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A MATHEMATICAL MODEL OF AN NTC THERMISTOR

***Abstract**: A mathematical model of a negative temperature coefficient thermistor was obtained using a unified approach to building a working mathematical model. This mathematical model has sufficient properties of fullness, accuracy, adequacy, productivity and economy for the purposes of this study. Applying such a model reduces the costs and time spent on research and makes efficient use of the mathematical modelling capabilities.*

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Introduction

Vast educational and scientific literature is devoted to the technical characteristics of negative temperature coefficient thermistors, basic principles of their operation and methods of circuit design using these thermistors. There are numerous examples of successful practical use of such devices in various fields.

The aim of this study is to build a working mathematical model of a negative temperature coefficient thermistor using a unified approach.

The dependence of the resistance R of such a thermistor on its temperature T is usually described by an expression (for an example, see [1]) which looks like this

$$R(T) = r\exp\left[\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)\right],$$

where r is the resistance of the thermistor at $T = T_0$; β is a coefficient that is constant for this specific thermistor. In a relatively narrow temperature range, however, it can be assumed that

$$R(T) = \frac{r}{1+\beta(T-T_0)T_0^{-2}}.$$

A unified approach to building a working mathematical model that has necessary properties for a specific study is described in [2; 3]. Some properties of mathematical models are formulated, for instance, in [4; 5]. An example of building a mathematical model with the necessary properties for a study is presented in [6]; some of the results of this study were published in [7–9]. The particular features of using a unified approach to building mathematical models are described, for example, in [10; 11].

Statement of the problem

Let the thermistor be a highly thermoconductive body. Its temperature T at the initial time point t_0 equals T_0, while $T \le T_1$. Convective heat exchange with the environment occurs on the thermistor surface with area S. The ambient temperature is equal to T_0, and the heat transfer coefficient is known and equal to α. For a relatively narrow temperature range from T_0 to T_1, let us assume that

$$R(T) = \frac{r}{1+\beta(T-T_0)T_0^{-2}},$$

$$C(T)=c\left[1+\gamma\left(T-T_0\right)\right],$$

where $R(T)$ and $C(T)$ are the resistance and total heat capacity of the thermistor; r and c are the resistance and total heat capacity of the thermistor at $T=T_0$; β and γ are positive constants. The electrical potential difference between the poles of the thermistor equals

$$U=\frac{rI}{1+\beta\left(T-T_0\right)T_0^{-2}}, \qquad (1)$$

where I is the strength of the direct electric current flowing through the thermistor.

Let U be the value of interest in the study. Let us design a working mathematical model of the object of study that has sufficient properties of fullness, adequacy, productivity and economy.

Solution

To solve the problem, we need to build a hierarchy of mathematical models for this object of study and determine the conditions under which we can calculate the sought value U with a relative error not exceeding δ_0.

If the difference $T-T_0$ is sufficiently small, then, according to (1), the sought value can be calculated using the formula

$$U_0=rI. \qquad (2)$$

Let us define the conditions under which the resulting formula is applicable. To do this, let us consider steady-state heat transfer. In this case, the heat output of the thermistor's material is equal to the heat flow from the thermistor, i.e.

$$R(T_*)I^2=\alpha\left(T_*-T_0\right)S,$$

where T_* is the steady-state thermistor temperature. The resulting equality allows us to easily calculate

$$T_*=T_0+\frac{T_0^2}{2\beta}\left(-1+\sqrt{1+\frac{4\beta rI^2}{\alpha ST_0^2}}\right),$$

and then find the sought steady-state value

$$U_*=\frac{2rI}{1+\sqrt{1+4\beta rI^2\alpha^{-1}S^{-1}T_0^{-2}}}, \qquad (3)$$

and for this temperature range

$$\frac{rI^2}{\alpha S\left(T_1-T_0\right)}\le 1+\beta\left(T_1-T_0\right)T_0^{-2}. \qquad (4)$$

The relative error of U_0 is

$$\delta\left(U_0\right)=\left|\frac{U-U_0}{U}\right|=\frac{U_0}{U}-1\le\frac{U_0}{U_*}-1.$$

If the condition

$$\frac{U_0}{U_*}-1\le\delta_0$$

is met, formula (2) may be used to find the sought value with a relative error not exceeding δ_0. Therefore, when the inequality

$$U_0\le\left(1+\delta_0\right)U_* \qquad (5)$$

is satisfied, mathematical model (2) sufficiently possesses the properties of fullness, accuracy, adequacy, productivity and economy.

Then let us define the conditions under which mathematical model (3) is applicable. To do this, we need to consider unsteady-state heat transfer. In this case, the change in thermistor temperature over time t is described by a first-order ordinary differential equation

$$C(T)\frac{dT}{dt}=R(T)I^2-\alpha\left(T-T_0\right)S,$$

and the initial condition is as follows:

$$T\left(t_0\right)=T_0.$$

Given that

$$U=\frac{U_0}{1+\beta\left(T-T_0\right)T_0^{-2}},$$

let us formulate a Cauchy problem

$$\frac{dU}{dt}=\frac{\beta U^2}{cU_0T_0^2}\,\frac{\alpha ST_0^2\left(U_0-U\right)-\beta IU^2}{\gamma T_0^2\left(U_0-U\right)+\beta U}, \qquad (6)$$

$$U\left(t_0\right)=U_0.$$

Then let us calculate the time point

$$t_*=t_0+\frac{c}{\alpha S}\left[\frac{\gamma T_0^2}{\beta}\left(\frac{U_*}{U_0}-1+\delta_0\right)\frac{U_0}{U_*}+\right.$$
$$+\left(\frac{U_0}{2U_0-U_*}+\frac{\gamma T_0^2}{\beta}\,\frac{U_0-U_*}{2U_0-U_*}\,\frac{U_0}{U_*}-1\right)\times$$
$$\times\ln\left(2-\frac{U_*}{U_0}-\delta_0\right)-\left(\frac{U_0}{2U_0-U_*}+\right.$$
$$\left.\left.+\frac{\gamma T_0^2}{\beta}\,\frac{U_0-U_*}{2U_0-U_*}\,\frac{U_0}{U_*}\right)\ln\left(\frac{U_0}{U_0-U_*}\delta_0\right)\right],$$

for which

$$U\left(t_*\right)=\frac{U_*}{1-\delta_0}.$$

Evidently, at $t\ge t_*$

$$\delta\left(U_*\right)=\left|\frac{U-U_*}{U}\right|=1-\frac{U_*}{U}\le\delta_0,$$

and the value U_* can be considered equal to $U(t)$ with a relative error not exceeding δ_0. Therefore, it is possible to use formula (3) to find the sought value with a relative error not exceeding δ_0.

If condition (5) is not met, mathematical model (3) at $t\ge t_*$ sufficiently possesses the properties of fullness, adequacy, productivity and economy.

Building a new mathematical model when creating a hierarchy of mathematical models for the object of study may lead to refining the previously determined conditions for the applicability of the constructed mathematical models. Indeed, using mathematical model (6), we can refine the condition of applicability for formula (2). For this let us calculate the time point

$$t^* = t_0 + \frac{c}{\alpha S}\left[\left(\frac{\gamma T_0^2}{\beta}\frac{U_0 - U_*}{2U_0 - U_*}\frac{U_0}{U_*} + \frac{U_0}{2U_0 - U_*} - 1\right)\ln\left(1 + \frac{U_*}{U_0}\delta_0\right) - \frac{\gamma T_0^2}{\beta}\delta_0 - \left(\frac{\gamma T_0^2}{\beta}\frac{U_0 - U_*}{2U_0 - U_*}\frac{U_0}{U_*} + \frac{U_0}{2U_0 - U_*}\right)\ln\left(1 - \frac{U_*}{U_0 - U_*}\delta_0\right)\right],$$

for which

$$U\left(t^*\right) = \frac{U_0}{1 + \delta_0}.$$

Evidently, at $t \le t^*$

$$\delta(U_0) = \left|\frac{U - U_0}{U}\right| = \frac{U_0}{U} - 1 \le \delta_0,$$

and the value U_0 can be considered equal to $U(t)$ with a relative error not exceeding δ_0. Therefore, it is possible to use formula (2) to find the sought value with a relative error not exceeding δ_0.

If condition (5) is met or $t \le t^*$, mathematical model (2) sufficiently possesses the properties of fullness, adequacy, productivity and economy.

Results

When inequality (4) is satisfied, the following statements are true; they allow us to identify a working mathematical model of the object of study.

(i) If condition (5) is met, or $t \le t^*$ within the scope of the study, then mathematical model (2) is considered the working mathematical model.

(ii) If condition (5) is not satisfied, then the mathematical model (3) at $t \ge t_*$ is chosen as the working mathematical model.

(iii) If inequality (5) is not satisfied, and the time interval from t^* to t_* is of interest, then mathematical model (6) is considered the working mathematical model.

Conclusion

Thus, a unified approach was used to formulate statements applicable to this study. They allow us to define a working mathematical model of a negative temperature coefficient thermistor. This mathematical model sufficiently possesses the properties of fullness, adequacy, productivity and economy.

It is evident that the use of such a mathematical model not only reduces the costs and time spent on research, but also makes efficient use of the mathematical modelling capabilities.

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