Oscillations in a Growth Model with Capital, Technology and Environment with Exogenous Shocks

Prof. Wei-Bin Zhang, PhD
College of Asia Pacific Management
Ritsumeikan Asia Pacific University, Japan

Abstract

This paper generalizes the dynamic growth model with wealth accumulation, technological change and environmental change by Zhang (2012) by making all the parameters as time-dependent parameters. The model treats physical capital accumulation, knowledge creation and utilization, and environmental change as endogenous variables. It synthesizes the basic ideas of the neoclassical growth theory, Arrow's learning-by-doing model and the traditional dynamic models of environmental change within a comprehensive framework. The behavior of the household is described with an alternative approach to household behavior. We simulated the model to demonstrate existence of equilibrium points, motion of the dynamic system, and oscillations due to different exogenous shocks.

Keywords: perturbations; economic oscillations; economic growth; technological change; wealth accumulation

Introduction

Economic fluctuations are commonly observed in empirical studies. There are a lot of theoretical and empirical research about mechanisms and phenomena of economic fluctuations. Zhang (1991, 2005, 2006) show how modern dynamic analysis can be applied to different economic systems, identifying existence of cycles, regular as well as irregular oscillations, and chaos in economic systems. There are also studies which empirically test validity of theories. For instance, Lucas (1977) suggests possible existence of some shocks that affect all sectors in an economy. Chatterjee and Ravikumar (1992) propose a neoclassical growth model with seasonal perturbations to taste and technology. They demonstrate how the economic system reacts to seasonal demand and supply perturbations. Gabaix (2011) holds that uncorrelated sectoral shocks are determinants of aggregate fluctuations (see also, Giovanni, et al. 2014; Stella, 2015). Nevertheless, there are only a few theoretical models which identify fluctuations due to dynamic interdependence among economic growth, technological change and environmental change is currently a main topic in economic theory. This study attempts to provide another contribution to the literature by identifying economic fluctuations in a model with dynamic interdependence between wealth, knowledge and environment under different environmental policies with a new approach to consumers' behavior with endogenous saving. The model is an extension of Zhang's model, which is on Solow's one-sector growth model, Arrow's learning by doing model, and some dynamic models in environmental economics. The main mechanisms of economic growth in these three models are integrated into a single framework.

The model is influenced by the neoclassical growth theory. The theory is developed initially by Solow (1956). The Solow model is has only one production sector without taking account of endogenous environmental and knowledge dynamics. The Solow model has been extended and generalized in numerous directions (Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). An important extension was the Uzawa two-sector growth model proposed by Uzawa (1961). The economic structure of our model is based on the Uzawa model. Our approach to technological change is based on Arrow's learning-by-doing. One of the first seminal attempts to render technical progress endogenous in growth models was made by Arrow in 1962. He emphasized one aspect of knowledge accumulation - learning by doing (Arrow, 1962). Theoretical economists had been relatively silent on the topic from the end of the 70s until the publication of Romer's 1986 paper. The literature on endogenous knowledge and economic growth have increasingly expanded since Romer re-examined issues of endogenous technological change and economic growth in his 1986's paper (Romer, 1986; Lucas, 1988; Grossman and Helpman, 1991; Aghion and Howitt, 1998). There are only a few models which have endogenous environmental change in growth models of capital and knowledge.

There are many studies on interdependence between economic growth and environmental change (Pearson, 1994; Bovenberg and Heijdra, 1998; Ayong Le Kama, 2001; Copeland and Taylor, 2004; Stern, 2004; Wirl, 2004; Dasgupta, *et al*. 2006; Kijima *et al*. 2010; Dam and Heidra, 2011; Tsurumi and Managi, 2010). It is well known that Kuznets (1955) postulated that economic growth and income inequalities follow an inverted U-curve. The environmental Kuznets curve refers to the same relation between environmental quality and per capita income. Nevertheless, a large number of empirical studies on the environmental Kuznets curve for various pollutants find different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, a monotonically increasing or monotonically decreasing relationship - between pollution and rising per capita income levels (Tsurumi and Managi, 2010). This study attempts examines interdependence among economic growth, technological change and dynamics of environment. It is an extension of the growth model with environment proposed by Zhang (2012). The paper is organized as follows. Section 2 introduces the basic model with wealth, technology and environment. Section 3 examines dynamic properties of the model and simulates the model, identifying the existence of a unique equilibrium and checking the stability conditions when all the parameters are constant. Section 4 studies effects of time-dependent perturbations in some parameters on the system. Section 5 concludes the study. The appendix proves the analytical results in Section 3.

The basic model

This paper basically follows Zhang's model (Zhang, 2012) except that all the parameters are time-dependent. This implies that the system can describe any exogenous changes over time. The model is more robust. The model is based on the two-sector economic model proposed by Uzawa (1961). We add endogenous environment and environmental sector to the model. The economy consists of one consumer goods, one capital goods and one environmental sectors. Households own capital of the economy and distribute their incomes to consume consumer goods and to save. Exchanges take place in perfectly competitive markets. We assume a homogenous but exogenously changeable population, denoted by $N(t)$. The labor force is distributed among the three sectors under perfect completion in labor market. We select commodity to serve as numeraire (whose price is normalized to 1), with all the other prices being measured relative to its price.

The production sectors

In the literature of environmental economics, pollution may affect productivity through the channel that pollution directly affects production technology or the productivity of any input. Let subscript index, i and s, stand for capital goods and consumer goods sectors respectively. We assume that production is to combine knowledge, $Z(t)$, labor force, $N_j(t)$, and physical capital, $K_j(t)$. We add environmental impact to the conventional production function. The production functions are specified as follows

$$F_j(t) = A_j(t)\Gamma_j(E)Z_j^{m_j(t)}(t)\, K_j^{\alpha_j(t)}(t)N_j^{\beta_j(t)}(t),$$
$$A_j(t),\, m_j(t),\, \alpha_j(t),\, \beta_j(t) > 0, \quad \alpha_j(t) + \beta_j(t) = 1, \quad j = i,\, s, \tag{1}$$

where $F_j(t)$ is the output level of sector j at time t, $\Gamma_j(E)$ is a function of the environmental quality measured by the level of pollution, $E(t)$, and $A_j(t), \alpha_j(t)$ and $\beta_j(t)$ are parameters. It is reasonable to assume that productivity is negatively related to the pollution level, i.e., $\Gamma_j(E) \le 0$. Here, we interpret $Z^{m_j(t)/\beta_j(t)}(t)$ as sector j's level of human capital. The term $Z^{m_j(t)/\beta_j(t)}(t)$ is sector j's human capital or qualified labor force. We see that the production function is a neoclassical one and homogeneous of degree one with the inputs, $N_j(t)$ and $K_j(t)$. Here, we call $m_j(t)$ sector j's knowledge utilization efficiency parameter. It should be noted that production functions with environment as a determinant of output are widely used in the literature of economic growth (e.g., Lucas, 1988; Ikefuji and Horri, 2012; and Kollenbach, 2105).

Markets are competitive; thus labor and capital earn their marginal products. The rate of interest, $r(t)$, and wage rate, $w(t)$, are determined by markets. Let $\tau_j(t)$ stand for the tax rate on sector j, $0 < \tau_j(t) < 1$, $j = i, s$. We don't fix the tax rate as the government may vary over time. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i(t)\bar{\tau}_i(t)F_i(t)}{K_i(t)} = \frac{\alpha_s(t)\bar{\tau}_s(t)p(t)F_s(t)}{K_s(t)},$$
$$w(t) = \frac{\beta_i(t)\bar{\tau}_i(t)F_i(t)}{N_i(t)} = \frac{\beta_s(t)\bar{\tau}_s(t)p(t)F_s(t)}{N_s(t)}, \quad (2)$$

where δ_k is the given depreciation rate of physical capital and $\bar{\tau}_j(t) \equiv 1 - \tau_j(t)$.

Consumer behaviors

Consumers make decisions on choice of consumption levels as well as on how much to save. $k(t)$, where $k(t) \equiv K(t)/N(t)$, stand for per capita wealth. We use $\tau_k(t)$ and $\tau_w(t)$ to stand for the fixed tax rates on, respectively, the wealth income and wage. Per capita current income from the interest payment $r(t)k(t)$ and the wage payment $w(t)$ are given by

$$y(t) = \bar{\tau}_k(t)r(t)k(t) + \bar{\tau}_w(t)w(t),$$

where $\bar{\tau}_k(t) \equiv 1 - \tau_k(t)$ and $\bar{\tau}_w(t) \equiv 1 - \tau_w(t)$. We call $y(t)$ the current income. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + k(t). \quad (3)$$

The disposable income is used for saving and consumption. The consumer distributes the total available budget between saving, $s(t)$, and consumption of services, $c(t)$. We use $\tau_c(t)$ to stand for a fixed tax rate on consumption. The budget constraint is

$$(1 + \tau_c(t))p(t)c(t) + s(t) = \hat{y}(t). \quad (4)$$

One might take account of consumers' awareness of environment. As Selden and Song (1995) show, at a lower level of pollution, the representative agent does not care much about environment and spends his resource on consumption; however, as the environment becomes worse and income becomes higher, more capital will be used for environmental improvement. There are many other important factors in describing consumers' behavior with endogenous environment (Bovenberg and Smulders, 1996; William, 2002, 2003; Mariani, 2010; Zhang *et al*., 2011; Pautrel, 2012; and Wang *et al*., 2015). At each point of time, consumers have two variables, $s(t)$ and $c(t)$, to decide. We assume that consumers' utility function is a function of $s(t)$, $c(t)$, and $E(t)$. For simplicity of analysis, we specify the utility function as follows

$$U(t) = c^{\xi_0(t)}(t)s^{\lambda_0(t)}(t)E^{-\chi_0(t)}(t), \quad \xi_0(t), \lambda_0(t), \chi_0(t) > 0,$$

where $\xi_0(t)$ is called the propensity to consume, and $\lambda_0(t)$ the propensity to own wealth. According to Balcao (2001) and Nakada (2004), utility depends negatively on pollution, which is a side product of the production process. Munro (2009: 43) argues that "environmental economics has been slow to incorporate the full nature of the household into its analytical structures. … [A]n accurate understanding household behavior is vital for environmental economics." Maximizing $U(t)$ subject to budget constraint (4) yields

$$p(t)c(t) = \xi(t)\hat{y}(t), \quad s(t) = \lambda(t)\hat{y}(t), \tag{5}$$

where

$$\xi(t) \equiv \frac{\rho(t)\xi_0(t)}{1 + \tau_c(t)}, \quad \lambda(t) \equiv \rho(t)\lambda_0(t), \quad \rho(t) \equiv \frac{1}{\xi_0(t) + \lambda_0(t)}.$$

The change in the household's wealth is given by

$$\dot{k}(t) = s(t) - k(t) - k(t)\frac{\dot{N}(t)}{N(t)}. \tag{6}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

Demand and supply balances

The demand for services equals supply of services

$$c(t)N(t) = F_s(t). \tag{7}$$

We assume full employment of labor and capital. That is

$$K_i(t) + K_s(t) + K_e(t) = K(t), \quad N_i(t) + N_s(t) + N_e(t) = N(t). \tag{8}$$

Environmental policy and environmental change

We consider that both production and consumption pollute environment. The dynamics of the stock of pollutants are described as follows

$$\dot{E}(t) = \theta_f(t)F_i(t) + \theta_c(t)C(t) - Q_e(t) - \theta_0(t)E(t), \tag{9}$$

in which $\theta_f(t)$, $\theta_c(t)$ and $\theta_0(t)$ are positive parameters and

$$Q_e(t) = A_e(t)\Gamma_e(E)Z^{m_e(t)}(t)K_e^{\alpha_e(t)}(t)N_e^{\beta_e(t)}(t), \quad A_e(t), \alpha_e(t), \beta_e(t), m_e(t) > 0, \tag{10}$$

where $N_e(t)$ and $K_e(t)$ are respectively the labor force and capital stocks employed by the environmental sector, $A_e(t)$, $\alpha_e(t)$, and $\beta_e(t)$ are positive parameters, and $\Gamma_e(E)$ (≥ 0) is a function of $E(t)$. We use $\theta_f(t)F(t)$ to describe the assumption that pollutants emitted during production processes are linearly positively proportional to the output level (Gutiérrez, 2008). We use $\theta_c(t)$ to reflect that in consuming one unit of the good the quantity $\theta_c(t)$ is left as waste (Prieur, 2009). The parameter $\theta_0(t)$ is called the rate of natural purification. The term $\theta_0(t)E(t)$ measures the rate that the nature purifies environment. The term, $K_e^{\alpha_e}(t)N_e^{\beta_e}(t)$, in $Q_e(t)$ means that the purification rate of environment is positively related to capital and labor inputs. The function, $\Gamma_e(E)$, implies that the purification efficiency is dependent on the stock of pollutants. For simplicity, we specify Γ_e as follows $\Gamma_e(E) = \theta_e(t)E^{\upsilon(t)}$, where $\theta_e(t) > 0$ and $\upsilon(t) > 0$ are parameters.

The environmental sector

The environmental sector determines the input factors. All the tax incomes are assumed for protecting environment. The total tax income is the sum of all the tax incomes on consumption and production. That is

$$Y_e(t) = \tau_i F_i(t) + \tau_s p(t)F_s(t) + \tau_c p(t)c_s(t)N_0 + \tau_k r(t)k(t)N_0 + \tau_w w(t)T(t)N_0. \tag{11}$$

Ono (2003) introduces tax on the producer and uses the tax income for environmental improvement in the traditional neoclassical growth theory. Dam and Heijdra (2011) examine how various environmental policies may affect behavior of households and firms within the traditional Ramsey growth model (Yang *et al.* 2008; Jackson *et al.* 2008; and Tsurumi and Managi, 2010). As there are only two input factors in the environmental sector, the environmental sector's budget

$$\left(r(t)+\delta_k\right)K_e(t)+w(t)N_e(t)=Y_e(t). \tag{12}$$

It is assumed that the environmental sector employs labor and capital in such a way that the purification rate achieves its maximum under the given budget constraint. The optimal problem is

$$\text{Max}\, Q_e(t) \quad \text{s.t.:}\ \left(r(t)+\delta_k\right)K_e(t)+w(t)N_e(t)=Y_e(t).$$

The optimal solution is given by

$$\left(r(t)+\delta_k\right)K_e(t)=\alpha(t)Y_e(t), \quad w(t)N_e(t)=\beta(t)Y_e(t), \tag{13}$$

where

$$\alpha(t)\equiv\frac{\alpha_e(t)}{\alpha_e(t)+\beta_e(t)}, \quad \beta(t)\equiv\frac{\beta_e(t)}{\alpha_e(t)+\beta_e(t)}.$$

Knowledge accumulation

This study uses knowledge in a highly aggregated sense. We assume that knowledge growth is through the so-called learning by doing (see also, Bovenberg and Smulders, 1995; Chakravorty *et al*. 2012; Henriet, 2012; Stafford, 2015). Following Arrow's learning-by-doing (Zhang, 2012), we propose the following equation for knowledge growth

$$\dot{Z}(t)=\frac{\tilde{\tau}(t)_i\, F_i(t)}{Z^{\varepsilon_i(t)}(t)}+\frac{\tilde{\tau}_s(t)F_s(t)}{Z^{\varepsilon_s(t)}(t)}-\delta_z(t)Z(t),$$

in which $\delta_z(t)\,(\geq 0)$ is the depreciation rate of knowledge, and $\varepsilon_j(t)$, and $\tilde{\tau}_j(t)$ are parameters. The term $\tilde{\tau}_j(t)F/Z^{\varepsilon_j(t)}$ measures the contribution to knowledge accumulation through learning by doing by sector $j's$ production sector.

We have thus built the dynamic model. We now examine dynamics of the model.

The dynamics and its properties

The system is nonlinear and involves many variables. Economists were not able to reveal its behavior even a few decades ago. Before demonstrating behavior of the system, we show that the dynamics of the economic system can be expressed by the three-dimensional differential equations system with $k_i(t)$, $E(t)$, $Z(t)$ and t as the variables. We also provide a computational procedure so that anyone with computer can observe the behavior of the system.

Lemma 1

The economy is governed by the following 3-dimensional differential equations

$$\dot{k}_i(t) = \Lambda_{k_i}(k_i(t),\ E(t),\ Z(t),\ t),$$

$$\dot{E}(t) = \Lambda_e(k_i(t),\ E(t),\ Z(t),\ t),$$

$$\dot{Z}(t) = \Lambda_e(k_i(t),\ E(t),\ Z(t),\ t), \qquad (14)$$

where the functions in (14) are dependent on $k_i(t)$, $E(t)$, $Z(t)$ and t which are given in the appendix. Moreover, all the other variables can be determined as functions of $k_i(t)$, $E(t)$ $Z(t)$ and t at any point in time by the following procedure: $k(t)$ by (A12) → $K(t) = k(t)\,N(t)$ → $k_s(t)$ and $k_e(t)$ by (A1) → $r(t)$, $w(t)$ and $p(t)$ by (A3) → $N_i(t)$, $N_s(t)$ and $N_e(t)$ by (A5) and (A9) → $K_i(t) = k_i(t)N_i(t)$, $K_s(t) = k_s(t)N_s(t)$ and $K_e(t) = k_e(t)N_e(t)$ → $F_i(t)$ and $F_s(t)$ by (1) → $Q_e(t)$ by (10) → $\hat{y}(t)$ by (4) → $c(t)$ and $s(t)$ by (5).

We determine the three variables with a given initial state of the three variables. The lemma allows us to calculate the motion of all the variables over time. As the expressions of the analytical results are tedious, it is difficult to explicitly interpret the results. For illustration we first deal with the case that all the parameters are constant. We specify the parameters as follows

$$b_i = 0.1,\ b_s = 0.05,\ b_e = 0.3,\ N_0 = 5,\ \alpha_i = 0.3,\ \alpha_s = 0.35,\ A_i = 1,\ A_s = 1.1,\ \alpha_e = 0.5,$$
$$\beta_e = 0.3,\ A_e = 0.5,\ \lambda_0 = 0.4,\ \xi_0 = 0.2,\ m_i = 0.4,\ m_s = 0.4,\ m_e = 0.3,\ \delta_z = 0.04,$$
$$\delta_k = 0.05,\ \tau_w = \tau_k = \tau_c = \ \tau_i = \tau_s = 0.05,\ \theta_f = 0.2,\ \theta_c = 0.25,\ \theta_0 = 0.05,\ \tilde{\tau}_i = 0.02,$$
$$\tilde{\tau}_i = 0.02,\ \tilde{\tau}_s = 0.01,\ \varepsilon_i = 0.1,\ \varepsilon_s = 0.2. \qquad (15)$$

The population is constant and equal to 5 units. The total factor productivities of the capital sector and the consumption sector are respectively 1 and 1.1. We specify α_i and α_s respectively with 0.3 and 0.35. The propensities to consume goods and to save are respectively specified at 0.2 and 0.4. The knowledge utilization parameter values of the capital sector, the consumption good sector and the environmental sector are respectively 0.4, 0.4, and 0.4. With $\varepsilon_i = 0.1$ and $\varepsilon_s = 0.2$ we mean that the knowledge creation of the two sectors exhibits decreasing returns to scale. The depreciation rate is specified at 0.05. The tax rates on the output of the capital sector, the output of the consumption good sector, consumption good, the wage rate, and the income from wealth are equally fixed at 5 percent. Under (15), the dynamic system has a unique equilibrium point. The equilibrium values are

$$K = 10.31,\ E = 11.50,\ Z = 1.86,\ F_i = 0.93,\ F_s = 6.30,\ Q_e = 1.20,\ N_i = 0.84,\ N_s = 3.87,$$
$$N_e = 0.29,\ K_i = 1.42,\ K_s = 8.29,\ K_e = 1.92,\ k_i = 1.70,\ k_s = 2.14,\ k_e = 6.63,\ f_i = 1.18,$$
$$f_s = 1.63,\ r = 0.15,\ w = 0.78,\ p = 0.78,\ k = 2.02,\ c = 1.26. \quad (16)$$

The three eigenvalues are -0.25, -0.09, and -0.03. Hence, the dynamic system has a unique stable steady state. With $k_i(0) = 1.5$, $E(0) = 13$, and $Z(0) = 1.5$, we plot the motion of the system as in Figure 1. The length of the simulation period is 100, which is long enough for the system approach its unique equilibrium point. The national capital stock and capital stocks employed by the three sectors rise over time. The current income, the output level of the capital sector, the wage rate and consumption level are all raised. The labor force shifts from the consumption sector to the environmental and capital sectors. The level of pollution is reduced. The price of consumption good falls. The current output of the environmental sector rises slightly and the pollution level falls initially and rises late on. The labor distribution changes slightly over time. The capital intensities of the two production sectors are increased. The rate of interest rises initially and falls later on. The price of consumption good changes slightly over time.

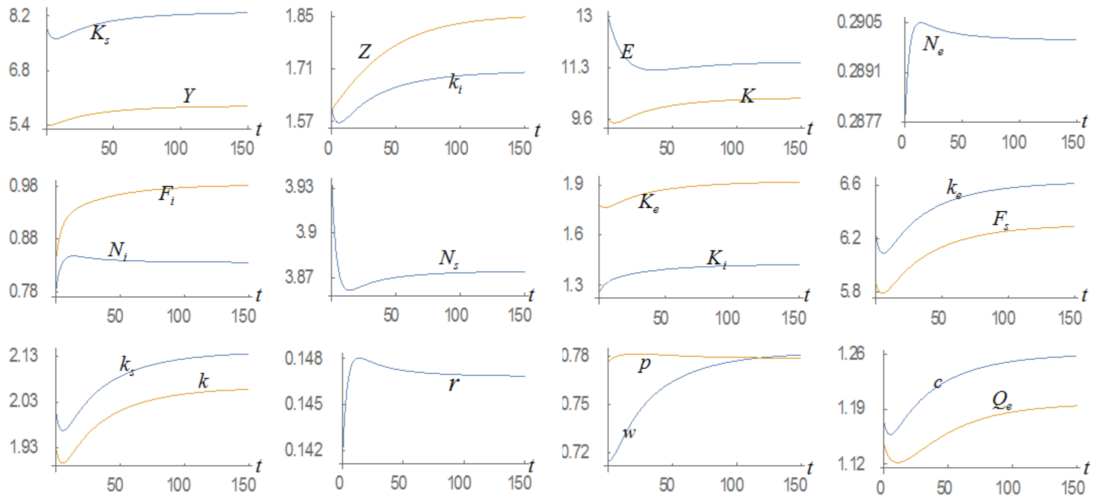

Figure 1. Motion of the Economic System

Comparative dynamic analysis

We now study effects of changes in some parameters on the motion of the economic system. Zhang (2012) shows how the system reacts to a once-for-all change in parameters. This section shows how the system reacts to time-dependent changes in parameters. For convenience we consider the parameters in (16) as the long-term average values. We make small perturbations around these long-term values. In this study we use $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable $x_j(t)$ due to changes in a parameter value.

Perturbations in the propensity to save

We specify perturbations in the propensity to save in the following way:

$$\lambda_0(t) = 0.4 + 0.05\sin(t).$$

We plot the simulation results in Figure 3. As the propensity to save oscillates around its trend value, the variables also show oscillatory behavior. The national output, national wealth, environment and price of consumer goods fluctuate slightly. The labor and capital distributions between the two sectors are oscillate their trend values. The rate of interest also shows large fluctuations due to the perturbations in the propensity to save. As the system is stable, we see that small exogenous perturbations don't lead the system to be far from its trend.

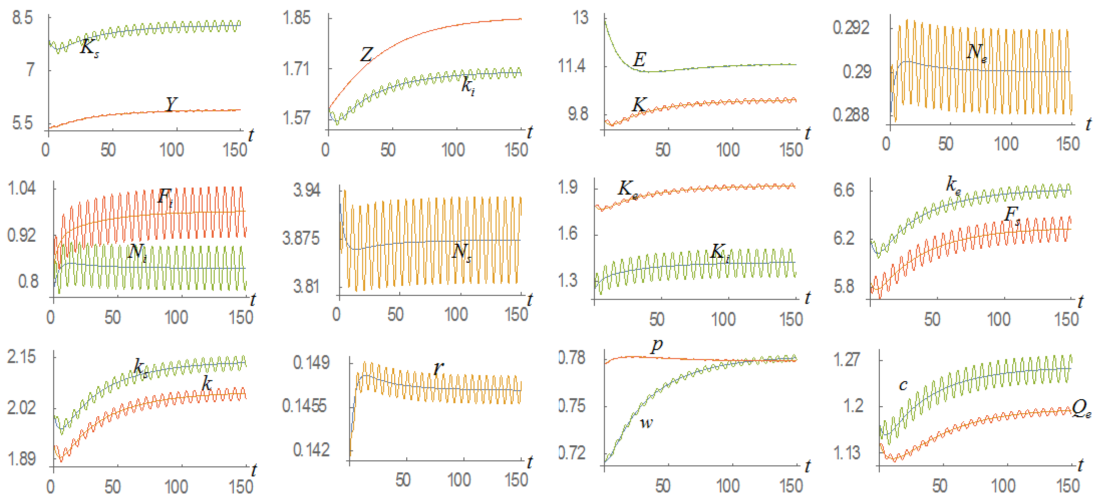

Figure 2. Perturbations in the Propensity to Save

Changes in how strongly consumption pollutes environment

We now study the case that the speed that consumption pollutes the environment is perturbed as follows

$$\theta_c(t) = 0.25 + 0.05\sin(t).$$

The simulation results are demonstrated in Figure 3. The environment is relatively strongly influenced by the fluctuations. As the environment has a strong impact on the capital sector, the output of the sector oscillates. The output of the consumer goods sector slightly change. The labor distribution and environment are sensitive to the exogenous perturbations. The consumption level, price, wage rate, and capital distribution are not strongly affected by the perturbations.

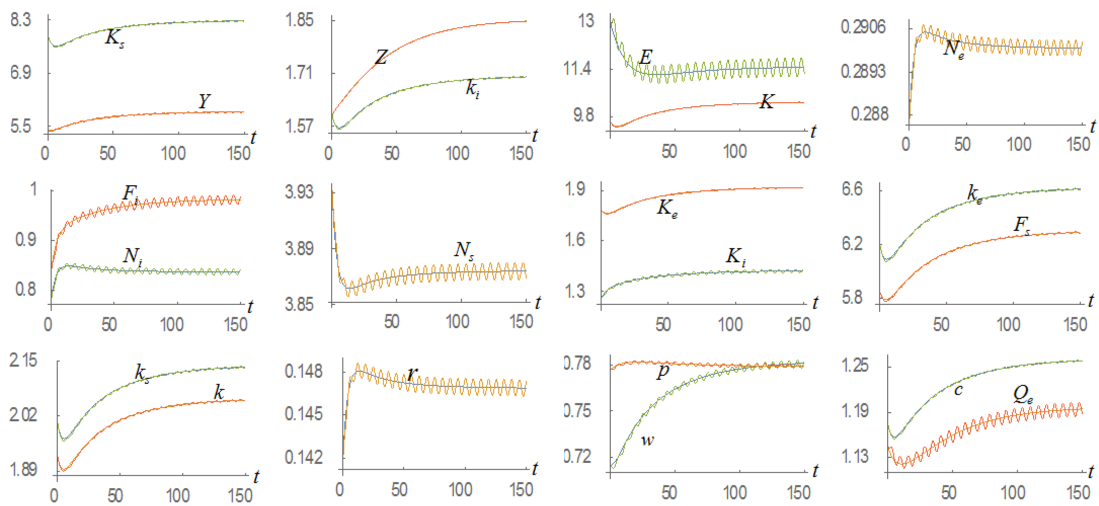

Figure 3. Changes in How Strongly Consumption Pollutes Environment

Fluctuations in the knowledge utilization efficiency parameter

We now examine how the system will react to the following fluctuations in the knowledge utilization efficiency in the capital goods sector

$$m_i(t) = 0.4 + 0.02\sin(t).$$

The simulation results are demonstrated in Figure 4. The environment and knowledge are slightly influenced by the fluctuations. The output level and labor distribution oscillate. The rate of interest, price of consumer goods and wage fluctuate. The consumption and wealth per household fluctuate relatively weak.

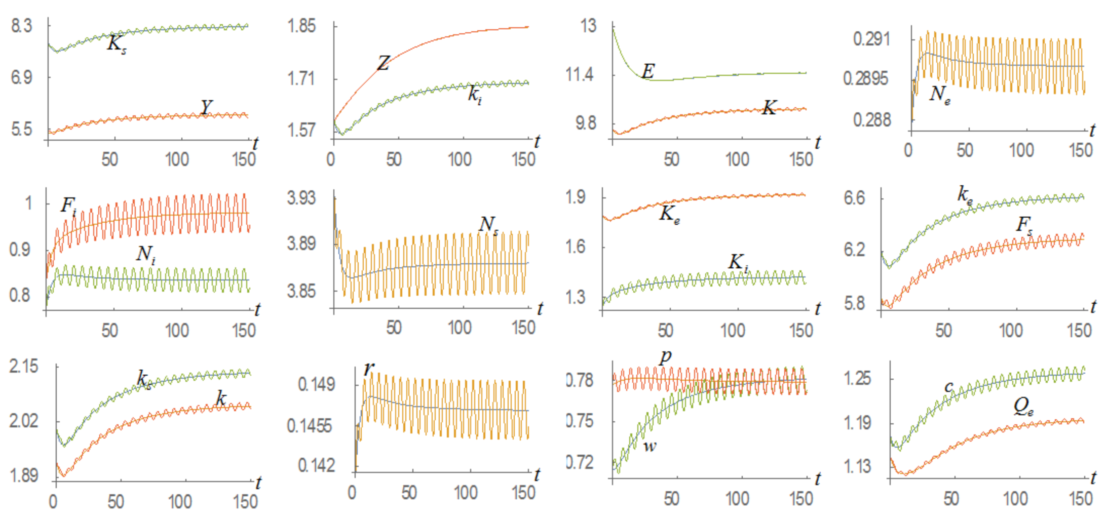

Figure 4. Fluctuations in the Knowledge Utilization Efficiency Parameter

Fluctuations in tax on the capital goods sector

We now examine how the system will react to the following fluctuations in the tax rate on the capital goods sector:

$$\tau_i(t) = 0.05 + 0.01\sin(t).$$

The simulation results are demonstrated in Figure 5. The environment and the output level of the environmental sector is strongly influenced by the fluctuations. The national output level and labor distribution oscillate widely. The rate of interest, price of consumer goods and wage fluctuate. The consumption and wealth per household fluctuate relatively weak.

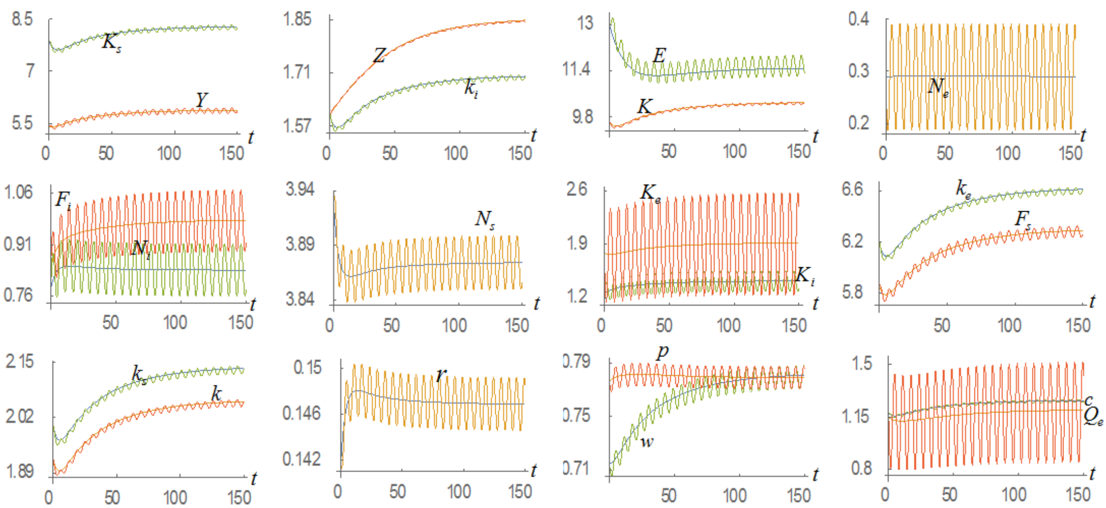

Figure 5. Fluctuations in the Tax Rate on the Capital Goods Sector

Fluctuations in environmental impact on the capital goods sector

We now examine how the system will react to the following fluctuations in the environmental impact on the capital goods sector:

$$b_i(t) = 0.1 + 0.01\sin(t).$$

The simulation results are demonstrated in Figure 6. The environment is slightly influenced by the fluctuations. The national output level, output level of the two sectors, and labor and capital distribution oscillates. The rate of interest, price of consumer goods and wage fluctuate.

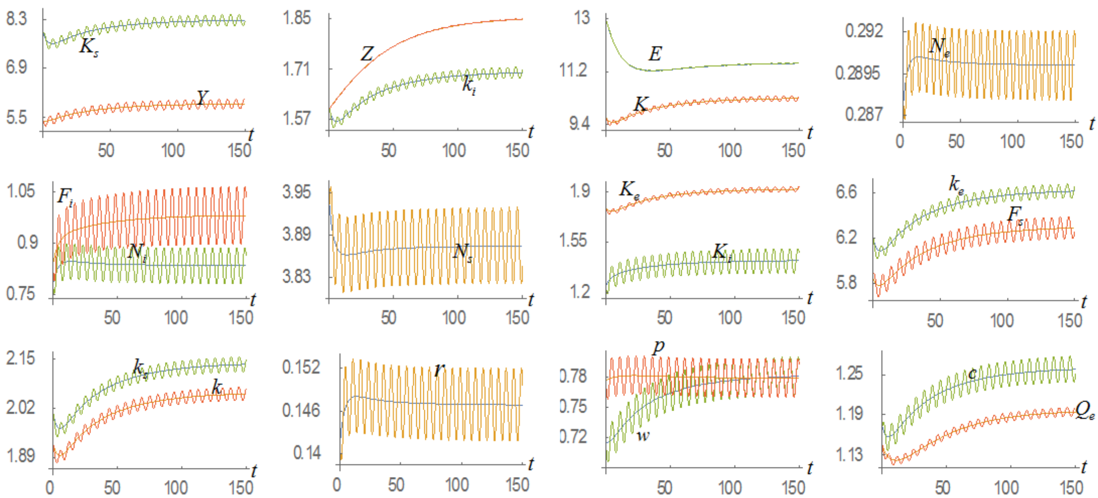

Figure 6. Fluctuations in Environmental Impact on the Capital Goods Sector

Concluding Remarks

This study shows economic oscillations due to periodic changes in some parameters in the economic model proposed by Zhang (2012). The model treats physical capital accumulation, knowledge creation and utilization, and environmental change as endogenous variables. It is based on the neoclassical .growth theory. The model is built by synthesizing the basic ideas of the neoclassical growth theory, Arrow's learning-by-doing model and the traditional dynamic models of environmental change within a comprehensive framework. The behavior of the household is described with an alternative approach to household behavior. We simulated the model to demonstrate existence of equilibrium points, motion of the dynamic system, and oscillations due to different exogenous shocks.

Appendix: Proving Lemma 1

We now prove the lemma. It should be noted that most of the proof are the same as the proof in in Zhang (2012). From (2) and (13), we obtain

$$k_i = \overline{\alpha}_i k_e = \overline{\alpha}_s k_s , \tag{A1}$$

where we omit time index and

$$k_i \equiv \frac{K_i}{N_i}, \;\; k_e \equiv \frac{K_e}{N_e}, \;\; k_s \equiv \frac{K_s}{N_s}, \;\; \overline{\alpha}_e \equiv \frac{\beta_e \, \alpha_i}{\alpha_e \, \beta_i}, \;\; \overline{\alpha}_s \equiv \frac{\beta_s \, \alpha_i}{\alpha_s \, \beta_i}. \tag{A2}$$

Insert (A1) in (2)

$$r = \alpha_i \, \overline{\tau}_i \, A_i \, Z^{m_i} \, \Gamma_i \, k_i^{-\beta_i} - \delta_k \, , \;\; w = \beta_i \, \overline{\tau}_i \, A_i \, Z^{m_i} \, \Gamma_i \, k_i^{\alpha_i} \, , \;\; p = \frac{\overline{\alpha}_s^{\alpha_s} \, w}{\beta_s \, \overline{\tau}_s \, A_s \, Z^{m_s} \, \Gamma_s \, k_i^{\alpha_s}}. \tag{A3}$$

We see that k_e, k_s, r, w and p are functions of k_i, Z, E, and t. From (11) and (7), we have

$$Y_e = \tau_i \, F_i + (\tau_s + \tau_c) \, p \, F_s + \tau_k \, r \, k \, N + \tau_w \, w \, N .$$

Insert $w \, N_e = \beta \, Y_e$ in the above equation

$$N_e = \beta \left[\tau_i \, \frac{F_i}{w} + (\tau_s + \tau_c) \frac{p \, F_s}{w} + \frac{\tau_k \, N \, r \, k}{w} + \tau_w \, N \right]. \tag{A4}$$

From (2) and (A4)

$$N_e = \beta \left[\frac{\tau_i \, N_i}{\beta_i \, \overline{\tau}_i} + \frac{(\tau_s + \tau_c) N_s}{\beta_s \, \overline{\tau}_s} + \frac{\tau_k \, N \, r \, k}{w} + \tau_w \, N \right]. \tag{A5}$$

Insert (A5) in (8)

$$a_1 \, N_i + a_2 \, N_s = (1 - \tau_w \, \beta) N - \phi \, k \, , \tag{A6}$$

where

$$a_1 \equiv 1 + \frac{\beta \, \tau_i}{\beta_i \, \overline{\tau}_i}, \;\; a_2 \equiv 1 + \frac{(\tau_s + \tau_c) \beta}{\beta_s \, \overline{\tau}_s}, \;\; \phi(k_i \, , E \, , Z) \equiv \frac{\tau_k \, \beta \, N \, r}{w}.$$

Insert (A1) and (A2) in $K_i + K_s + K_e = K$

$$N_i + \frac{N_s}{\overline{\alpha}_s} + \frac{N_e}{\overline{\alpha}_e} = \frac{K}{k_i}. \tag{A7}$$

Substitute (A5) into (A7)

$$b_1 N_i + b_2 N_s = \phi_0 k - \tau_w N, \tag{A8}$$

where

$$b_1 \equiv \frac{\overline{\alpha}_e}{\beta} + \frac{\tau_i}{\beta_i \overline{\tau}_i}, \quad b_2 \equiv \frac{\overline{\alpha}_e}{\beta \overline{\alpha}_s} + \frac{\tau_s + \tau_c}{\beta_s \overline{\tau}_s}, \quad \phi_0(k_i, E, Z) \equiv \left(\frac{\overline{\alpha}_e}{\beta k_i} - \frac{\tau_k r}{w} \right) N.$$

From (A7) and (A8), we solve the labor distribution as functions of k, E, Z, k_i, and t as follows

$$N_i = \overline{\theta}_i k - \theta_i, \quad N_s = \theta_s - \overline{\theta}_s k, \tag{A9}$$

where

$$\overline{\theta}_i \equiv (a_1 \phi_0 + b_1 \phi)\theta, \quad \theta_i \equiv [a_1 \tau_w + (1 - \tau_w \beta) b_1] \theta N,$$

$$\theta_s \equiv [(1 - \tau_w \beta) b_2 + a_2 \tau_w] \theta N, \quad \overline{\theta}_s \equiv (b_2 \phi + a_2 \phi_0)\theta, \quad \theta \equiv (a_1 b_2 - a_2 b_1)^{-1}.$$

From $p c = \xi \hat{y}$ and the definition of $\overline{y}$

$$p F_s = (\overline{\tau}_k r + 1) \xi N k + \overline{\tau}_w \xi N w, \tag{A10}$$

where we use $c N = F_s$. Insert (1) in (A10)

$$\frac{w N_s}{\overline{\tau}_s \beta_s} = (\overline{\tau}_k r + 1) \xi N k + \overline{\tau}_w \xi N w. \tag{A11}$$

Substitute N_s in (A9) into (A11)

$$k = \Omega(k_i, E, Z, t) \equiv (\theta_s - \bar{\tau}_w \bar{\tau}_s \beta_s \xi N)\left[\left(\frac{\bar{\tau}_k r + 1}{w}\right)\xi \bar{\tau}_s \beta_s N + \bar{\theta}_s\right]^{-1}. \tag{A12}$$

We can thus express k as a function of $k_i(t)$, $E(t)$, $Z(t)$, and t. From (A2), k_s and k_e are functions of k_i and t. From (A7) and (A9), N_i, N_s and N_e are functions of $k_i(t)$, $E(t)$, $Z(t)$, and t. By the following procedure, we can express other variables as functions of $k_i(t)$, $E(t)$, $Z(t)$, and t at any point in time: r, w and p by (A3) → $\hat{y}$ by (4) → c and s by (5). By these results and from (9) we get the following differential equation

$$\dot{E}(t) = \Lambda_e(k_i(t), E(t), Z(t), t). \tag{A13}$$

Similarly, from the equation for knowledge dynamics, we have

$$\dot{Z}(t) = \Lambda_z(k_i(t), E(t), Z(t), t) \equiv \frac{\tilde{\tau}_i(t) F_i(t)}{Z^{\varepsilon_i}(t)} + \frac{\tilde{\tau}_s(t) F_s(t)}{Z^{\varepsilon_s}(t)} - \delta_z Z(t). \tag{A14}$$

Taking derivatives of (A12) with respect to t yields

$$\dot{k} = \frac{\partial \Omega}{\partial k_i}\dot{k}_i + \Lambda_e \frac{\partial \Omega}{\partial E} + \Lambda_z \frac{\partial \Omega}{\partial Z} + \frac{\partial \Omega}{\partial t}, \tag{A15}$$

where we also use (A13) and (A14). From (5) and (6), we have

$$\dot{k} = \lambda \hat{y} - k - k\frac{\dot{N}}{N}. \tag{A16}$$

Insert (A15) and (A12) in (A16)

$$\dot{k}_i = \Lambda_{k_i}(k_i(t), E(t), Z(t), t) \equiv \left[\lambda \hat{y} - k - k\frac{\dot{N}}{N} - \Lambda_e \frac{\partial \Omega}{\partial E} - \Lambda_z \frac{\partial \Omega}{\partial Z} - \frac{\partial \Omega}{\partial t}\right]\left(\frac{\partial \Omega}{\partial k_i}\right)^{-1}. \tag{A17}$$

We have thus proved Lemma 1.

References

1. Aghion, P. and Howitt, P. (1998) Endogenous Growth Theory. Mass., Cambridge: The MIT Press.
2. Arrow, K.J. (1962) The Economic Implications of Learning by Doing. Review of Economic Studies 29, 155-173.
3. Ayong Le Kama, A.D. (2001) Sustainable Growth, Renewable Resources and Pollution. Journal of Economic Dynamics & Control 25, 1911-18.
4. Azariadis, C. (1993) Intertemporal Macroeconomics. Oxford: Blackwell.
5. Barro, R.J. and Sala-i-Martin, X. (1995) Economic Growth. New York: McGraw-Hill, Inc.
6. Balcao, A. (2001) Endogenous Growth and the Possibility of Eliminating Pollution, Journal of Environmental Economics and Management 42, 360-73.
7. Bovenberg, A.L. and Heijdra, B.J. (1998) Environmental Tax Policy and Intergenerational Distribution. Journal of Public Economics 67, 1-24.
8. Bovenberg, A.L. and Smulders, S. (1995) Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model. Journal of Public Economics 57, 369–91.
9. Bovenberg, A.L. and Smulders, S. (1996) Transitional Impacts of Environmental Policy in an Endogenous Growth Model. International Economic Review 37 (4), 861–93.
10. Burmeister, E. and Dobell, A.R. (1970) Mathematical Theories of Economic Growth. London: Collier Macmillan Publishers.
11. Chakravorty, U., Leach, A., and Moreaux, M. (2012) Cycles in Nonrenewable Resource Prices with Pollution and Learning-By-Doing. Journal of Economic Dynamics and Control 36 (10), 1448–61.
12. Chatterjee, B. S. and Ravikumar, B. (1992) A Neoclassical Model of Seasonal Fluctuations. Journal of Monetary Economics 29 (1), 59–86.
13. Copeland, B. and Taylor, S. (2004) Trade, Growth and the Environment. Journal of Economic Literature 42, 7-71.
14. Dam, L. and Heijdra, B.J. (2011) The Environmental and Macroeconomic Effects of Socially Responsible Investment. Journal of Economic Dynamics & Control (forthcoming).
15. Dasgupta, S., Hamilton, K., Pandey, K.D., Wheeler, D. (2006) Environment during Growth: Accounting for Governance and Vulnerability. World Development 34, 1597-611.
16. Gabaix, X. (2011) The Granular Origins of Aggregate Fluctuations. Econometrica 79, 733–72.

17. Giovanni, J. Di., Levchenko, A., and Mejean, I. (2014) Firms, Destinations, and Aggregate Fluctuations. Econometrica 82 (4), 1303–40.
18. Grossman, G.M. and Helpman, E. (1991) Innovation and Growth in the Global Economy. Mass., Cambridge: The MIT Press.
19. Gutiérrez, M. (2008) Dynamic Inefficiency in an Overlapping Generation Economy with Pollution and Health Costs. Journal of Public Economic Theory 10, 563-94.
20. Henriet, F. (2112) Optimal Extraction of a Polluting Nonrenewable Resource with R&D toward a Clean Backstop Technology. Journal of Public Economics 14 (2), 311–47.
21. Ikefujia, M. and Horiib, R. (2012) Natural Disasters in a Two-Sector Model of Endogenous Growth. Journal of Public Economics 96 (9), 784-96.
22. Jackson, T., Illsley, B., Curry, J., and Rapaport, E. (2008) Amenity Migration and Sustainable Development in Remote Resource-Based Communities: Lessons from Northern British Columbia. International Journal of Society Systems Science 1, 26-48.
23. Kijima, M., Nishide, K. and Ohyama, A. (2010) Economic Models for the Environmental Kuznets Curve: A Survey. Journal of Economic Dynamics & Control 34, 1187-201.
24. Kollenbach, G. (2105) Abatement, R&D and Growth with a Pollution Ceiling. Journal of Economic Dynamics and Control 54, 1-16.
25. Kuznets, S. (1955) Economic Growth and Economic Inequality. American Economic Review 45, 1-28.
26. Lucas, R.E. (1977) Understanding Business Cycles. Carnegie-Rochester Conference Series on Public Policy 5, 7-29.
27. Lucas, R.E. (1988) On the Mechanics of Economic Development. Journal of Monetary Economics 22, 3-42.
28. Mariani, F., Perez-Barahona, A., and Raffin, N. (2010) Life Expectancy and the Environment. Journal of Economic Dynamics and Control 34 (4), 798–815.
29. Munro, A. (2009) Introduction to the Special Issue: Things We Do and Don’t Understand About the Household and the Environment. Environmental and Resources Economics 43, 1-10.
30. Nakada, M. (2004) Does Environmental Policy Necessarily Discourage Growth? Journal of Economics 81, 249-68.
31. Ono, T. (2003) Environmental Tax Policy and Long-Run Economic Growth. The Japanese Economic Review 53, 202-17.
32. Pautrel, X. (2012) Pollution, Private Investment in Healthcare, and Environmental Policy. Scandinavia Journal of Economics 114, 334–57.

33. Pearson, P.J. (1994) Energy, Externalities and Environmental Quality: Will Development Cure the Ills It Creats? Energy Studies Review 6, 199-216.
34. Prieur, F. (2009) The Environmental Kuznets Curve in a World of Irreversibility. Economic Theory 40, 57-90.
35. Romer, P.M. (1986) Increasing Returns and Long-Run Growth. Journal of Political Economy 94, 1002-1037.
36. Selden, T.M. and Song, D. (1995) Neoclassical Growth, the J Curve for Abatement and the Inverted U Curve for Pollution. Journal of Environmental Economics and Management 29, 162-68.
37. Solow, R. (1956) A Contribution to the Theory of Growth. Quarterly Journal of Economics 70, 65-94.
38. Stafford, T.M. (2015) Indoor air quality and academic performance. Journal of Environmental Economics and Management 70, 34-50.
39. Stella, A. (2015) Firm Dynamics and the Origins of Aggregate Fluctuations. Journal of Economic Dynamics and Control (forthcoming).
40. Stern, D.I. (2004) The Rise and Fall of the Environmental Kuznets Curve. World Development 32, 1419-39.
41. Tsurumi, T. and Managi, S. (2010) Decomposition of the Environmental Kuznets Curve: Scale, Technique, and Composition Effects. Environmental Economics and Policy Studies 11, 19-36.
42. Uzawa, H. (1961) On a Two-Sector Model of Economic Growth. Review of Economic Studies 29, 47-70.
43. Wang, M., Zhao, J.H., and Bhattacharyad, J. (2015) Optimal Health and Environmental Policies in a Pollution-Growth Nexus. Journal of Environmental Economics and Management 71, 160–79.
44. William, R.C. (2002) Environmental Tax Interactions When Pollution Affects Health or Productivity. Journal of Environmental Economics and Management 44, 261–70.
45. William, R.C. (2003) Health Effects and Optimal Environmental Taxes. Journal of Public Economics 87, 323–35.
46. Wirl, F. (2004) Sustainable Growth, Renewable Resources and Pollution: Thresholds and Cycles. Journal of Economic Dynamics & Control 28, 1149-57.
47. Yang, F., Ang, S., Liang, L., and Zhai, D.J. (2008) Conditional Eco-Efficiency Measure from the Perspective of Pollution Emission Firms. International Journal of Sustainable Society 3, 276-91.
48. Zhang, J.J., Nie, T.F., and Du, S.F. (2011) Optimal Emission-Dependent Production Policy with Stochastic Demand. International Journal of Society Systems Science 3, 21-39.

49. Zhang, W.B. (1991) Synergetic Economics. Springer-Verlag, Heidelberg.
50. Zhang, W.B. (2005) Differential Equations, Bifurcations, and Chaos in Economics. Singapore: World Scientific.
51. Zhang, W.B. (2006) Discrete Dynamical Systems, Bifurcations and Chaos in Economics. Elsevier: Amsterdam.
52. Zhang, W.B. (2012) Capital Accumulation, Technological Progress and Environmental Change in a Three-Sector Growth Model, International Journal of Information Systems and Social Change 3 (3), 1-18.