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Ch.B. Narmuradov
Termez State University
Termez, Uzbekistan
choribegaliyevich@mail.ru

A.X. Toyirov
Termez State University
Termez, Uzbekistan
akbartoyirov@gmail.com

Sh.M. Yuldashev
Termez State University
Termez, Uzbekistan
shamsiddin4836@gmail.com

F.B. Xolliev
Termez State University
Termez, Uzbekistan
surxon88@bk.ru

MATHEMATICAL MODELING OF MOVEMENT OF A VISCOUS INCOMPRESSIBLE LIQUID BY THE SPECTRAL-GRID METHOD

Abstract: The article analyzes the existing numerical methods for solving the hydrodynamic stability problem. We consider such methods as: finite-difference, step-by-step integration, local collocation, determinant, pre-integration method, spectral, spectral-grid. The spectral-grid method was used for mathematical modeling of the problem by stability hydrodynamics, the results obtained allow us to establish the convergence of the spectral-grid method, estimate the convergence rate and develop an effective algorithm that significantly reduces the order of complex matrices in the algebraic system being solved in comparison with the known spectral methods.

Key words: hydrodynamic stability, Reynolds number, wave numbers, eigenvalues and eigenvectors, Chebyshev polynomials of the first kind, conditions of orthogonality and continuity, grave conditions, generalized and standard eigenvalue problem, complex matrices.

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Introduction

Numerical methods are increasingly used to study mathematical models of hydrodynamic systems. At the same time, their application to the solution of the basic equations — the Navier-Stokes equations — for large Reynolds numbers encounters serious difficulties. They are mainly associated with the

presence of a small parameter at the highest derivative and, as a consequence, the appearance of strong spatial inhomogeneity in the solution. Therefore, the requirements for the approximation properties of numerical methods increase sharply. The stability problem of single-phase hydrodynamic systems reduces to the eigenvalue problem for an ordinary

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fourth-order differential equation with a small parameter at the highest derivative. Mathematical modeling and the construction of numerical methods for solving this class of equations are devoted to [1-3]. Existing methods for modeling the stability problem of hydrodynamic systems make it possible to accurately calculate individual eigenvalues of the stability problem and to obtain a solution in the areas of heterogeneity. However, when modeling the spectrum of eigenvalues, their effectiveness is insufficient. In more complex multi-parameter problems of hydrodynamic stability (for example, in

mathematical modeling of the stability of two-phase flows), the situation is aggravated by a decrease in efficiency and becomes practically unacceptable.

FORMULATION OF THE PROBLEM

One of the important problems in the numerical simulation of hydrodynamic systems is the problem of hydrodynamic stability. In a linear formulation for plane-parallel flows of a single-phase flow, this problem reduces to the eigenvalue problem for the Orr-Sommerfeld equation [1-6]:

$$\frac{1}{ikRe} D^2 \psi - [(U(\eta) - \lambda)D + \frac{d^2 U}{d\eta^2}] \psi = 0, \eta_0 < \eta < \eta_l \quad (1)$$

with uniform boundary conditions

$$\begin{aligned} \psi(\eta_0) = \frac{d\psi}{dy}(\eta_0) &= 0; \\ \psi(\eta_l) = \frac{d\psi}{dy}(\eta_l) &= 0 \end{aligned} \quad (2)$$

which depend on the type of currents studied.

In equation (1), the parameter $\lambda = \lambda_r + i\lambda_i$ is the eigenvalues, λ_r is the phase velocity, λ_i is the rise coefficient, $D = \frac{d^2}{d\eta^2} - k^2$ is the differential operator, η is the coordinate of the direction across the main stream, k is the wave number, is the Reynolds number, ρ is the density, μ is the viscosity, $Re = \frac{\rho UL}{\mu}$ is the maximum velocity of the main flow, L is the characteristic length, $U(\eta)$ is the profile of the velocity of the main flow, $\psi(\eta)$ is the amplitude of the stream function for perturbations.

The investigated flow will be stable or unstable depending on the value of the imaginary part of the eigenvalue $\lambda = \lambda_r + i\lambda_i$. If $\lambda_i > 0$, then the flow under consideration is unstable if $\lambda_i < 0$ -stable. If $\lambda_i = 0$, then the oscillations are neutrally stable, the curve in which $\lambda_i = 0$ is called the neutral stability curve.

Effective methods are required to determine the eigenvalues in problem (1) - (2).

THE SOLUTION OF THE PROBLEM

Equation (1) contains a small parameter $(kRe)^{-1}$ for the highest derivative, therefore, considerable difficulties arise in obtaining approximate solutions close to exact. Existing numerical methods [2,4] for numerical modeling of problem (1) - (2) can be divided into several groups: 1) finite-difference methods; 2) methods of step-by-step integration; 3) the method of exclusion and differential sweep; 4) spectral methods; 5) pre-integration method.

1. The use of finite-difference methods for solving equation (1) - (2) was proposed in [7]. The essence of these methods is to approximate the

derivatives involved in (1) by finite differences, and to solve the resulting system using linear algebra methods. Such a scheme, however, requires a fairly small step. For $k k Re \sim 10^4$, for example, to obtain sufficiently accurate (three signs) results, a uniform difference grid containing 100 nodes was used. Another disadvantage of this method is that with its help there is only one eigenvalue.

In order to reduce the number of computational nodes in [8], it is proposed to use a difference grid with a variable step to solve equation (1) (2). The construction of such a grid depends on several parameters, the choice of which is associated with certain difficulties. The data presented in this work show that the number of nodes increases with the parameter kRe . The method defines one eigenvalue.

A method for constructing a non-uniform grid for the numerical solution of second-order equations with a small parameter with the highest derivative was proposed in [9, 10]. According to this technique, in [11, 12], to find the eigenvalues for equation (1) - (2), an uneven grid is constructed using a special mapping. The mapping is specified so that the gradient modulus of the desired function is estimated by a value independent of kRe . Selecting the parameters of this display allows you to adjust the distance between the nodes of the grid in accordance with the size of the transition zone of the boundary layer and the critical point. The use of an uneven grid makes it possible to calculate one eigenvalue with high accuracy for a small number of grid nodes.

2. The difference calculation of the hydrodynamic stability problem using two-dimensional grids is described in [13]. An analysis of the necessary conditions imposed on the grid parameters for the correct description of the hydrodynamic properties is described in [14, 15]. Studies have shown that with an increase in the parameter kRe , a smaller step is required.

3. In [16], a more effective method for solving ordinary differential equations is proposed — the local collocation method. Its essence is that the integration

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region is divided into parts, in each of which an approximate local solution is calculated using the collocation method. As a result, a solution in the entire domain is obtained using a system of linear algebraic equations with a tape matrix. The convergence of the method is proved and estimates of the rate of convergence of the approximate solution to the exact one are obtained. It is shown that the local collocation method is equivalent to a finite-difference method of high order of accuracy. This method allows you to localize and determine the most dangerous eigenvalues, it is also effective for determining one eigenvalue.

4. Along with the methods described above, in the problems of hydrodynamic stability, the method of step-by-step integration is used. The essence of this method is to reduce boundary value problems to Cauchy problems and integrate the latter with an arbitrary variable step. Among them, the shooting method was the first to be used for hydrodynamic stability problems [17]. The calculations showed that if the eigenvalues of the matrix differ greatly in the value of the material part, and this is characteristic of equation (1) - (2), then during the integration process, due to numerical errors, almost no correct sign remains in the eigenvalues.

5. To overcome this difficulty, an orthogonalization method was proposed in [18]. It consists in dividing the integration interval into rather short sections, the length of which should be the smaller, the greater the difference between the values of different eigenvalues. Then, in each section, two solutions are orthogonalized and normalized. The disadvantage of this method is that it requires a large amount of computation to determine one eigenvalue.

6. In [19], a method is described for constructing basic solutions using the elimination method, which differs little from the orthogonalization method, but in the elimination method the integration and orthogonalization procedures are combined, which leads to savings in arithmetic calculations. In some cases, the differential sweep method is more convenient [20]. Its application for solving the hydrodynamic stability problem is described in [21]. The essence of the method is that in the process of direct sweep, solutions are found that satisfy the boundary conditions at one of the ends of the interval. After solving the eigenvalue problems, the eigenfunctions are found by inverse sweeping. Close to the differential sweep method is the determinant method [22]. In [23], this method was applied to localize the eigenvalues in studying the stability of the boundary layer. These methods also provide for the definition of one eigenvalue.

7. The use of spectral methods for the numerical simulation of problem (1) (2) was described in [24], where Chebyshev polynomials of the first kind were used as basis functions. It is shown that in this case the convergence in the number of basis functions is

exponential. The main advantage of these methods is that you can immediately find all spectral values and choose the most unstable among them. However, finding all the eigenvalues of a filled matrix of a high order is a very time-consuming process, associated with high costs of computer time. In addition, with an increase in the parameter kRe , the size of the matrices necessary for sufficiently accurate determination of the eigenvalues increases, and this imposes an additional memory requirement.

8. In the article [25], for the numerical integration of equation (1) - (2), the preliminary integration method is used. The essence of this method is that the solution of equation (1) - (2) is expanded in a series according to Chebyshev polynomials of the first kind, the derivatives of the solution are also represented as a series in these polynomials. Then, equations (1) are preliminarily fourfold integrated and four integration constants appear in the obtained equations. These constants are selected from the condition of satisfying four boundary conditions. The result is a generalized algebraic eigenvalue problem. Solving this system, all eigenvalues of equation (1) - (2) are determined. The complex matrices of the algebraic system will be filled, and with an increase in the number of basis functions, their order increases sharply, which leads to a deterioration in the accuracy of calculating the eigenvalues.

It follows from the above analysis that almost all of the listed methods, except the spectral method and the method of preliminary integration, are designed to find one eigenvalue.

ANALYSIS OF RESULTS

The development of effective numerical methods for immediately determining all the eigenvalues of the hydrodynamic stability problem (1) - (2) is of undoubted interest. The need to create such methods is especially evident in mathematical modeling of more complex problems, in particular, in the study of the stability problem of two-phase and multiphase flows. In this regard, it is necessary to have such a numerical solution method that would ensure high accuracy of calculations with a small amount of memory and time. In addition, it is desirable that the method under consideration allows us to simultaneously calculate all the eigenvalues of the problem. The spectral-grid method (CCM) [26-32] meets all these requirements and combines the high accuracy of spectral methods with the efficiency of the uneven grid method. Using this method, all eigenvalues of the hydrodynamic stability problem are determined. Theoretical substantiations and numerical results for solving the problem on the eigenvalues of single-phase and two-phase flows are given in [26]. The equivalence of the spectral-grid method with the method of inhomogeneous spline approximation was established in [27]. In [28], convergence theorems for the spectral-grid method were proved and estimates of

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the convergence rate of the method were obtained. The rationale for the efficiency of the spectral-grid method is described in [29]. The use of the spectral-grid method for the numerical simulation of more complex problems of hydrodynamic stability for two-phase flows is described in [30–32]. The spectral-grid method (CCM) consists in the fact that the integration region is divided into a grid, in each of whose elements an approximate solution is sought in the form of a series of Chebyshev polynomials of the first kind. At the internal nodes of the grid, continuity of the solution of the stability equations and their derivatives up to the $(m - 1)$ th order, where m is the order of the highest derivative of the differential equation, is required. At the boundary of the integration interval, the satisfaction of the

corresponding boundary conditions of the stability problem is required. An approximate solution in the entire grid domain is determined by solving the generalized eigenvalue problem for a system of linear algebraic equations with a special block-diagonal matrix.

We turn to the presentation of the SSM algorithm for the numerical simulation of problem (1) (2). To do this, on the integration interval $[\eta_0, \eta_1]$ we introduce a grid and get N different elements:

$$[\eta_0, \eta_1], [\eta_1, \eta_2], \dots, [\eta_j, \eta_{j+1}], \dots, [\eta_{N-1}, \eta_N], \\ j = 1, 2, \dots, N - 1$$

Differential equation (1) on each of these elements takes the form

$$D^2 \psi_j - ik Re [(U_j(\eta) - \lambda) D - U_j''] \psi_j = 0, j = 1, 2, \dots, N - 1. \quad (3)$$

The boundary conditions (2) are written at the points η_0 and η_N :

$$\psi_1(\eta_0) = \frac{d\psi_1}{d\eta}(\eta_0) = 0; \\ \psi_N(\eta_N) = \frac{d\psi_N}{d\eta}(\eta_N) = 0 \quad (4)$$

At the partition points, we require continuity of the solution of equation (3) and its derivatives up to 3rd order. These conditions are of the form:

$$\eta = \frac{m_j}{2} + \frac{l_j}{2} y, m_j = \eta_j + \eta_{j-1}, l_j = \eta_j - \eta_{j-1},$$

l_j denotes the length of the j th element. After this transformation, equations (3) take the form

$$D_j^2 \psi_j - ik_j Re_j [(U_j(y) - \lambda) D - U_j''] \psi_j = 0, j = 1, 2, \dots, N - 1, \quad (6)$$

$$D_j = \frac{d^2}{dy_j^2} - k_j^2, k_j = \frac{l_j}{2} k, Re_j = \frac{l_j}{2} Re.$$

From conditions (4) - (5) we have

$$\psi_1(-1) = 0, \frac{d\psi_1}{dy}(-1) = 0 \\ l_j^{(-t)} \psi_j^{(t)}(+1) = l_{j+1}^{(-t)} \psi_{j+1}^{(t)}(-1), t = 0, 1, 2, 3, j = 1, 2, \dots, N - 1, \\ \psi(+1) = 0, \frac{d\psi_N}{dy}(+1) = 0. \quad (7)$$

We seek an approximate solution to problem (6) - (7) on each of the grid elements in the form

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$$\psi_j(y) = \sum_{n=0}^{p_j} a_n^{(j)} T_n(y)$$

$$U_j(y_i^j) = \sum_{n=0}^{p_j} b_n^{(j)} T_n(y_i^j),$$

$$y_l^j = \cos\left(\pi \frac{l}{p_j}\right), \quad l = 0, 1, 2, \dots, p_j, \quad j = 1, 2, \dots, N, \tag{8}$$

where $T_n(y)$ -are Chebyshev polynomials of the first kind, y_l^j their nodes, and p_j is the number of polynomials used to approximate the solution on the j th element of the grid.

The expansion coefficients $b_n^{(j)}$ [26] for the function $U_j(y)$ (8) are determined by the following inverse transformation:

$$b_n^{(j)} = \frac{2}{p_j c_n} \sum_{l=0}^{p_j} \frac{1}{c_l} U_j(y_l^{(j)}) T_n(y_l^{(j)}), \quad c_0 = c_{p_j} = 2, \quad c_m = 1$$

$$m \neq 0, p_j, j = 1, 2, \dots, N.$$

For the convenience of exposing the SSM, we write equation (6) in the operator form, i.e.

$$L_j \psi_j = 0, j = 1, 2, \dots, N,$$

where the L_j -differential operator defined by the formula

$$L_j = D_j^2 - ik_j Re_j [(U_j(y) - \lambda) D_j - U_j''(y)]$$

Substituting series (8) into equation (7), we require that the left-hand side of (6) on each of the grid elements be orthogonal to the first $(p_j - 4)$ Chebyshev polynomials:

$$\sum_{n=0}^{p_1} (-1)^n a_n^{(1)} = 0,$$

$$\sum_{n=0}^{p_1} (-1)^{n-1} n^2 a_n^{(1)} = 0,$$

$$\sum_{n=0}^{p_j} a_n^{(j)} = \sum_{n=0}^{p_{j+1}} (-1)^n a_n^{(j+1)},$$

$$\frac{1}{l_j} \sum_{n=0}^{p_j} (n)^2 a_n^{(j)} = \frac{1}{l_{j+1}} \sum_{n=0}^{p_{j+1}} (-1)^{n-1} n^2 a_n^{(j+1)}$$

$$\frac{1}{l_j^2} \sum_{n=0}^{p_j} a_n^{(j)} T_n''(+1) = \frac{1}{l_{j+1}^2} \sum_{n=0}^{p_{j+1}} T_n''(-1),$$

$$\frac{1}{l_j^3} \sum_{n=0}^{p_j} a_n^{(j)} T_n'''(+1) = \frac{1}{l_{j+1}^3} \sum_{n=0}^{p_{j+1}} T_n'''(-1),$$

$$j = 1, 2, 3, \dots, N - 1, \quad ,$$

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$$\sum_{n=0}^{p_N} a_n^{(N)} = 0$$

$$\sum_{n=0}^{p_N} n^2 a_n^{(N)} = 0. \tag{10}$$

In formula (10), derivatives of Chebyshev polynomials are calculated by the following recurrence formulas:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), T_0(x) = 1, T_1(x) = x, n = 1, 2, 3, \dots,$$

$$T_{n+1}(x) = 2xT_n(x) + 2T_n(x) - T_{n-1}(x), T_0(x) = 0, T_1(x) = 1, n = 1, 2, 3, \dots,$$

$$T_{n+1}(x) = 2xT_n(x) + 4T_n(x) - T_{n-1}(x), T_0(x) = 0, T_1(x) = 0, n = 1, 2, 3, \dots,$$

$$T_{n+1}(x) = 2xT_n(x) + 6T_n(x) - T_{n-1}(x), T_0(x) = 0, T_1(x) = 1, n = 1, 2, 3, \dots,$$

In the general case, when different numbers of Chebyshev polynomials are given on different elements, we obtain

$$m^- = (p_1 + p_2 + \dots + p_N + N)$$

equations to determine the same number of unknowns. It is convenient to write the resulting system in matrix form:

$$(A - \lambda B)X = 0, \tag{11}$$

where the complex matrices A and B have a block-diagonal structure of a special form, and the vector x contains the coefficients $a_n^{(j)}$ in expansion (8), i.e.

$$x^T = (a_0^{(1)}, a_1^{(1)}, \dots, a_{p_1}^{(1)}, a_0^{(2)}, a_1^{(2)}, \dots, a_{p_2}^{(2)}, \dots, a_0^{(N)}, a_1^{(N)}, \dots, a_{p_N}^{(N)})$$

In system (11), the matrix B is degenerate and contains 4N zero rows corresponding to boundary conditions and continuity conditions, since they are independent of λ . The corresponding rows of the matrix A will contain integers obtained from the values of the Chebyshev polynomials and their derivatives to the third order at the points -1 or 1. Using non-degenerate transformations of the columns of the matrices A and B, we reduce system (11) to the form

$$(AQ - \lambda BQ)(Q^{-1}x) = 0, \tag{12}$$

where Q is the corresponding non-degenerate transformation [26]. With this transformation Q, the zero rows of the matrix B do not change, and the nonzero ones are transformed according to the transformation Q. Then, from each block of the matrices AQ and BQ, the first four rows and the first four columns can be excluded. From the remaining equations we obtain an algebraic system

$$(T - \lambda W)Y = 0, \tag{13}$$

where W is in the general case a non-degenerate square matrix. Then the order of the matrices T and W will be: $(m^- - 4N) \times (m^- - 4N)$, where m^- is the total number of polynomials in the CCM, i.e.

$$\bar{m} = \sum_{(j-1)}^N (p_j + 1)$$

Multiplying (13) on the left by the matrix $W - I$, we obtain

$$(D - \lambda E)Y = 0, D = TW^{-1} \tag{14}$$

standard eigenvalue problem. The eigenvalues of system (14) can be found by standard methods. In this paper, they are determined using the QR algorithm.

CONCLUSION

From an analysis of methods for solving the hydrodynamic stability problem, it can be determined that the spectral method (SM), the preliminary integration method (MPI) and the spectral-grid method allow one to determine all eigenvalues of the problem (1) - (2), and the remaining methods are designed to determine only one own value. Therefore, we compare SM, MPI and SSM in accuracy and in the number of arithmetic operations. In the table. 1. data are presented showing the accuracy of the calculations of the unstable mode (harmonic with the largest value λ_i for the Poiseuille flow $U(y) = 1 - y^2$ for various numbers of basis functions. The parameters Re, k, are fixed: $Re = 10^4, k = 1$. Their value is selected in accordance with the results of [24.25]. Calculations of the quantity $\lambda = \lambda_r + i\lambda_i$, corresponding to $\max(\lambda_i)$ are given in Table 1.

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Table 1

m^-	Methods	$\lambda = \lambda_r + i\lambda_i$
	CM	0.23690887+0.00365515i
15	MPI	0.23690886+0.00365513i
	SSM	0.23690887+0.00365517i
	SM	0.23752649+0.00373967i
32	MPI	0.23752650+0.00373953i
	SSM	0.23752649+0.00373967i
Exact Value [24,25]		0.23752649+0.00373967i

Now compare these methods in terms of the number of arithmetic operations. To solve the eigenvalue problem of the form (14) with a complex matrix D, the implementation of a single step of the

QR algorithm requires approximately $\bar{Q} = \frac{20}{3}n^3$ arithmetic operations, where n -indicates the order of the matrix D. The comparison results are given in table. 2.

Table 2

m^-	MPI	SM	SSM	
	Q^-	Q^-	N	Q^-
5	837	7	1	6
20	53600	27443	4	426
50	837500	652150	10	6666
100	6700000	5927730	20	426666

In tables 1 and 2, m^- denotes the total number of Chebyshev polynomials of the first kind for approximating the solution of problem (1) - (2), and N denotes the number of grid elements in the spectral-

grid method. The results presented illustrate that the spectral-grid method is economical and has high accuracy.

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