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A METHOD FOR SOLVING THE PROCUREMENT OPTIMIZATION PROBLEM BASED ON INVERSE CALCULATIONS

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Procurement optimization; Inverse calculations; Inverse task; Quadratic programming.

A B S T R A C T

The paper describes a method for solving the procurement optimization problem based on inverse calculations. The method involves solving the unconstrained optimization problem and adjusting the obtained values of arguments subject to the constraint. Compared to conventional nonlinear optimization methods, the proposed method is easier to implement with computer software, since it does not require any determination of additional variables and multiple solutions to an unconstrained optimization problem. A solution to a two-constraint problem using inverse calculations has been considered. In this case, the optimization problem is transformed into a single-constraint problem. Therefore, when constructing a system of equations for determining increments of arguments, the transformed form of the objective function and the constraint are taken into account. This paper discusses a solution to a procurement portfolio development problem for a confectionery company with a limited budget and a target value of contribution margin. The obtained solution was compared with the solution produced by the mathematical software package and the penalty method. The result of this comparison is presented in the paper. The proposed algorithm can be used in systems intended to support the procurement decision making process. The algorithm can also be used to solve quadratic programming problems of the considered form in other fields of research.

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1. INTRODUCTION

In the course of its activities, a company needs to use various resources. Therefore, the procurement management has a significant impact on the overall performance. When operating in a competitive environment and having a variety of proposals to choose from, the company needs to make decisions regarding procurement parameters, such as a product mix, a list of suppliers, etc. However, since its financial resources are limited, the company needs to use them as efficiently as possible. Thus, the company faces a procurement optimization problem, i. e. the need to choose the best option among the existing alternatives, which would maximize the value of procurement by using available resources. In different studies, the procurement optimization means optimization of supplier selection, optimization of supplies, and optimization of material costs. There are a number of models and methods to solve this kind of problems. Let us review some studies in the field of enterprise resource management.

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A large number of studies are dedicated to problems of inventory management in which the costs connected with delivery, storage and shortage serve as an objective function (Cretu, Fontes, & Homayouni, 2019). In case of constant demand and constant time between deliveries, the classical Wilson model can be used. Some studies propose some modifications of this model to take into account more complex conditions, e. g., the presence of several product mixes (Chen, 2003; Haksever & Moussourakis, 2005). The paper (Chen, 2003) considers the case where the demand is random. Chang, Ouyang, and Teng (2003) proposed a model in which the supplier grants a delay in payment to the buyer for large orders. In (Dewi, Baihaqi, & Widodo, 2015), the objective function includes seven types of costs: procurement, ordering, storage, delivery delays, penalty, and operating costs. Classical optimization methods (penalty method, Lagrangian method), the fuzzy-set theory, the queueing theory, transformations based on Cauchy inequality (Teng, 2009), etc. are used to solve inventory management problems. The authors of (Krichena, Laabidia, & Abdelazizb, 2011) use the game theory to search for forms of cooperation in procurement between retailers with a single supplier who grants quantity discounts and delays in payment. For more complex problems, such as optimization of multi-level inventory management systems, the simulation method (Chu, You, Wassick, & Agarwal, 2015) is used. The paper (Sauvageau & Frayret, 2015) discusses the use of an agent-based approach for optimizing waste paper procurement. Storage costs, quality of purchased products, and average stock level are considered as indicators of procurement activities. The most common heuristic optimization methods. The paper (Liu & Tao, 2015) presents a multi-criteria optimization model designed to minimize the time gap between deliveries, purchase and delivery costs. In order to solve the problem, the particle swarm optimization algorithm was used. The study (Yao & Chiou, 2004) discusses a model with a single supplier and several customers. The optimization problem is to minimize supplier costs with limited customer costs. It proposes a solving algorithm based on enumerating values of the original variables: the production interval and the replenishment interval (how often the product stock is replenished by customers).

There are also studies (Castro, Aguirre, Zeballos, & Mendez, 2011) that solve the problem on the basis of a hybrid algorithm combining simulation and mathematical programming.

Suppler selection is an important aspect of procurement activities. It can be influenced by various factors, such as product quality and reliability of a supplier, as well as its pricing policy. The study (Tu et al., 2018) presents a supplier selection model aimed at minimizing procurement costs and maximizing the quality of service. Constraints of the problem determine the minimum number of selected suppliers, and the agreement between the purchase volume and the existing demand. The study (Yadav & Sharma, 2016) discusses the supplier ranking process based on a set of characteristics (price, discounts offered, meeting delivery deadlines, quality of packaging, reputation, etc.) and their weighting factors. The higher the value of the integrated indicator, the more preferable the selection of the respective supplier.

When solving product mix optimization problems, it is necessary to determine a range of products based on their individual parameters (marginal profit, price, unit costs, etc.). For example, the quantity of each product can be determined in such a way as to maximize the total profit for a given demand and the operating time of equipment. The business profile of a company can be taken into account as well; for example, in the paper by Manakhov (2016), the products are sold on credit.

In reference (Buravlev & Ivancov, 2012), a procurement optimization problem for military equipment is discussed. The total costs for a given number of years was used as an objective function to be minimized. The compliance of the total purchase volume with the requirements of the state arms program was taken as a constraint. In order to solve the problem, dynamic programming was used. The paper (Capliced & Sheffi, 2003) considers the procurement optimization for a carrier. It presents a problem aimed at minimizing the cost of procurement while meeting the existing demand.

This study is dedicated to solving the procurement optimization problem, i. e. the choice of the type and quantity of goods ordered by a firm with a limited budget from suppliers (Farmanov, 2008). Thus, the problem considers two basic processes: purchasing products from a supplier and selling products to customers. The purchase volume of a certain product is used as a controlled variable.

2. METHODS FOR SOLVING THE OPTIMIZATION PROBLEM

Optimization methods are widely used in solving the economic problems related to optimal resource management. The statement of the optimization problem involves identifying an objective function to be minimized or maximized, and constraints on its arguments. Linear programming problems (objective function and constraints are linear) and nonlinear programming problems (objective function and/or constraints are nonlinear) can be distinguished according to the type of the objective function and constraints.

This study is dedicated to solving quadratic programming problems with two constraints in the form of equality:

$$\begin{aligned} & f(x) \to \min, \\ & h_1(x) = 0, \\ & h_2(x) = 0, \end{aligned} \tag{1}$$

where $f(x)$ is the quadratic function;
$h(x)$ is the linear function.

The paper considers the case where the partial derivatives of the objective function depend on a single argument and are linear, i. e. second-order partial derivatives of the objective function are constant.

Two classical methods, the penalty method and the Lagrangian method, are most commonly used for solving nonlinear programming problems. Both methods are based on reducing a constrained optimization problem to an unconstrained optimization problem. The penalty method introduces two concepts: the penalty function formed from the initial objective function and the penalty function of a constraint and a penalty parameter. At each iteration, the unconstrained optimization problem of the penalty function is solved for a given penalty parameter, the value of which gradually increases. The algorithm stops working when the elements of the iteration sequences change slightly from step to step. If the sequence of function arguments is valid, the penalty method is called internal; otherwise, it is called external. For example, when using the quadratic penalty applicable in the presence of a constraint (equality), the solution to the minimization problem will be reduced to finding the minimum of the penalty function for different values of the penalty parameter R:

$$P(x,R) = f(x) + R\sum_{l=1}^{k} h_l^2(x), \qquad (2)$$

where $P(x,R)$ is the penalty function;
$f(x)$ is the objective function;
$h(x)$ is the constraint function;
k is the number of constraints.

The Lagrangian method implies transforming the constrained optimization problem into the unconstrained optimization problem with some unknown parameters — Lagrange multipliers (Trunov, 2014). When solving the problem, it is necessary to form the Lagrange function to be minimized:

$$L(x,\lambda) = f(x) + \sum_{l=1}^{k} \lambda_l h_l(x), \qquad (3)$$

where $L(x,\lambda)$ is the Lagrange function;
λ is the Lagrange multiplier.

Unconstrained optimization problems can be solved by using zero-order methods, which use only values of the function and arguments at calculated points (Hooke-Jeeves, simplex, template-based search), first-order methods (gradient descent, Cauchy, Fletcher-Reeves, etc.), which involve calculating the first partial derivative of the optimized function at test points, and second-order (Newton) methods, which require the existence of the first and second partial derivatives of the function to be optimized.

Such problems can be also solved by methods modifying or combining the algorithmic aspects of penalties and Lagrange multipliers. For example, this approach is used by (Hosobe, 2015).

Computer implementation of the existing methods is a time-consuming process. The penalty method requires multiple solutions to the unconstrained optimization problem, determining an increment of the parameter R. The Lagrange multiplier method introduces additional variables λ and, therefore, increases the dimension of the problem.

Also, evolutionary algorithms are used to solve nonlinear programming problems. As an example of these algorithms, a genetic algorithm can be given. It is used to generate populations of solutions, and then perform mating, mutation, selection and generation of a new population (Isaev, 2005). In reference (Isaev, 2005), a penalty function is generated on the basis of an objective function and constraints. The advantage of such methods is that they can provide a solution to global optimization problems in cases where the optimization of a function by classical methods is difficult due to its behavior.

However, the resulting solution will be suboptimal and vary from implementation to implementation. The disadvantages of such methods include the high cost of computing resources arising from the need to generate multiple solutions and perform multiple iterations to improve them. Also, the implementation of such algorithms, sets of solution correction rules, is a time-consuming process. Therefore, the use of such methods is not always reasonable.

The paper is aimed at developing a method based on inverse calculations for solving a procurement optimization problem with a limited budget and a target value of total contribution margin. This method is easier to implement with computer software compared to conventional techniques.

3. INVERSE CALCULATIONS IN THE ECONOMIC DECISION-MAKING PROCESS

Activities of socio-economic entities can be analyzed using various indicators which are related to each other through additive, multiplicative, multiple, and mixed relationships. Based on the causal relationship of variables, problems can be divided into direct and inverse ones. The direct problem is aimed at determining the resulting indicator using the available values of original variables and the type of relationship in order to assess the status of an entity. Determining the profit of an enterprise using the target values of income and expenses can be given as an example.

The inverse problem is more complex than the direct one and aims to select values of original variables so that the target value of the resulting variable is obtained. The ultimate goal of solutions to these problems is to make optimal management decisions, for example, to determine the level of income and expenses that would provide a target profit growth.
Such problems are widely used in astronomy, physics, economics, etc.

The complexity of solving inverse problems is due to the fact that they are not correct. The concept of correctness was defined by J. Hadamard, and means that the solution to the problem exists, is unique on a certain set, and continuously depends on input data. A valuable contribution to the study of inverse problems was made by A. Tikhonov, who demonstrated that the specification of additional conditions for a solution provides a stable problem. He proposed a way to regularize an incorrect problem, i.e. reducing the original problem of solving some operator equation to a problem of finding the minimum of some functional.

A set of inverse calculation tools was developed to solve inverse problems in the field of economics using expert information specified by an analyst. A solution based on inverse calculations (Odincov, 2004) is understood to be finding the increments of function arguments using the following information: initial values of arguments and functions, a new value of the function, coefficients of relative importance of arguments, and direction of changes in arguments.

The problem is presented as a system of equations where the ratio of changes in the arguments is equal to the ratio of the relative priority coefficients. In the case of two arguments, the system can be expressed as:

$$\begin{cases} \dfrac{\Delta x_1}{\Delta x_2} = \dfrac{\alpha_1}{\alpha_2}; \\ h(x_1 + \Delta x_1, x_2 + \Delta x_2) = y + \Delta y. \end{cases} \tag{4}$$

where x_1 , x_2 are the initial values of the arguments;
Δx_1 , Δx_2 are the changes in the arguments;
α_1, α_2 are the relative priority coefficients of the arguments x_1 , x_2 respectively;
y is the initial value of the resulting indicator;
Δy is the change in the resulting indicator;
h is the function of relationship between the arguments.

By solving the system, analytical formulas for determining changes in the arguments can be obtained.
So, let us consider the procurement costing problem. Input data:

- the purchase volume of the first product (x_1) is equal to 11 kg;
- the purchase volume of the second product (x_2) is equal to 16 kg;
- the purchase price of the first product is 125 units of money;
- the purchase price of the second product is 105 units of money.

The total procurement cost is equal to:
$125 \cdot 11 + 105 \cdot 16 = 3{,}055$ units of money.

It is necessary to reduce the number of the first and second products so that the cost of procurement is equal to 2,500. Figure 1 shows point Q, corresponding to the initial values of the variables, and a graph showing the values of the variables x_1 and x_2 at which the procurement cost is 2,500. It can be seen that the target result can be achieved by many combinations of values of x_1 and x_2. Therefore, in order to solve the problem, it is necessary to involve some expert information: to specify whether the variables will decrease or increase, as well as the degree of their change which can be determined using the relative priority coefficients.

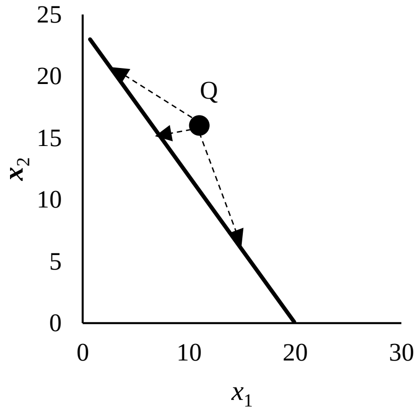

Figure 1. Line of the target level of procurement cost

Assume that the relative priority coefficient for the first product be 0.4, for the second one — 0.6. The target value of the resulting indicator should be achieved by reducing the purchase volume for each product, i.e., the change in the total purchase volume is mainly due a decrease in the purchase volume of the second product.

The system of equations has the form:

$$\begin{cases} \dfrac{\Delta x_1}{\Delta x_2} = \dfrac{0.4}{0.6}; \\ 125(11 - \Delta x_1) + 105(16 - \Delta x_2) = 2500. \end{cases}$$

Solution to the system: Δx_1=2.743, Δx_2=2.02.

Solving problems of this kind is relevant, because it helps to answer the question "how to do it so that ...?", and determine control actions to achieve the desired state of an economic entity, which is the essential function of systems supporting optimal decision-making. Thus, it is possible to solve the most important problem, synthesizing management by objectives with the balanced scorecard, where the root of the tree represents the strategic objective, while operational indicators are located at terminal nodes (Figure 2) (Odintsov, 2014).

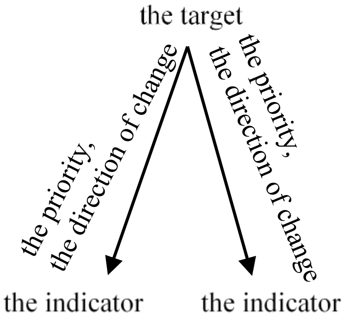

Figure 2. Elements of the inverse problem

This method has been adopted in solving socio-economic problems. In particular, it is used to generate a value of a certain integrated quantity by determining its component indicators. For example, the paper (Vishtak & Shtyrova, 2014) considers the development of the integrated quality indicator of continuing education at university, which is determined by a set of groups of indicators at a lower hierarchy level: indicators of learning outcomes, quality indicators of the learning process, management quality indicators of an organizational unit; indicators of resource availability. In turn, each of the above groups of indicators is determined by indicators of a lower hierarchy level. Thus, the problem is solved from top to bottom of the hierarchical structure presented in the tree-like form. In reference (Barmina & Kvyatkovskaya, 2010), the integrated performance indicator of a business company is developed using a hierarchical structure as well. It presents methods for determining integrated performance indicators and a set of measurable quality indicators based on cognitive modeling. The inverse calculation mechanism is used to solve a problem of developing recommendations on how to improve the performance of a company.

The paper (Blyumin & Borovkova, 2018) discusses the use of inverse calculations to determine performance indicators for employees at a university department so that the desired department's rating is reached. A further analysis of the final changes identified employees who could make the greatest inputs to the overall department's rating.

In most cases, the integrated indicator is developed using additive convolution, in which increments of the arguments can be determined using the following formulas (u are the weighting factors of the indicators that constitute the integrated characteristic y):

$$u_1(x_1+\Delta x_1)+u_2(x_2+\Delta x_2)=y+\Delta y,$$
$$\Delta x_1=\frac{\Delta y}{u_1+u_2\dfrac{\alpha_2}{\alpha_1}}, \qquad (5)$$
$$\Delta x_2=\frac{\Delta y}{u_2+u_1\dfrac{\alpha_1}{\alpha_2}}.$$

Some studies are aimed not only at using the inverse calculation tools for solving applied problems, but also at modifying these tools for individual tasks. For example, the paper (Odintsov & Romanov, 2014) considers the existence of constraints on values of indicators.

A separate area of research is dedicated to the development of methods and algorithms based on inverse calculations which do not require any expert information or require it to a lesser extent. Relying on expert opinions has its positive aspects: several possible solutions to the problem can be considered; relative priority coefficients can be set taking into account the real possibility of a direction of change in the arguments. However, when using expert information, an analyst needs to take some additional steps to identify and justify it. The resulting solution is subjective and depends on the analyst's experience and competence. In addition, the statement of the problem may imply the optimization of individual parameters instead of using expert information or using the existing relationship between indicators.

The problems that can be solved without involving expert information include, but not limited to, a problem aimed at finding a solution as close as possible to the original one, i.e. with a minimal change in arguments. Here, the initial values characterize the current state of an entity of interest. Therefore, the less they change, the less effort is required to achieve the goal.

Classical metrics (Euclidean metric, squared Euclidean metric, or Manhattan distance) can be considered as a measure of deviation of the obtained solution from the original one. For example, the paper (Gribanova, 2018) discusses a solution to a problem in the case where the sum of squares of argument increments is minimized. Using geometry, the authors determined systems of equations to be solved by finding a change in the arguments. So, for the problem presented in Figure 3 the shortest distance from point Q to the straight line $x_2=\frac{2500-105x_1}{125}$ is the length of the perpendicular QD. Thus, when moving from point Q to point D, the change in the arguments will be minimal. The change in the first argument (Δx_1) is equal to the length of segment QB; the change in the second argument (Δx_2) is equal to the length of segment BD ($QD^2=DB^2+QB^2$).

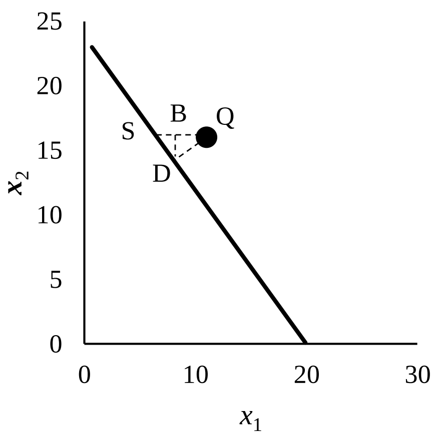

Figure 3. Solving the problem while minimizing changes in the arguments

As can be seen from Figure 3, angle QDB is equal to angle DSB. Since the tangent of the angle is equal to the slope, the ratio of the increments is equal to the slope with the variable x_1 with a minus sign, while values of the increments can be determined by the following system:

$$\begin{cases} \dfrac{\Delta x_2}{\Delta x_1} = 0.84; \\ 125(11+\Delta x_1)+105(16+\Delta x_2) = 2500. \end{cases}$$

Solution to the system: Δx_1= –2.603, Δx_2= –2.187.

4. METHOD FOR SOLVING THE PROBLEM BASED ON INVERSE CALCULATIONS

This approach can be used to solve quadratic programming problems.

The study (Gribanova, 2019) describes an algorithm to solve a quadratic programming problem with a single linear constraint using inverse calculations:

$$\begin{gathered} f(x) \to \min, \\ h(x) = \sum_{i=1}^{n} a_i x_i = A, \end{gathered} \tag{6}$$

where a is numerical values with the arguments;
A is the specified value of the constraint.

This algorithm includes the following main steps.

Step 1. Solve the unconstrained optimization problem: determine the minimum point of the objective function $f(x)$. As a result, values of $\hat{x}$ are obtained.

Step 2. Calculate values of $\eta_{i,j}$ as the ratio of second-order partial derivatives of the objective function (j – index of the variable used as the basic index; k_i is the value of the second-order partial derivative with respect to the variable x_i, $i = 1..n, i \neq j$; n is the number of variables):

$$\eta_{i,j} = \frac{k_i}{k_j}. \tag{7}$$

Step 3. Values of $r_{i,j}$ are calculated as a ratio of numerical values with the arguments in the constraint (which are equal to the partial derivatives for the corresponding variables):

$$r_{i,j} = \frac{a_i}{a_j}. \tag{8}$$

Step 4. Solve the system of equations:

$$\begin{cases} \dfrac{\Delta x_i}{\Delta x_j}\eta_{i,j} = r_{i,j}, i = 1..n, i \neq j; \\ \sum_{i=1}^{n} a_i(x_i + \Delta x_i) = A. \end{cases} \tag{9}$$

The following general formulas for the increments, obtained by solving the system of equations, can be used for the considered problem:

$$\begin{gathered} \Delta x_j = \frac{A - \sum_{i=1}^{n} a_i \hat{x}_i}{\sum_{i=1, i\neq j}^{n} a_i \dfrac{r_{i,j}}{\eta_{i,j}} + a_j}, \\ \Delta x_i = \frac{r_{i,j}}{\eta_{i,j}} \Delta x_j, i = 1..n, i \neq j. \end{gathered} \tag{10}$$

Let us consider the use of this apparatus for solving the following quadratic programming problem:

$$\begin{gathered} f(x) = 4x_1^2 + x_2^2 \to \min, \\ x_1 + 2x_2 = 3. \end{gathered} \tag{11}$$

Figure 4 shows a contour plot and line $x_1 = 3 - 2x_2$. The objective function reaches its minimum at point Q (0;0). The problem is reduced to moving from the minimum point Q to a point on the constraint line, while minimizing change in the arguments and (Figure 1). The shortest distance from point Q to the straight line $x_1 = 3 - 2x_2$ is the length of the perpendicular QD.

Values of the increments can be determined by the following system:

$$\begin{cases} \dfrac{\Delta x_2}{\Delta x_1} = 2; \\ 0+\Delta x_1 + 2(0+\Delta x_2) = 3. \end{cases}$$

Solution to the system: Δx_1 =0.6, Δx_2=1.2.

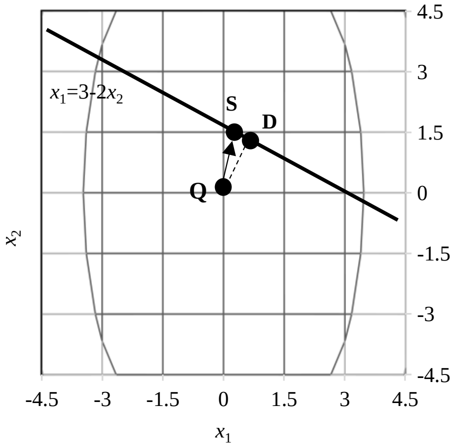

Figure 4. Graphic representation of the problem

However, it is now necessary to take into account the difference in influence of the arguments on the change in the function $f(x)$. To do this, we calculate the second-order partial derivatives (first-order partial derivatives will be linear functions, the change rate of which is to be calculated):

$$\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2} = 8, \frac{\partial^2 f(x_1,x_2)}{\partial x_2^2} = 2.$$

The ratio of partial derivatives is equal to 4. Let us adjust the system of equations taking into account the calculated ratio of second-order partial derivatives:

$$\begin{cases} \dfrac{\Delta x_2}{\Delta x_1}\dfrac{2}{8} = 2; \\ 0 + \Delta x_1 + 2(0 + \Delta x_2) = 3. \end{cases}$$

Solution to the system: Δx_1=0.176, Δx_2=1.412 (point S). The obtained solutions match the solution to the quadratic programming problem obtained using the Mathcad package.

This paper considers the optimization problem with two constraints:

$$\begin{aligned} h_1(x) &= \sum_{i=1}^{n} a_i x_i = A, \\ h_2(x) &= \sum_{i=1}^{n} c_i x_i = C. \end{aligned} \tag{12}$$

where c is numerical values with the arguments;
C is the specified value of the constraint.

In this case, it is necessary to derive a mathematical expression of the variable x_s from the equation h_1.
As a result, the optimization problem is given by:

$$\begin{aligned} & f\left(x_2,\ldots,\frac{A - \sum\limits_{i=1, i\neq s}^{n} a_i x_i}{a_s},\ldots,x_n\right) \to \min, \\ & h_2\left(x_1,\ldots,\frac{A - \sum\limits_{i=1, i\neq s}^{n} a_i x_i}{a_s},\ldots,x_n\right) = C. \end{aligned} \tag{13}$$

5. SOLUTION TO THE PROCUREMENT OPTIMIZATION PROBLEM

Let us consider the application of the considered method for solving the procurement optimization problem, based on the data provided by a confectionery company (Gribanova, 2018). The company makes a daily demand forecast based on available statistical data for previous periods. The company needs to purchase goods in such a way as to best meet the demand with limited financial resources. The input data for the model include:

- b_i is the forecast value of average demand for the product i ($i = 1..n$, n is the number of product items);
- a_i is the purchase price of the ith product;
- A is the amount of the procurement budget.

The resulting problem is a quadratic programming problem with a linear constraint in the form of the equality:

$$\begin{aligned} f(x) &= \sum_{i=1}^{n} (x_i - b_i)^2 \to \min, \\ h(x) &= \sum_{i=1}^{n} a_i x_i = A. \end{aligned} \tag{14}$$

This problem also allows the contribution margin to be taken into account and a constraint on its specified value to be added:

$$\begin{aligned} f(x) &= \sum_{i=1}^{n} (x_i - b_i)^2 \to \min, \\ h_1(x) &= \sum_{i=1}^{n} a_i x_i = A, \\ h_2(x) &= \sum_{i=1}^{n} c_i x_i = C. \end{aligned} \tag{15}$$

where C is the value of total contribution margin;
c_i is the contribution margin of the ith product.

Details of three confectionery products are given in Table 1. The procurement budget is 3,000 rubles. The target value of total contribution margin is 1,150 rubles.

The single-constraint quadratic programming problem can be presented as follows:

$$f(x) = (x_1 - 11)^2 + (x_2 - 16)^2 + (x_3 - 8)^2 + \\ +(x_4 - 5)^2 \to \min \qquad (16) \\ 125x_1 + 105x_2 + 170x_3 + 160x_4 = 3000.$$

The minimum of the objective function $f(x)$ represents the forecast demand b: $\hat{x}_1 = 11$, $\hat{x}_2 = 16$, $\hat{x}_3 = 8$, $\hat{x}_4 = 5$. Thus, when solving the procurement optimization problem, values of the forecast demand can be taken as the solution obtained in the first step of the considered algorithm, i. e. there is no need to solve the unconstrained optimization problem.

Table 1. Input data

Indicator	Product number, i			
	1	2	3	4
Forecast demand, kg	11	16	8	5
Purchase price, rubles per kg	125	105	170	160
Contribution margin, rubles	50	40	40	55

Since in this case the arguments equally influence the change in the function, all values of $\eta_{i,j}$ are equal to 1. Let us calculate the values of $r_{i,j}$ using the purchase price data a_i: $r_{1,4} = \frac{a_1}{a_4} = 0.781$, $r_{2,4} = \frac{a_2}{a_4} = 0.656$, $r_{3,4} = \frac{a_3}{a_4} = 1.063$.

Next, using inverse calculations, we can determine changes in order quantities by solving the system of equations:

$$\begin{cases} 125(11 + \Delta x_1) + 105(16 + \Delta x_2) + 170(8 + \Delta x_3) + \\ +160(5 + \Delta x_4) = 3000; \\ \dfrac{\Delta x_1}{\Delta x_4} = 0.781; \\ \dfrac{\Delta x_2}{\Delta x_4} = 0.656; \\ \dfrac{\Delta x_3}{\Delta x_4} = 1.063. \end{cases}$$

The resulting increment values are equal to $\Delta x_1 = -3.412$, $\Delta x_2 = -2.866$, $\Delta x_3 = -4.64$, $\Delta x_4 = -4.367$. Therefore, the following values (kg) will be the solution to the problem:

$$x_1 = 11 - 3.412 = 7.588, \\ x_2 = 16 - 2.866 = 13.134, \\ x_3 = 8 - 4.64 = 3.36, \\ x_4 = 5 - 4.367 = 0.633.$$

Table 2 presents the results obtained using the penalty method (quadratic penalty).

For a variant of problem (16), the Lagrange function will have the form:

$$L(x, \lambda) = (x_1 - 11)^2 + (x_2 - 16)^2 + (x_3 - 8)^2 + \\ +(x_4 - 5)^2 + \lambda(125x_1 + 105x_2 + 170x_3 + 160x_4 - \qquad (17) \\ -3000).$$

Table 2. The results obtained using the penalty method

Penalty parameter R	Arguments of the function				$f(x)$
	x_1	x_2	x_3	x_4	
0	11	16	8	5	0
50	7.717	13.169	3.118	0.766	60.553
100	7.588	13.134	3.36	0.633	60.459

To solve the problem, it is necessary to calculate the partial derivatives of the function with respect to the variables x:

$$\frac{\partial L(x,\lambda)}{\partial x_1} = 125\lambda + 2x_1 - 22, \\ \frac{\partial L(x,\lambda)}{\partial x_2} = 105\lambda + 2x_2 - 32, \\ \frac{\partial L(x,\lambda)}{\partial x_3} = 170\lambda + 2x_3 - 16, \\ \frac{\partial L(x,\lambda)}{\partial x_4} = 160\lambda + 2x_4 - 10.$$

Then it is necessary to solve the system of equations:

$$\begin{cases} 125\lambda + 2x_1 - 22 = 0, \\ 105\lambda + 2x_2 - 32 = 0, \\ 170\lambda + 2x_3 - 16 = 0, \\ 160\lambda + 2x_4 - 10 = 0, \\ 125x_1 + 105x_2 + 170x_3 + 160x_4 - 3000 = 0. \end{cases}$$

The solution to the system: x_1=7.588, x_2=13.134, x_3=3.36, x_4=0.633, λ=0.055. The value of the objective function f is 60.459.

With the obtained values of the arguments, the total contribution margin is 1,074 rubles. For example, we need to find a solution at which the total contribution margin would reach 1,150 rubles. The optimization task is given by:

$$f(x) = (x_1 - 11)^2 + (x_2 - 16)^2 + (x_3 - 8)^2 + \\ +(x_4 - 5)^2 \to \min, \qquad (18) \\ 125x_1 + 105x_2 + 170x_3 + 160x_4 = 3000; \\ 50x_1 + 40x_2 + 40x_3 + 55x_4 = 1150.$$

Let us solve this problem using the proposed approach. In the first equation, we select a variable with the highest numerical value and express it as:

$$x_3 = \frac{3000-(125x_1+105x_2+160x_4)}{170}. \quad (19)$$

Then the optimization problem is given by:

$$\begin{gathered} f(x)=(x_1-11)^2+(x_2-16)^2+ \\ (\frac{3000-(125x_1+105x_2+160x_4)}{170}-8)^2+ \\ +(x_4-5)^2 \to \min, \quad (20) \\ 50x_1+40x_2+40\frac{3000-(125x_1+105x_2+160x_4)}{170}+ \\ +55x_4=1150. \end{gathered}$$

The minimum of the objective function $f(x)$ is defined as: $\hat{x}_1 = 7.588$, $\hat{x}_2 = 13.13$, $\hat{x}_4 = 0.633$.

The second derivatives of the objective function: $\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2} = 3.081$, $\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2} = 2.763$, $\frac{\partial^2 f(x_1,x_2)}{\partial x_4^2} = 3.772$. The partial derivatives of the constraint function: $\frac{\partial h_2}{\partial x_1} = 20.588$, $\frac{\partial h_2}{\partial x_2} = 15.294$, $\frac{\partial h_2}{\partial x_4} = 17.353$.

Then we obtain the following system of equations:

$$\begin{cases} \frac{\Delta x_1}{\Delta x_2}\frac{3.081}{2.763} = \frac{20.588}{15.294}; \\ \frac{\Delta x_1}{\Delta x_2}\frac{3.081}{3.772} = \frac{20.588}{17.353}; \\ 50(7.588+\Delta x_1)+40(13.13+\Delta x_2)+ \\ 40\frac{3000-(125(7.588+\Delta x_1)+105(13.13+ +\Delta x_2)+160(0.633+\Delta x_4))}{170}+ \\ +55(0.633+\Delta x_4)=1150. \end{cases}$$

The solution to the system: Δx_1=1.682, Δx_2=1.393, Δx_4=1.158. Thus, the target values of the arguments are equal to: x_1 = 9.27, x_2 =14.527, x_4 =1.791, x_3 =0.172. The value of the objective function f is 76.731. When solving the optimization problem using the standard MathCad function, the value of the objective function is 76.401.

Table 3 presents the results obtained using the penalty method (quadratic penalty).

Table 3. The results obtained using the penalty method (the problem with two constraints)

Penalty parameter R	Arguments of the function				$f(x)$
	x_1	x_2	x_3	x_4	
0	11	16	8	5	0
100	8.391	14.209	-0.23	3.114	81.305
1000	9.65	14.452	0.301	1.407	76.401
10000	9.649	14.453	0.301	1.407	76.401

For a variant of problem (18), the Lagrange function will have the form:

$$\begin{gathered} L(x,\lambda)=(x_1-11)^2+(x_2-16)^2+(x_3-8)^2+ \\ +(x_4-5)^2+\lambda_1(125x_1+105x_2+170x_3+160x_4- \quad (21) \\ -3000)+\lambda_2(50x_1+40x_2+40x_3+55x_4-1150). \end{gathered}$$

To solve the problem, it is necessary to calculate the partial derivatives of the function with respect to the variables x:

$$\frac{\partial L(x,\lambda)}{\partial x_1} = 125\lambda_1+50\lambda_2+2x_1-22,$$
$$\frac{\partial L(x,\lambda)}{\partial x_2} = 105\lambda_1+40\lambda_2+2x_2-32,$$
$$\frac{\partial L(x,\lambda)}{\partial x_3} = 170\lambda_1+40\lambda_2+2x_3-16,$$
$$\frac{\partial L(x,\lambda)}{\partial x_4} = 160\lambda_1+55\lambda_2+2x_4-10.$$

Then it is necessary to solve the system of equations:

$$\begin{cases} 125\lambda_1+50\lambda_2+2x_1-22=0, \\ 105\lambda_1+40\lambda_2+2x_2-32=0, \\ 170\lambda_1+40\lambda_2+2x_3-16=0, \\ 160\lambda_1+55\lambda_2+2x_4-10=0, \\ 125x_1+105x_2+170x_3+160x_4-3000=0, \\ 50x_1+40x_2+40x_3+55x_4-1150=0. \end{cases}$$

The solution to the system: x_1=9.658, x_2=14.454, x_3=0.305, x_4=1.396, λ_1=0.189, λ_2= –0.419. The value of the objective function f is 76.401.

As it can be seen, the solution to the problem obtained using the method based on inverse calculations is consistent with the solution arrived at by the penalty method. Furthermore, a combination of these methods can be used to increase their accuracy: the solution obtained using the method based on inverse calculations can be considered as a starting point for solving the problem by the penalty method.

5.1 Solving the problem with a nonlinear constraint

Let us consider the case where the dependence of the company's contribution margin on the purchase volume of the ith product is expressed by a quadratic function:

$$\begin{gathered} c_1 = 287.65-3.84(x_1-9)^2, \\ c_2 = 310.34-3.5(x_2-10)^2, \\ c_3 = 285.74-3.09(x_3-10)^2, \\ c_4 = 388.58-6.15(x_4-8)^2. \end{gathered} \quad (22)$$

The profit value is limited to 1,000.
Then the optimization problem is given by:

$$\begin{gathered} f(x) = (x_1 - 11)^2 + (x_2 - 16)^2 + (x_3 - 8)^2 + \\ +(x_4 - 5)^2 \to \min, \\ 125x_1 + 105x_2 + 170x_3 + 160x_4 = 3000; \\ 287.65 - 3.84(x_1 - 9)^2 + 310.34 - 3.5(x_2 - 10)^2 + \\ +285.74 - 3.09(x_3 - 10)^2 + 388.58 - 6.15(x_4 - 8)^2 = \\ = 1000. \end{gathered} \tag{23}$$

The solution to the unconstrained optimization problem has been obtained previously (when expressing x_3 from the first equation): $\hat{x}_1 = 7.588, \hat{x}_2 = 13.13, \hat{x}_4 = 0.633$. However, in this case, the system of equations should be generated iteratively due to the fact that the partial derivatives of the constraint depend on the values of x. Graphically, such a problem-solving process can be represented as an approximation to a target value of the constraint function at a certain step. For example, Figure 5 (the problem-solving process for a nonlinear relationship) presents the initial point Q. Its coordinates change with values of the partial derivatives of the constraint function and the second derivatives of the objective function. At a large step, the solution can significantly differ from the optimal one. At a small step, a large number of iterations will be required to achieve the target value of the constraint function.

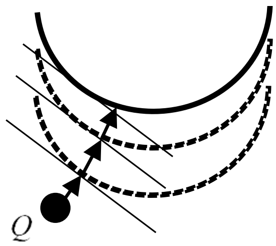

Figure 5. The problem-solving process for a nonlinear relationship

Then, at the first iteration, the values of the arguments change as follows ($\propto = 0.05$, a is some small number that enables a motion towards the target value of the constraint A at a certain step):

$$\breve{x}_1 = 7.588 + 0.05\frac{-19.33}{3.08} = 7.27,$$
$$\breve{x}_2 = 13.13 + 0.05\frac{-46.4}{2.76} = 12.294,$$
$$\breve{x}_4 = 0.633 + 0.05\frac{56.353}{3.77} = 1.38.$$

Table 4 presents the results of further iterations.

Table 4. The problem solution at $\propto = 0.05$

The number of iteration	Arguments of the function				$f(x)$
	x_1	x_2	x_3	x_4	
1	7.274	12.294	3.406	1.380	61.823
2	7.003	11.562	3.473	2.001	65.157
3	6.771	10.925	3.549	2.520	69.602
4	6.573	10.372	3.628	2.953	74.573
5	6.406	9.894	3.704	3.317	79.683
6	6.265	9.479	3.776	3.623	84.682

Table 5 presents the results produced by the mathematical software package, the Lagrange multiplier method and the penalty method (the change in the penalty parameter = 500; accuracy = 10^{-4}).

Thus, the solution with the lowest value of the objective function was obtained using the Lagrange multiplier method.

Table 5. Solutions of the problem using various methods

The method	Arguments of the function				$f(x)$
	x_1	x_2	x_3	x_4	
Lagrange multiplier method	6.35	9.676	3.606	3.608	82.859
The penalty method	5.913	9.822	3.817	3.63	83.424
The standard MathCad function	6.349	9.677	3.605	3.609	82.859

6. DISCUSSION

A method for solving the procurement optimization problem based on inverse calculations was proposed. A solution to the problem with one and two linear constraints, as well as with one linear and one nonlinear constraint, was considered. The constraints of the problem are given by an equation, which corresponds to a limit on the amount of available funds and the need to fully expend them, as well as to the achievement of the profit target. In the case of two constraints, the problem is reduced to a single-constraint problem using the variable replacement method.

The method is developed on the basis of the previously proposed method for solving inverse problems while minimizing the sum of squares of argument increments. The difference is that the proposed method offers the possibility to solve optimization problems with several constraints. Its modification for solving a problem by successively changing the arguments can provide a solution even with a nonlinear constraint. The obtained results agree with the solutions produced by a mathematical software package and classical methods (penalties and Lagrange multipliers).

The highest degree of agreement was achieved with a single linear constraint. When solving the problem with a single nonlinear constraint, the iterative procedure was applied by successively changing values of the arguments, taking into account the partial derivatives of the constraint and the second partial derivatives of the objective function.

In contrast to classical methods, the proposed method does not require multiple optimization of the modified function, including the objective function and the constraint. Also, this method does not require determining additional variables that increase the dimension of the problem. Thus, it is possible to solve the problem in a shorter time and simplify the computer implementation of the method.

The disadvantage of this method is the need to calculate the second derivatives: in the case of two linear constraints, computational experiments produced a solution with a higher value of the objective function as compared to classical methods. This method can be used in combination with other methods, i.e. the solution provided by the method can be considered as a starting point for the further problem-solving process.
The further research can be associated with a modification of the developed method for solving problems with inequality constraints, and application of this method to other fields of research.

7. CONCLUSION

The paper considers a solution to the procurement optimization problem using the proposed method based on inverse calculations. It should be noted that the discussed problem has a constraint on the cost of procurement and contribution margin. The method based on inverse calculations is easier to implement with computer software. It is founded on unconstrained optimization and a system of equations to be solved (the penalty method requires multiple unconstrained optimizations by changing the penalty parameter, while in the Lagrangian method, it is necessary to determine the gradient of the Lagrange function, set up and solve the system of equations). Based on the data from the confectionery company, some computational experiments were conducted to solve the procurement optimization problem using the penalty method and the inverse calculation method. The obtained results are consistent with the solution produced by the standard function of the MathCad software package. The proposed method based on inverse calculations can be used by organizations in their decision support systems for procurement planning. This method is also applicable to other quadratic programming optimization problems of the kind presented in this paper.

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