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A WORKING MATHEMATICAL MODEL OF AN PTC THERMISTOR

Abstract: *A mathematical model of a positive temperature coefficient thermistor was obtained using a unified approach to building a working mathematical model. This mathematical model has sufficient properties of fullness, accuracy, adequacy, productivity and economy for the purposes of this study. Applying such a model reduces the costs and time spent on research and makes efficient use of the mathematical modelling capabilities.*

Key words: *PTC thermistor, working mathematical model, properties of mathematical models, principles of mathematical modeling.*

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Introduction

Vast educational and scientific literature is devoted to the technical characteristics of positive temperature coefficient thermistors, basic principles of their operation and methods of circuit design using these thermistors. There are numerous examples of successful practical use of such devices in various fields.

The aim of this study is to build a working mathematical model of a positive temperature coefficient thermistor using a unified approach.

The dependence of the resistance R of such a thermistor on its temperature T is not linear over a broad temperature range (for an example, see [1; 2]). In a relatively narrow temperature range, however, it can be assumed that

$$R(T) = r\left[1 + \beta(T - T_0)\right],$$

where r is the thermistor resistance at $T = T_0$; β is a positive constant.

A unified approach to building a working mathematical model that has necessary properties for a specific study is described in [3; 4]. Some properties of mathematical models are formulated, for instance, in [5; 6]. An example of building a mathematical model with the necessary properties for a study is presented in [7]; some of the results of this study were published in [8–10]. The particular features of using a unified approach to building mathematical models are described, for example, in [11; 12].

Statement of the problem

The thermistor is considered to be a body with high thermal conductivity, i.e. the dependence of the temperature of the body on the spatial coordinates at any time point is disregarded. Its temperature T at the initial time point t_0 equals T_0, while $T \le T_1$. Convective heat exchange with the environment occurs on the thermistor surface with area S. The ambient temperature is equal to T_0, and the heat transfer coefficient is known and equal to α. For a relatively narrow temperature range from T_0 to T_1, let us assume that

$$R(T) = r\left[1 + \beta(T - T_0)\right],$$

$$C(T)=c\left[1+\gamma\left(T-T_0\right)\right],$$

where $R(T)$ and $C(T)$ are the resistance and total heat capacity of the thermistor; r and c are the resistance and total heat capacity of the thermistor at $T=T_0$; β and γ are positive constants. An electric current flows through the thermistor; its strength equals

$$I=\frac{U}{r\left[1+\beta\left(T-T_0\right)\right]}, \qquad (1)$$

where U is the constant electrical potential difference between the poles of the thermistor.

Let I be the value of interest in the study. Let us design a working mathematical model of the object of study that has sufficient properties of fullness, adequacy, productivity and economy.

Solution

To solve the problem, we need to build a hierarchy of mathematical models for this object of study and determine the conditions under which we can calculate the sought value I with a relative error not exceeding δ_0.

If the difference $T-T_0$ is sufficiently small, then, according to (1), the sought value can be calculated using the formula

$$I_0=\frac{U}{r}. \qquad (2)$$

Let us define the conditions under which the resulting formula is applicable. To do this, let us consider steady-state heat transfer. In this case, the heat output of the thermistor's material is equal to the heat flow from the thermistor, i.e.

$$\frac{U^2}{R(T_*)}=\alpha\left(T_*-T_0\right)S,$$

where T_* is the steady-state thermistor temperature. The resulting equality allows us to easily calculate

$$T_*=T_0+\frac{1}{2\beta}\left(-1+\sqrt{1+\frac{4\beta U^2}{\alpha S r}}\right),$$

and then find the sought steady-state value

$$I_*=\frac{2U}{r\left[1+\sqrt{1+4\beta U^2\alpha^{-1}S^{-1}r^{-1}}\right]}, \qquad (3)$$

and for this temperature range

$$\frac{U^2}{\alpha S r\left(T_1-T_0\right)}\le 1+\beta\left(T_1-T_0\right). \qquad (4)$$

The relative error of I_0 is

$$\delta\left(I_0\right)=\left|\frac{I-I_0}{I}\right|=\frac{I_0}{I}-1\le\frac{I_0}{I_*}-1.$$

If the condition

$$\frac{I_0}{I_*}-1\le\delta_0$$

is met, formula (2) may be used to find the sought value with a relative error not exceeding δ_0. Therefore, when the inequality

$$I_0\le\left(1+\delta_0\right)I_* \qquad (5)$$

is satisfied, mathematical model (2) sufficiently possesses the properties of fullness, accuracy, adequacy, productivity and economy.

Then let us define the conditions under which mathematical model (3) is applicable. To do this, we need to consider unsteady-state heat transfer. In this case, the change in thermistor temperature over time t is described by a first-order ordinary differential equation

$$C(T)\frac{dT}{dt}=\frac{U^2}{R(T)}-\alpha\left(T-T_0\right)S,$$

and the initial condition is as follows:

$$T\left(t_0\right)=T_0.$$

Given that

$$I=\frac{I_0}{1+\beta\left(T-T_0\right)},$$

let us formulate a Cauchy problem

$$\frac{dI}{dt}=\frac{\beta I^2}{cI_0}\frac{\alpha S\left(I_0-I\right)-\beta U I^2}{\gamma\left(I_0-I\right)+\beta I}, \qquad (6)$$

$$I\left(t_0\right)=I_0.$$

Then let us calculate the time point

$$t_*=t_0+\frac{c}{\alpha S}\left[\frac{\gamma}{\beta}\left(\frac{I_*}{I_0}-1+\delta_0\right)\frac{I_0}{I_*}+\left(\frac{I_0}{2I_0-I_*}+\frac{\gamma}{\beta}\frac{I_0-I_*}{2I_0-I_*}\frac{I_0}{I_*}-1\right)\times\ln\left(2-\frac{I_*}{I_0}-\delta_0\right)-\left(\frac{I_0}{2I_0-I_*}+\frac{\gamma}{\beta}\frac{I_0-I_*}{2I_0-I_*}\frac{I_0}{I_*}\right)\ln\left(\frac{I_0}{I_0-I_*}\delta_0\right)\right],$$

for which

$$I\left(t_*\right)=\frac{I_*}{1-\delta_0}.$$

Evidently, at $t\ge t_*$

$$\delta\left(I_*\right)=\left|\frac{I-I_*}{I}\right|=1-\frac{I_*}{I}\le\delta_0,$$

and the value I_* can be considered equal to $I(t)$ with a relative error not exceeding δ_0. Therefore, it is possible to use formula (3) to find the sought value with a relative error not exceeding δ_0.

If condition (5) is not met, mathematical model (3) at $t\ge t_*$ sufficiently possesses the properties of fullness, adequacy, productivity and economy.

Building a new mathematical model when creating a hierarchy of mathematical models for the object of study may lead to refining the previously determined conditions for the applicability of the constructed mathematical models. Indeed, using mathematical model (6), we can refine the condition of applicability for formula (2). For this let us calculate the time point

$$t^* = t_0 + \frac{c}{\alpha S}\left[\left(\frac{\gamma}{\beta}\frac{I_0 - I_*}{2I_0 - I_*}\frac{I_0}{I_*} + \frac{I_0}{2I_0 - I_*} - 1\right)\ln\left(1 + \frac{I_*}{I_0}\delta_0\right) - \left(\frac{I_0}{2I_0 - I_*} + \frac{\gamma}{\beta}\frac{I_0 - I_*}{2I_0 - I_*}\frac{I_0}{I_*}\right) \times \ln\left(1 - \frac{I_*}{I_0 - I_*}\delta_0\right) - \frac{\gamma}{\beta}\delta_0\right],$$

for which

$$I\left(t^*\right) = \frac{I_0}{1+\delta_0}.$$

Evidently, at $t \le t^*$

$$\delta\left(I_0\right) = \left|\frac{I - I_0}{I}\right| = \frac{I_0}{I} - 1 \le \delta_0,$$

and the value I_0 can be considered equal to $I(t)$ with a relative error not exceeding δ_0. Therefore, it is possible to use formula (2) to find the sought value with a relative error not exceeding δ_0.

If condition (5) is met or $t \le t^*$, mathematical model (2) sufficiently possesses the properties of fullness, adequacy, productivity and economy.

Results

When inequality (4) is satisfied, the following statements are true; they allow us to identify a working mathematical model of the object of study.

If condition (5) is met, or $t \le t^*$ within the scope of the study, then mathematical model (2) is considered the working mathematical model.

If condition (5) is not satisfied, then the mathematical model (3) at $t \ge t_*$ is chosen as the working mathematical model.

If inequality (5) is not satisfied, and the time interval from t^* to t_* is of interest, then mathematical model (6) is considered the working mathematical model.

Conclusion

Thus, a unified approach was used to formulate statements applicable to this study. They allow us to define a working mathematical model of a positive temperature coefficient thermistor. This mathematical model sufficiently possesses the properties of fullness, adequacy, productivity and economy.

It is evident that the use of such a mathematical model not only reduces the costs and time spent on research, but also makes efficient use of the mathematical modelling capabilities.

References:

1. Sze, S. M., & Ng, K. K. (2006). *Physics of Semiconductor Devices*. Hoboken, New Jersey: John Wiley & Sons.
2. Macklen, E. D. (1979). *Thermistors*. Ayr: Electrochemical Publications Ltd.
3. Markelov, G. E. (2015). On Approach to Constructing a Working Mathematical Model. *ISJ Theoretical & Applied Science, 04 (24)*, 287–290. Soi: http://s-o-i.org/1.1/TAS*04(24)52 Doi: http://dx.doi.org/10.15863/TAS.2015.04.24.52
4. Markelov, G. E. (2015). Constructing a Working Mathematical Model. *ISJ Theoretical & Applied Science, 08 (28)*, 44–46. Soi: http://s-o-i.org/1.1/TAS-08-28-6 Doi: http://dx.doi.org/10.15863/TAS.2015.08.28.6
5. Myshkis, A. D. (2011). *Elements of the Theory of Mathematical Models* [in Russian]. Moscow: URSS.
6. Zarubin, V. S. (2010). *Mathematical Modeling in Engineering* [in Russian]. Moscow: Izd-vo MGTU im. N. E. Baumana.
7. Markelov, G. E. (2012). Peculiarities of Construction of Mathematical Models. *Inzhenernyi zhurnal: nauka i innovatsii, No. 4*, http://engjournal.ru/catalog/mathmodel/hidden/150.html
8. Markelov, G. E. (2000). Effect of initial heating of the jet-forming layer of shaped-charge liners on the ultimate elongation of jet elements. *J. Appl. Mech. and Tech. Phys., 41, No. 2*, 231–234.
9. Markelov, G. E. (2000). Effect of initial heating of shaped charge liners on shaped charge penetration. *J. Appl. Mech. and Tech. Phys., 41, No. 5*, 788–791.
10. Markelov, G. E. (2000). *Influence of heating temperature on the ultimate elongation of shaped-*

charge jet elements. Proc. of the 5th Int. Conf. “Lavrentyev Readings on Mathematics, Mechanics and Physics”. (p. 170). Novosibirsk: Lavrentyev Institute of Hydrodynamics.

11. Markelov, G. E. (2015). Particular Aspects of Teaching the Fundamentals of Mathematical Modeling. *ISJ Theoretical & Applied Science, 05 (25),* 69–72. Soi: http://s-o-i.org/1.1/TAS*05(25)14 Doi: http://dx.doi.org/10.15863/TAS.2015.05.25.14
12. Markelov, G. E. (2016). Teaching the Basics of Mathematical Modeling. Part 2. *ISJ Theoretical & Applied Science, 01 (33),* 72–74. Soi: http://s-o-i.org/1.1/TAS-01-33-15 Doi: http://dx.doi.org/10.15863/TAS.2016.01.33.15