(t) + \nu q_{12} \gamma_m U_{0,m}(t) - \bar{\partial}_2 W_{1,m} - \gamma_m U_{1,m}(t) = 0,$$

$$\nu q_1 \gamma_m W_{0,m}(t) + \bar{\partial}_1 U_{0,m}(t) + \gamma_m W_{1,m}(t) - U_{1,m}(t) = 0,$$

$$q_1 W_{0,m}(t) + 2\nu q_1 \gamma_m U_{0,m}(t) +$$

$$+ \left[ \left( q_2 \ln \frac{r_2}{r_1} - \frac{1}{2} \right) \bar{\partial}_2 + 2 \frac{r_1^2}{r_2^2} \right] W_{1,m}(t) -$$

$$- \left( \frac{q_2}{q_1} \ln \frac{r_2}{r_1} + \frac{1}{2} \right) \gamma_m U_{1,m}(t) = 0,$$

$$- 2\nu q_1 \gamma_m W_{0,m}(t) + \frac{q_1}{q_2} \bar{\partial}_1 U_{0,m}(t) -$$

$$- \left[ 2\nu q_2 \bar{\partial}_2 \ln \frac{r_2}{r_1} + 2 \frac{r_1^2}{r_2^2} \right] \gamma_m W_{1,m}(t) + \quad (4)$$

$$+ \left[ \left( 2q_2 \gamma_m^2 + \bar{\partial}_2 \right) \ln \frac{r_2}{r_1} - 2 \frac{r_1^2}{r_2^2} \right] U_{1,m}(t) = 0,$$

where  $\bar{\partial}_1 = (1 - q_1) \frac{\partial^2}{\partial t^2} + \gamma_m^2$ ,  $\bar{\partial}_2 = \frac{\partial^2}{\partial t^2} + \gamma_m^2$ .

In order for system (4) to have a nonzero solution, it is necessary that its main determinant, composed of the coefficients of the unknown functions  $U_{i,m}(t)$  and  $W_{i,m}(t)$  ( $i=1,2$ ), be equal to zero. Denoting this determinant by  $\Delta_1$  and introducing the following notation

$$\omega_1 = \left( q_2 \ln \frac{r_2}{r_1} - \frac{1}{2} \right) \bar{\partial}_2 + 2 \frac{r_1^2}{r_2^2}; \quad \omega_2 = \frac{q_2}{q_1} \ln \frac{r_2}{r_1} + \frac{1}{2};$$

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$$\omega_3 = 2\nu q_2 \bar{\partial}_2 \ln \frac{r_2}{r_1} + 2 \frac{r_1^2}{r_2^2};$$

$$\omega_4 = (2q_2 \gamma_m^2 - \bar{\partial}_2) \ln \frac{r_2}{r_1} - 2 \frac{r_1^2}{r_2^2}. \quad (5)$$

$$\Delta_1 = \begin{vmatrix} q_1 & 2\nu q_1 \gamma_m & -\left(2 - \frac{1}{2} \bar{\partial}_2\right) & -\frac{1}{2} \gamma_m \\ -\gamma_m 2\nu q_1 & \frac{q_1}{q_2} \bar{\partial}_1 & 2\gamma_m & -2 \\ q_1 & 2\nu q_1 \gamma_m & \omega_1 & \omega_2 \gamma_m \\ -2\nu q_1 \gamma_m & \frac{q_1}{q_2} \bar{\partial}_1 & \omega_3 \gamma_m & \omega_4 \end{vmatrix}$$

Expanding this determinant by the elements of the third row, we get

$$\Delta_1 = q_1 A_{31} + 2\nu q_1 \gamma_m A_{32} + \omega_1 A_{33} + \omega_2 \gamma_m A_{34}$$

where  $A_{3i} = (-1)^{3+i} D_{3i}$  is an algebraic complement,  $D_{3i}$  are minors of the third row elements  $a_{3i}$ .

The resulting expression for the determinant can be rewritten as

$$\Delta_1 = q_1 D_{31} - 2\nu q_1 \gamma_m D_{32} + \omega_1 D_{33} - \omega_2 \gamma_m D_{34}.$$

Putting instead of  $\omega_i$  ( $i=1,4$ ) their expressions (5), we finally obtain

$$\Delta_1 = a_1 \bar{\partial}_1 \bar{\partial}_2^2 - \bar{\partial}_2 + a_3 \bar{\partial}_1 - a_5 \bar{\partial}_2 + a_6 \quad (6)$$

where

$$a_1 = \frac{q_1^2}{q_2} \ln \left( \frac{r_2}{r_1} \right) \left( \frac{3}{2} - q_2 \ln \frac{r_2}{r_1} \right);$$

$$a_4 = -4\nu^2 q_1^2 \ln \left( \frac{r_2}{r_1} \right) \left( q_2 \ln \frac{r_2}{r_1} - 1 \right);$$

$$a_2 = q_2 \gamma_m^2 (q_2 + \nu) - (\nu q_1 \gamma_m^2 + \nu q_2 \gamma_m) \ln \frac{r_2}{r_1};$$

$$a_3 = 4q_2 \gamma_m^2 \ln \frac{r_2}{r_1} + 2\gamma_m^2 - 4\nu \gamma_m^2 + 2\nu \gamma_m;$$

$$a_5 = 2\nu^2 q_1^2 \left[ 2\nu q_2 \gamma_m^2 + \gamma_m^2 \ln \frac{r_2}{r_1} + 4 \ln \frac{r_2}{r_1} + 4\gamma_m \frac{r_1^2}{r_2^2} \ln \frac{r_2}{r_1} + 4\nu q_2 \gamma_m^3 \left( \ln \frac{r_2}{r_1} + \frac{1}{2} \right) \ln \frac{r_2}{r_1} \right];$$

$$a_6 = 2\nu^2 q_1^2 2\gamma_m^2 \left( 1 - \frac{r_1^2}{r_2^2} \right) + 8 \left( 1 + \frac{r_1^2}{r_2^2} \right).$$

Putting into (6), the values of the differential operators  $\bar{\partial}_1$  and  $\bar{\partial}_2$  by formulas (4), we finally obtain

$$\Delta_1 = a_1 \frac{\partial^6}{\partial t^6} + [a_1 \gamma_m^2 (3 - 2q_1) - a_2 - a_4] \frac{\partial^4}{\partial t^4} + [a_1 \gamma_m^4 (3 - q_1) - \gamma_m^2 + a_3 q_1] \frac{\partial^2}{\partial t^2} + a_1 \gamma_m^6 - \quad (7)$$

$$-(a_2 + a_4) \gamma_m^4 + (a_3 - a_5) \gamma_m^2.$$

Hence, each of the functions  $U_{0,m}(t)$ ,  $U_{1,m}(t)$ ,

$W_{0,m}(t)$ , and  $W_{1,m}(t)$  must satisfy the equation

$$\Delta_1 \zeta_m(t) = 0, \quad (8)$$

where  $\zeta_m(t)$  is any of the above functions. Then, based on (3), the displacements  $U_r$  and  $U_z$  must satisfy the same equation.

In equation (8), we put  $\zeta_m(t) = A_m e^{\omega t}$  and obtain the following frequency equation

$$a_1 (1 - q_1) \omega^6 + [a_1 \gamma_m^2 q_1 - a_2 (1 - q_1) - a_4] \omega^4 + [a_1 \gamma_m^4 (3 - q_1) - \gamma_m^2 (a_2 q_1 + 2a_4) + a_3 q_2] \omega^2 + a_1 \gamma_m^6 - (a_2 + a_4) \gamma_m^4 + (a_3 - a_5) \gamma_m^2 + a_6 = 0 \quad (9)$$

The obtained equation is general for both the layer and the shell. To obtain a simpler frequency equation for the shell in the expressions for the coefficients  $a_i$ , it is sufficient to take  $\ln(r_2/r_1) = 0$ . In this case, the coefficients  $a_i$  ( $i=1,6$ ) take the form:

$$a_1 = 0; \quad a_2 = -\frac{q_1^2}{2q_2} \nu \gamma_m (2\nu \gamma_m - \gamma_m - 1);$$

$$a_3 = \frac{2q_1^2}{q_2} \left[ \gamma_m^2 \frac{l_1^2}{l_2^2} - 2\nu \gamma_m^2 + \nu \gamma_m \left( 2 - \frac{r_1^2}{r_2^2} \right) + 2 \frac{r_1^2}{r_2^2} \right];$$

$$a_4 = 0; \quad a_5 = 2 \left( 1 + \gamma_m - \frac{r_1^2}{r_2^2} \right);$$

$$a_6 = \gamma_m^4 + 6 \frac{r_1^2}{r_2^2} \gamma_m^3 + 2\gamma_m^2 + 8 \left( 1 + \frac{r_1^2}{r_2^2} \right).$$

The equation in (9) takes the following form:

$$a_2 (1 - q_1) \omega^4 + [a_2 (2 - q_1) \gamma_m^2 - q_1 a_3] \omega^2 + (a_2 + a_4) \gamma_m^4 + (a_5 - a_3) \gamma_m^2 + a_6 = 0 \quad (10)$$

For a comparative analysis, let us calculate the frequencies of natural longitudinal-radial oscillations of an elastic circular cylindrical shell based on the oscillation equations of various theories. As such theories, we will take the classical Kirchhoff-Love theory [6] and the refined theories of Herman-Mirsky [4] and Fillipov-Khudoinazarov [5]. The corresponding frequency equations have the following forms:

Fillipov-Khudoinazarov

$$\omega^6 + [(g_2 + g_3) \gamma_m^2 + g_5 \omega_1] \omega^4 + [g_2 g_3 \gamma_m^4 + (1 + g_2) g_5 \omega_1 \gamma_m^2 + \omega_1] \omega^2 + g_2 g_4 \gamma_m^6 + g_2 g_5 \omega_1 \gamma_m^4 + g_2 g_6 \omega_1^2 \gamma_m^2 = 0, \quad (11)$$

Herman-Mirsky

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$$\omega^6 + \frac{2y_1}{1-\nu} [k_T + y_1 y_4 + y_1(g+k)] \gamma_m^4 + \frac{4y_1}{(1-\nu)^2} \left[ y_1 k_T \gamma_m^4 + 2 \left( y_1 y_4 + \frac{k_T - k_E^2}{2} \right) \gamma_m^2 - y_2 \gamma_m^2 + y_4 k_T \right] \omega^2 + y_1 k_T \gamma_m^6 + (k_T + y_1 y_4) \gamma_m^6 + (y_4 k_T - y_3) \gamma_m^2 = 0, \quad (12)$$

Kirchhoff-Love

$$\omega^4 + \frac{2}{1-\nu_0} \left( \frac{1}{3} \gamma_m^4 + \gamma_m^2 + \frac{1}{\xi^2} \right) \omega^2 + \left[ \frac{4}{3(1-\nu)^2} \gamma_m^6 + \frac{4(1+\nu) \cdot \gamma_m^2}{(1-\nu)\xi^2} \right] = 0, \quad (13)$$

where

$$g_1 = \frac{3-4\nu}{8(1-2\nu)(1-\nu^2)}; \quad g_2 = \frac{3(1-\nu)(1+4\nu)}{3-4\nu};$$

$$g_3 = 3-2\nu; \quad g_4 = 2-2\nu; \quad g_5 = 3+2\nu;$$

$$g_6 = 2+2\nu; \quad y_1 = \frac{1}{3} \left( 1 - \frac{1}{\xi^2} \right); \quad y_3 = k_T \frac{\nu^2}{\xi^2};$$

$$k_T = \frac{1-\nu}{2} k^2;$$

$$y_2 = \frac{3k_T \nu}{3\xi^2} + k_T^2 + \frac{\nu^2}{3\xi^2};$$

$$y_4 = \frac{1}{\xi^2} \left( 1 + \frac{1}{3\xi^2} \right); \quad \xi = \frac{R}{h};$$

$R$  -radius of the middle surface,  $h$  -shell thickness,  
 $k_T$  -Tymoshenko correction coefficient.

**Results and Discussions.**

Frequency equations (9) - (13) were solved numerically using the MAPLE application programs with the following geometric data of the shell  $r_1 = 1,0; r_2 = 1,1; h = 0,1$ . Poisson's ratio was taken to be  $\nu = 0,2$ . The Timoshenko coefficient was taken to be  $5/6$ . Table 1 shows the numerical values of the shell frequencies depending on the values of the waveforms calculated on the basis of equations (9) and (10). In Fig. 2, based on the obtained numerical values, the dependences of the frequency  $\omega$  on the waveform  $\gamma_m$  are plotted. Table 2 shows the numerical values of the shell frequencies depending on the values of the waveforms calculated on the basis of equations (11) - (13). In Fig. 3, on the basis of the obtained numerical values, the dependences of the frequency  $\omega$  on the waveform  $\gamma_m$  are plotted according to the classical and refined equations of various theories. On the given Tables 1, 2 it can be observed that the real parts of the roots of the frequency equations, except for the Hermann-Mirsky equations, are negative. From the physical meaning of the problem on the basis of the Hurwitz criterion [26] it follows that the roots of the frequency equations (respectively, cubic and quadratic equations with respect to  $\omega^2$ ) must be purely imaginary.

The obtained numerical values of the roots of the equations show that in reality such results follow from all equations, except for the Hermann-Mirsky equation (Table 2) up to a certain value of the

**Table 1**

$\gamma_m$		Equation (9)			Equation (10)	
		$\omega_1$	$\omega_2$	$\omega_3$	$\omega_1$	$\omega_2$
0,1	D	0	0	0	0	0
	M	$\pm 0,1624$	$\pm 2,3557$	$\pm 19,9508$	$\pm 1,0539$	$\pm 0,1549$
0,3	D	0	0	0	0	0
	M	$\pm 0,4837$	$\pm 2,3659$	$\pm 19,9511$	$\pm 1,0539$	$\pm 0,4647$
0,5	D	0	0	0	0	0
	M	$\pm 0,8062$	$\pm 2,3859$	$\pm 19,9517$	$\pm 1,0539$	$\pm 0,7745$
0,7	D	0	0	0	0	0
	M	$\pm 1,1287$	$\pm 2,4154$	$\pm 19,9527$	$\pm 1,0844$	$\pm 1,0539$
0,9	D	0	0	0	0	0
	M	$\pm 1,4512$	$\pm 2,4542$	$\pm 19,9539$	$\pm 1,3942$	$\pm 1,0539$
1,1	D	0	0	0	0	0
	M	$\pm 1,7736$	$\pm 2,5019$	$\pm 19,9554$	$\pm 1,7041$	$\pm 1,0539$
1,3	D	0	0	0	0	0
	M	$\pm 2,0961$	$\pm 2,5578$	$\pm 19,9572$	$\pm 2,0139$	$\pm 1,0539$
1,5	D	0	0	0	0	0
	M	$\pm 2,4186$	$\pm 2,6214$	$\pm 19,9594$	$\pm 2,3237$	$\pm 1,0539$
1,7	D	0	0	0	0	0
	M	$\pm 2,6921$	$\pm 2,7411$	$\pm 19,9619$	$\pm 2,6336$	$\pm 1,0539$
1,9	D	0	0	0	0	0

**Impact Factor:** ISRA (India) = 6.317    SIS (USA) = 0.912    ICV (Poland) = 6.630  
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	M	±2,7694	±3,0636	±19,9647	±2,9434	±1,0539
2,1	Д	0	0	0	0	0
	M	±2,8526	±3,3861	±19,9679	±3,2533	±1,0539
2,3	Д	0	0	0	0	0
	M	±2,9412	±3,7086	±19,9714	±3,5631	±1,0539
2,5	Д	0	0	0	0	0
	M	±3,0347	±4,0311	±19,9753	±3,8729	±1,0539

Table 2

		Equation (9)			Equation Kirchoff-Love		Equation Hermann-Mirsky			Equation Filippov-Khudoynazarov		
$\gamma_m$		$\omega_1$	$\omega_2$	$\omega_3$	$\omega_1$	$\omega_2$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_1$	$\omega_2$	$\omega_3$
0,1	Д	0	0	0	0	0	0	0	0	0	0	0
	M	±0,1	±2,3	±19,9	±0,01	±0,16	±0,04	±0,24	±0,97	±0,25	±1,9	±3,21
0,3	Д	0	0	0	0	0	0	±0,04	∓0,04	0	0	0
	M	±0,4	±2,3	±19,9	±0,08	±0,50	±0,04	±0,74	±0,74	±0,77	±2,0	±3,23
0,5	Д	0	0	0	0	0	0	±0,39	∓0,39	0	0	0
	M	±0,8	±2,3	±19,9	±0,24	±0,84	±0,07	±0,89	±0,89	±1,28	±2,0	±3,27
0,7	Д	0	0	0	0	0	0	±0,57	∓0,57	0	0	0
	M	±1,1	±2,4	±19,9	±0,47	±1,18	±0,12	±1,05	±1,05	±1,79	±2,0	±3,33
0,9	Д	0	0	0	0	0	0	±0,73	∓0,73	0	0	0
	M	±1,4	±2,4	±19,9	±0,79	±1,52	±0,19	±1,21	±1,21	±2,14	±2,3	±3,40
1,1	Д	0	0	0	0	0	0	±0,87	∓0,87	0	0	0
	M	±1,7	±2,5	±19,9	±1,18	±1,85	±0,27	±1,39	±1,39	±2,21	±2,8	±3,50
1,3	Д	0	0	0	0	0	0	±1,02	∓1,02	0	0	0
	M	±2,0	±2,5	±19,9	±1,64	±2,19	±0,35	±1,56	±1,56	±2,29	±3,3	±3,60
1,5	Д	0	0	0	0	0	0	±1,16	∓1,16	0	0	0
	M	±2,4	±2,6	±19,9	±2,19	±2,53	±0,43	±1,75	±1,75	±2,39	±3,7	±3,85
1,7	Д	0	0	0	0	0	0	±1,31	∓1,31	0	0	0
	M	±2,6	±2,7	±19,9	2,82	2,87	±0,51	±1,94	±1,94	±2,50	±3,8	±4,36
1,9	Д	0	0	0	0	0	0	±1,45	∓1,45	0	0	0
	M	±2,7	±3,0	±19,9	3,21	3,52	±0,59	±2,13	±2,13	±2,62	±4,0	±4,88
2,1	Д	0	0	0	0	0	0	±1,60	∓1,60	0	0	0
	M	±2,8	±3,3	±19,9	3,54	4,30	±0,67	±2,32	±2,32	±2,75	±4,1	±5,39
2,3	Д	0	0	0	0	0	0	±1,74	∓1,74	0	0	0
	M	±2,9	±3,7	±19,9	3,88	5,16	±0,75	±2,51	±2,51	±2,89	±4,3	±5,90
2,5	Д	0	0	0	0	0	0	±1,89	∓1,89	0	0	0
	M	±3,0	±4,03	±19,9	4,22	6,09	±0,82	±2,71	±2,71	±3,03	±4,4	±6,42

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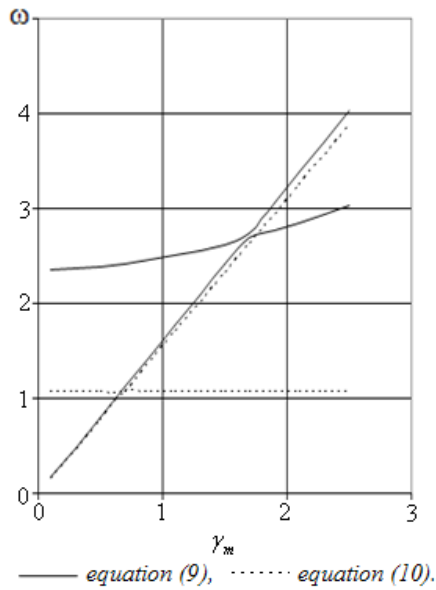


Fig.1. Comparison of natural frequencies theories (9) and (10).

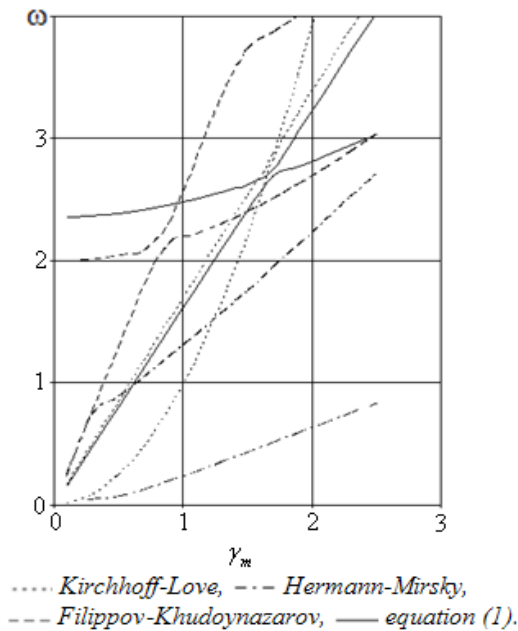


Fig.2. Comparison of natural frequencies of vibrations according to different longitudinal-radial according to equations.

parameter  $\gamma_m$  and at certain values of the transverse dimensions of the shell. As for the Hermann-Mirsky equations, such results can be obtained for separate values of the Timoshenko coefficient  $k_T > 1$ , which is impossible in principle.

### Conclusions

From a comparison of the numerical results obtained by equations (9) and (10), it follows that in the case of equation (9) we have six frequency values, and for the equation for the shell four frequencies. When passing from equation (9) to equation (10) for the shell, two frequencies are lost. The obtained numerical results, at  $r_1 = 1, 0; r_2 = 1, 1; h = 0, 1; \nu = 0, 2; k_T = \frac{5}{6}$ , are shown in Table 2 and presented in the form of curves in Fig. 3. From Table 2 and Fig. 3 the following conclusions follow:

- the Hermann-Mirsky equation does not obey the Hurwitz criterion and gives imprecise results. More accurate results can be obtained that are consistent with the Hurwitz criterion only at certain values of the Timoshenko correction coefficient  $k_T$ . This conclusion completely coincides with the same conclusion of work [9];

from the graphs in Fig. 2 and Fig. 3 it follows that equations (1) describe well the wave process, like equations (11), in long shells ( $l \gg m\pi$ ) regardless of the values of the number  $m$ , i.e. with sufficiently low and high forms of wave formation;

- equations (1) are suitable for solving dynamic problems in shells of medium length with sufficiently low waveforms;

- these equations are unsuitable for describing wave processes in short shells, the lengths of which are commensurate with the transverse dimensions of the shells  $l \gg m\pi$ .

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