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OAJI (USA) = 0.350

SOI: [1.1/TAS](#) DOI: [10.15863/TAS](#)

International Scientific Journal Theoretical & Applied Science

p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online)

Year: 2019 Issue: 12 Volume: 80

Published: 30.12.2019 <http://T-Science.org>

QR – Issue



QR – Article



Ismoil Ibragimovich Safarov

Tashkent Institute of Chemistry and Technology
Doctor of Physical and Mathematical Sciences,
Professor to department of Advanced Mathematics, Uzbekistan
Safarov54@mail.ru

Muxsin Khudoyberdiyevich Teshaev

Bukhara Engineering-Technological Institute
Doctor of Physical and Mathematical Sciences,
Professor to department of Advanced Mathematics,
Bukhara, Republic of Uzbekistan,

Zafar Ixtiyorovich Boltaev

Bukhara Engineering-Technological Institute
Senior Lecturer to Department of Technology Engineering, docent,
Bukhara, Republic of Uzbekistan,

Nurillo Raximovich Kulmuratov

Navoi State Mining Institute
Senior Lecturer to Department of Technology Engineering,
nurillo.Kulmuratov.64@mail.ru

Nurbek Nurilloevich Hamroev

Navoi State Mining Institute
Senior Lecturer to Department of Technology Engineering,
Magistrate, Uzbekistan

OWN WAVES IN A SPATIAL VISCOELASTIC CYLINDER WITH RADIAL CRACK

Abstract: This work considers the propagation of natural waves by an infinite viscoelastic cylinder with a radial crack. The task is posed in cylindrical coordinate systems. Using the Navier equation and the physical equation, a system of six differential equations is obtained. After not complicated transformations, a spectral boundary-value problem was obtained for a system of ordinary and partial differential equations with complex coefficient equations, which is further solved by the direct and orthogonal Godunov sweep method with a combination of the Mueller and Gauss methods. The dispersion relation is obtained for a viscoelastic cylinder with a radial crack. It was found that, in the case of a cylinder with a radial crack, the first mode has a cutoff frequency, and the phase velocity tends to infinity. At large wavenumbers, the limiting phase velocity of this mode coincides with the velocity of the Rayleigh wave. At the cutoff frequency, the axial displacements are equal to zero and the oscillations of the cylinder occur in a plane deformed state.

Key words: crack, viscoelastic cylinder, freezing procedure, Navier equation, orthogonal sweep, ordinary differential equation.

Language: English

Citation: Safarov, I. I., Teshaev, M. K., Boltaev, Z. I., Kulmuratov, N. R., & Hamroev, N. N. (2019). Own waves in a spatial viscoelastic cylinder with radial crack. *ISJ Theoretical & Applied Science*, 12 (80), 341-345.

Soi: <http://s-o-i.org/1.1/TAS-12-80-67> **Doi:**  <https://dx.doi.org/10.15863/TAS.2019.12.80.67>

Introduction.

In a number of works, for the control of linearly extended objects, the use of a rod wave is proposed to give rise to the minimum velocity dispersion and torsion wave mode, in which there is no dispersion [1,2]. As an informative parameter in the waveguide control of linearly extended objects, as a rule, the reflection coefficient is used. The specified parameter does not allow to identify longitudinal defects [3,4]. Taking into account the damping ability of the waveguide material plays an important role in the dynamic behavior of the structure [5,6]. It leads to a noticeable weakening of natural oscillations, a significant decrease in amplitudes during forced oscillations and smoothing of stresses in the concentration zone during oscillations. The complexity of their solution is explained by many reasons, for example, the rheological properties of real waveguides, non-classical geometric shapes, etc., which causes a wide variety of schematized models to describe real phenomena in one approximation or another and makes it difficult to create a unified mathematical model of a mechanical system [7].

The dispersion dependences having a certain number of traveling wave modes in the frequency range were obtained in [8, 9]. In this paper, we consider one of the problems of this type on the propagation of Eigen waves in an isotropic viscoelastic cylindrical waveguide with a radial crack. The viscoelastic properties of materials are described using the Boltzmann – Voltaire integral [10, 11]. A solution technique and an algorithm have been developed to study the propagation of waves in a viscoelastic cylinder with a radial crack.

2. Statement of the problem of wave propagation in an infinite cylinder with a radial crack.

A viscoelastic isotropic cylindrical waveguide is considered which occupies a region in

$$V = \{r_0 < r \leq R, 0 < \theta \leq \theta_0, -\infty < z < \infty\},$$

a dimensionless system of cylindrical coordinates (r, ϕ, z) . The waveguide has a collinear direction Oz axis. Let the natural (harmonic) waves propagate along the Oz axis in an infinite viscoelastic cylinder with a radial crack. The relationship between stress and strain is as follows [7]

$$\sigma_{ik} = \tilde{\lambda}\theta\delta_{ik} + 2\tilde{\mu}\varepsilon_{ik}.$$

Here σ_{ik} - is the stress tensor, ε_{ik} - is the strain tensor, θ - is the volumetric strain, $\tilde{\lambda}$ and $\tilde{\mu}$ is the operator modulus of elasticity [3,5]

$$\tilde{\lambda}\phi(t) = \lambda_{01} \left[\phi(t) - \int_0^t R_\lambda(t - \tau)\phi(\tau)d\tau \right];$$

$$\tilde{\mu}\phi(t) = \mu_{01} \left[\phi(t) - \int_0^t R_\mu(t - \tau)\phi(\tau)d\tau \right], \quad (2)$$

$\phi(t)$ – a random function of time; $R_\lambda(t - \tau)$ and $R_\mu(t - \tau)$ - relaxation nuclei and λ_{01}, μ_{01} - instant elastic moduli.

We take the integral terms in (2) as small, then the functions $\phi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$ - is a slowly varying function of time, ω_R - is a real constant. Further, applying the freezing procedure [5], we note relations (2) with approximate forms

$$\begin{aligned} \tilde{\lambda}\phi &= \lambda_{01} [1 - \Gamma_\lambda^C(\omega_R) - i\Gamma_\lambda^S(\omega_R)]; \tilde{\mu}\phi \\ &= \mu_{01} [1 - \Gamma_\mu^C(\omega_R) - i\Gamma_\mu^S(\omega_R)]\phi, \end{aligned}$$

Where

$$\begin{aligned} \Gamma_\lambda^C(\omega_R) &= \int_0^\infty R_\lambda(\tau) \cos \omega_R \tau d\tau; \Gamma_\lambda^S(\omega_R) \\ &= \int_0^\infty R_\lambda(\tau) \sin \omega_R \tau d\tau, \end{aligned}$$

$$\begin{aligned} \Gamma_\mu^C(\omega_R) &= \int_0^\infty R_\mu(\tau) \cos \omega_R \tau d\tau; \Gamma_\mu^S(\omega_R) \\ &= \int_0^\infty R_\mu(\tau) \sin \omega_R \tau d\tau \end{aligned}$$

- respectively, the cosine and sine Fourier images of the core relaxation of the material. As an example of a viscoelastic material, we take three parametric relaxation nuclei

$$R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}.$$

The basic equations of motion of a viscoelastic cylindrical mechanical waveguide occupying region V, which are defined by the following equations [2]:

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (3)$$

where ρ - is the density of the material.

Relations (1) and (3) after identical algebraic transformations are reduced to a system of six differential equations resolved with respect to the first derivative with respect to the radial coordinate

$$\frac{\partial u_r}{\partial r} = \frac{1}{K_\lambda} \sigma_{rr} - \frac{\tilde{\lambda}}{K_\lambda} \left(\frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right);$$

$$\frac{\partial u_\phi}{\partial r} = \frac{1}{\tilde{\mu}} \sigma_{r\phi} - \frac{1}{r} \left(\frac{\partial u_r}{\partial \phi} - u_\phi \right);$$

$$\frac{\partial u_z}{\partial r} = \frac{1}{\tilde{\mu}} \sigma_{rz} - \frac{\partial u_r}{\partial z};$$

$$\frac{\partial \sigma_{rr}}{\partial r} = \rho \frac{\partial^2 u_r}{\partial t^2} - \frac{\tilde{A}}{r} - \frac{1}{r} \frac{\partial \sigma_{r\phi}}{\partial \phi} - \frac{\partial \sigma_{rz}}{\partial z};$$

$$\frac{\partial \sigma_{r\phi}}{\partial r} = \rho \frac{\partial^2 u_\phi}{\partial t^2} - \frac{1}{r} \frac{\partial}{\partial \phi} [\sigma_{rr} - \tilde{A}] - \frac{2\sigma_{r\phi}}{r} - \frac{\partial}{\partial z} \tilde{B};$$

$$\begin{aligned} \frac{\partial \sigma_{rz}}{\partial r} &= \rho \frac{\partial^2 u_z}{\partial t^2} - \frac{\partial}{\partial z} \left[\sigma_{rr} - 2\tilde{\mu} \left(\frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial z} \right) \right] - \frac{\sigma_{rz}}{r} \\ &\quad - \frac{1}{r} \frac{\partial}{\partial \phi} \tilde{B}, \end{aligned}$$

$$\tilde{A} = 2\tilde{\mu} \left[\frac{\partial u_r}{\partial r} - \frac{1}{r} \left(\frac{\partial u_\phi}{\partial \phi} + u_r \right) \right];$$

$$\tilde{B} = \tilde{\mu} \left(\frac{\partial u_\phi}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \phi} \right), K_\lambda = \tilde{\lambda} + 2\tilde{\mu}.$$

Where $\sigma_{rr}, \sigma_{r\phi}, \sigma_{rz}, \sigma_{\phi\phi}, \sigma_{\phi z}, \sigma_{zz}$ are the components of the stress tensor, respectively; $\varepsilon_{rr}, \varepsilon_{r\phi}, \varepsilon_{rz}, \varepsilon_{\phi\phi}, \varepsilon_{\phi z}, \varepsilon_{zz}$ - respectively, the components of the strain tensor. The relationship between stress and strain is given in (1). The boundary conditions are set in the form:

$$r = r_0 \rightarrow 0, R: \sigma_{rz} = \sigma_{rr} = \sigma_{r\phi} = 0; \quad (5)$$

$$\phi = 0, 2\pi: u_\phi = 0; \sigma_{\phi z} = \sigma_{\phi r} = 0, \quad (6)$$

conditions (5) at $r = 0$, in the physical plane it can be interpreted as the result of the passage to the limit, from a hollow cylinder with a free inner surface to a solid, when the inner radius tends to zero.

3. Solution Methods.

In the case of traveling harmonic waves along the z axis, the solution of the boundary value problem (7), (8), (9) allows separation of variables [8]

$$u_r = w(r) \cos \frac{\phi}{2} \cos(kz - \omega t);$$

$$u_\phi = v(r) \sin \frac{\phi}{2} \cos(kz - \omega t);$$

$$u_z = u(r) \cos \frac{\phi}{2} \sin(kz - \omega t);$$

$$\sigma_{rr} = \sigma(r) \cos \frac{\phi}{2} \cos(kz - \omega t);$$

$$\sigma_{r\phi} = \tau_\phi(r) \sin \frac{\phi}{2} \cos(kz - \omega t);$$

$$\sigma_{rz} = \tau_z(r) \cos \frac{\phi}{2} \sin(kz - \omega t), \quad (7)$$

Where $w(r), v(r), u(r)$ - are the displacement amplitudes, $\sigma(r), \tau_\phi(r), \tau_z(r)$ - are the stress amplitudes of the stress tensor components, $\omega = \omega_R + i\omega_I$ - is the complex natural frequency, ω_R -is the natural wave propagation frequency, ω_I - is the damping coefficient, κ - is the wave number, $C = \omega/k$ -is the phase velocity.

Substituting (7) into (4), (5), (6), we obtain the spectral boundary value problem. The problem is reduced to a system of ordinary differential equations with complex coefficients

$$w' = \frac{\sigma}{K_\lambda} - \frac{\tilde{\lambda}}{K_\lambda} \left(ku + \frac{v}{r} + \frac{w}{r} \right);$$

$$v' = \frac{\tau_\phi}{\mu} + \frac{v}{r} + \frac{w}{2r};$$

$$u' = \frac{\tau_z}{\mu} + kw;$$

$$\sigma' = -\omega^2 \rho w + \frac{\tilde{a}}{r} - \frac{\tau_\phi}{2r} - k\tau_z;$$

$$\tau_\phi' = -\omega^2 \rho v - \frac{2\tau_\phi}{r} + (\sigma + \tilde{a}) \frac{1}{2r} - k\tilde{b};$$

$$\tau_z' = -\omega^2 \rho u - \frac{\tau_z}{r} - \frac{\tilde{b}}{2r} + k \left(\sigma + 2\mu \left(ku - w' \right) \right), \dots' = \frac{d}{dr}. \quad (8)$$

Here

$$\tilde{a} = 2\mu \left(\frac{v+w}{2r} - w' \right); \tilde{b} = \mu \left(-\frac{u}{2r} - kv \right), t$$

the boundary conditions

$$r = r_0 \rightarrow 0: \sigma = \tau_\phi = \tau_z = 0;$$

$$r = R: \sigma = \tau_\phi = \tau_z = 0. \quad (9)$$

Thus, the spectral boundary-value problem (8), (9) is formulated that describes the propagation of harmonic waves in an infinite cylinder with a radial crack. We note that the choice of boundary conditions on the faces of the slit in the form of (6) was determined primarily by the possibility of separation of variables along the coordinates r and ϕ , which greatly simplifies the solution of the original problem. Separation of variables is also possible in the case of the following boundary conditions:

$$\phi = 0: \sigma_{\phi\phi} = 0; u_r = u_z = 0;$$

$$\phi = 2\pi: \sigma_{\phi\phi} = 0; u_r = u_z = 0. \quad (10)$$

Indeed, performing a change of variables in (7), (8) so that conditions (10) are satisfied

$$u_r = \tilde{w}(r) \sin \frac{\phi}{2} \cos(kz - \omega t);$$

$$u_\phi = \tilde{v}(r) \cos \frac{\phi}{2} \cos(kz - \omega t);$$

$$u_z = \tilde{u}(r) \sin \frac{\phi}{2} \sin(kz - \omega t);$$

$$\sigma_{rr} = \tilde{\sigma}(r) \sin \frac{\phi}{2} \cos(kz - \omega t);$$

$$\sigma_{r\phi} = \tilde{\tau}_\phi(r) \cos \frac{\phi}{2} \cos(kz - \omega t);$$

$$\sigma_{rz} = \tilde{\tau}_z(r) \sin \frac{\phi}{2} \sin(kz - \omega t), \quad (11)$$

we obtain a spectral boundary-value problem having complex coefficients and roots

$$\tilde{w}' = \frac{\tilde{\sigma}}{K_\lambda} - \frac{\lambda}{K_\lambda} \left(k\tilde{u} - \frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r} \right);$$

$$\tilde{u}' = \frac{\tilde{\tau}_\phi}{\mu} + \frac{\tilde{v}}{r} - \frac{\tilde{w}}{2r};$$

$$\tilde{u} = \frac{\tilde{\tau}_z}{\mu} + k\tilde{w}; \quad (12)$$

$$\sigma' = \frac{2\mu}{r} \left(-\frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r} - \tilde{w}' \right) \frac{\tilde{\tau}_\phi}{2r}$$

$$\tau_\phi' = \frac{2\tilde{\tau}_\phi}{r} \frac{1}{2r} \left(\tilde{\sigma} + 2\mu \left(-\frac{\tilde{v}}{2r} + \frac{\tilde{w}}{r} - \tilde{w}' \right) - k \left(\frac{\tilde{u}}{2r} - k\tilde{v} \right) \right)$$

$$\tilde{\tau}_z' = -\rho\omega^2 \tilde{u} - \frac{\tilde{\tau}_z}{r} + \frac{\mu}{2r} \left(\frac{u}{2r} - k\tilde{v} \right) + k \left(\tilde{\sigma} + 2\mu \left(k\tilde{u} - \tilde{w}' \right) \right).$$

With boundary conditions

$$r = r_0 \rightarrow 0: \tilde{\sigma} = \tilde{\tau}_\phi = \tilde{\tau}_z = 0;$$

$$r = R: \tilde{\sigma} = \tilde{\tau}_\phi = \tilde{\tau}_z = 0 \quad (13)$$

It is easy to see that problem (12), (13) reduces to problem (8), (9)

Using replacement

$$\tilde{\tau}_z = \tau_z, \tilde{\tau}_\phi = -\tau_\phi, \tilde{\sigma} = \sigma, \tilde{w} = w, \tilde{u}_\phi = -u_\phi, \tilde{u}_z = u_z.$$

The solution of problem (8), (9) was carried out by the orthogonal method Runs of Godunov, Muller and Gauss [9, 10].

At the cutoff frequency, the axial displacements are equal to zero and the oscillations of the cylinder occur in a plane deformed state.

In the second mode, cutoff frequencies are observed at $0 \leq k \leq 0.075$. Only real and part opinion axial displacements, annular and radial displacements are equal to zero. The curves are numbered in the order of growth of k . Note the strong dependence of the

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forms on the wave number. With an increase in the wave number in the first mode, localization of oscillations near the outer surface of the cylinder takes place. It is characteristic that the second mode, which

at small wavenumbers is a form of predominantly axial vibrations, gradually grows into a form of predominantly radial vibrations with increasing k .

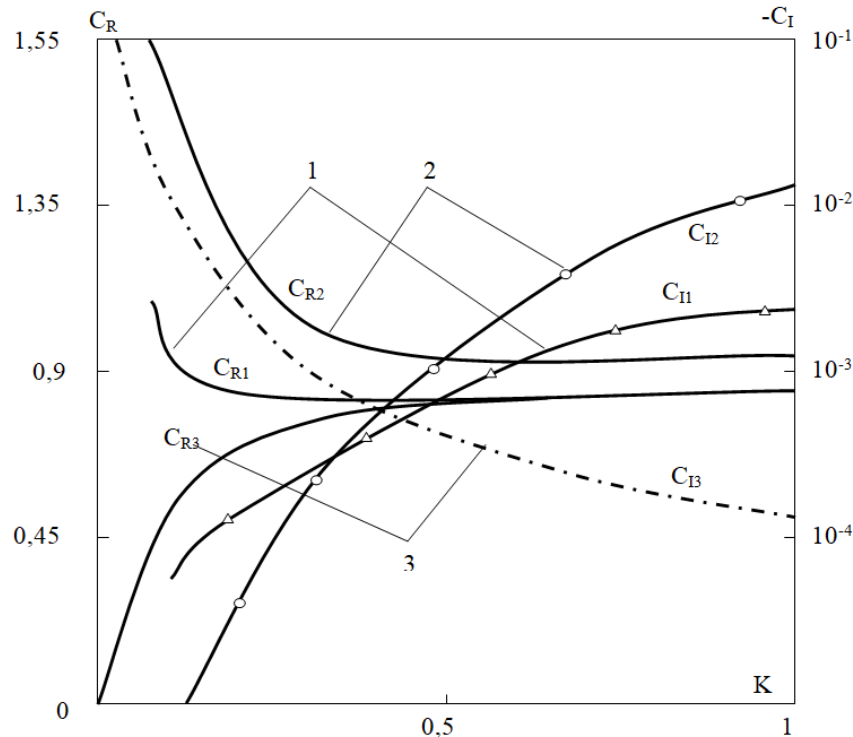


Fig. 1. Change the real and imaginary parts of the oscillation frequency depending on k .

1. C_{R1}, C_{I1} , - real and imaginary parts of the first mode of the complex phase velocity of the cylinder by a radial crack;
2. C_{R2}, C_{I2} , - real and imaginary parts of the second mode of the complex phase velocity of the cylinder by a radial crack;
3. C_{R3}, C_{I3} - real and imaginary parts of the first mode of the complex phase velocity of a continuous cylinder.

Findings.

1. It was found that in an elastic cylinder with a radial crack there are no waves having real parts of the phase velocity localized near the axis of the cylinder.

2. Taking into account the viscoelastic properties of the wedge material reduces the real parts of the wave propagation velocity by 10-15%, and also allows you

to evaluate the damping capabilities of the system as a whole.

This work was supported by the Foundation for Basic Research F-4-17 of the Republic of Uzbekistan.

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