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Stock Price Forecasting Using Univariate Singular Spectral Analysis through Hadamard Transform

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Abstract: The stock price prediction or forecasting is one of the important information in the stock market. Investors need stock price information before buying shares, while shareholders need stock price information for selling shares. There are many stock price prediction techniques have been proposed in the time series analysis. One of the simplest and powerful techniques is singular spectrum analysis, which works on the time series decomposition that is constructed using sub-series or window of the initial time series. However, choosing the exact window length is not easy because it depends on the time series characteristics. In this paper, the Hadamard transform of time series is proposed as an alternative technique to choose the window length in time series embedding. Technically, the length of the window for time series embedding is determined directly based on the size of the Hadamard spectrum. The experimental results show that the proposed method not only facilitates the determination of window length in time series embedding but can also improve the performance of the standard singular spectrum analysis method. The error rates of the proposed and the baseline methods (the standard singular spectrum analysis and the standard singular spectrum analysis with minimum description length) are 0.0088, 0.0194, and 0.1441, respectively.

Keywords: Singular spectrum analysis, Stock price forecasting, Time series embedding, Time series decomposition, Hadamard transform.

1. Introduction

The stock market has several basic components, which are traders, buyers, and bargaining activities. Traders can be companies, or stock agents and dealers. Buyers are generally investors who intend to invest capital through company shares. Shareholders might sell their shares to get a fresh of funds when they need additional capital. They also might sell their shares when their shares have an upward trend in prices since the purchase of shares. Instead, investors will buy shares when the share price is low, and/or expect future profits if the stock price tends to rise. The stock prices generally change many times in minutes to hours daily depending on the market supply and demand [1]. Almost all of the stock stakeholders need quality information to make a decision about whether they want to buy or sell their shares.

The needs of stock price information are very necessary because there are many factors that affect stock price volatility because they always want a profit and do not expect any loss in the stock market. The stock price volatility can be affected by corporate action (e.g., acquisition, merger, right issue, divested shares, etc.), company performance estimation (e.g., debt dividends, debt ratios, price to book values, earning per share, profit level, etc.), government policies (e.g., export-import policy, debt policy, foreign investment policy, etc.), currency exchange rates, macroeconomics condition (e.g., inflation, interest rates, unemployment level, etc.), market rumors and sentiments, and also market manipulation. It is not easy to determine what certain factors that cause the stock price changes today, tomorrow, or in the future. There is no certain theory that can explain all of the conditions in the exact manner as each stock has different characteristics among others [1].

The stock price prediction is the most popular technique to analyze the stock price. Many stock prediction techniques have been introduced in many references that can be classified into three categories: fundamental analysis, technical analysis, and traditional time series forecasting [2]. In recent years, the time series forecasting has been an active field of study in the time series analysis. The autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroscedasticity (GARCH), and multivariate regression are the earliest popular models for time series analysis [3].

Hereafter, the time series forecasting in stock price prediction growth fast since data mining techniques introduced as the stock price prediction. Hsu proposed a self-organizing map (SOM) neural network and genetic programming (GP) for stock price prediction [2]. Kim and Han proposed a genetic algorithm (GAs) and artificial neural networks (ANN) to predict the stock price index [4]. The neural network approach is also used by Rajakumar et al. [5]. Kim proposed a support vector machine (SVM) and comparing to the backpropagation neural network (BPNN) and case-based reasoning (CBR) to evaluate the stock price prediction [6]. Pai and Lin proposed a hybrid method to predict stock price through ARIMA and SVM [7]. Tsang et al. [8] proposed a feed-forward backpropagation neural network for stock buying/selling alert system. Chang and Liu proposed a linear combination consequence using Takagi Sugeno Kang (TSK) fuzzy system to predict stock prices [9]. Huang and Tsai proposed a hybrid method using support vector regression (SVR), self-organizing feature map (SOFM), and filter-based feature selection to predict the stock prices [10].

The other approach that has more attention in stock forecasting is the singular spectrum analysis (SSA) as it is powerful to present the time series analysis [11, 12]. SSA is not just a time series analysis technique but also integrates multivariate statistics, multivariate geometry, dynamical systems, and signal processing [12-14]. SSA is also a non-parametric approach that is no need any statistically assumptions [15, 16]. Technically, SSA works through the time series decomposition into a sum of component series that consist of either trend, cycle, or noise [12, 15, 17-19].

One of the popular issues in the SSA implementation is related to the choice of window length to construct a trajectory matrix in time series embedding. Some literature suggests the window length should be large but not larger than half of the length of the time series. Choosing the best window length is not easy because it needs prior information about the time series characteristics [12, 20]. This study is motivated to address the window length selection issue by utilizing the time series embedding through the Hadamard transform to represent the time series spectrum. The objectives of this study are not only to provide an alternative solution on window length selection but also to improve the SSA method performance on the time series analysis.

This paper is presented in several sections, as follows. Section 2 presents many various methods based on SSA for time series analysis, specifically in stock price prediction or forecasting. Section 3 presents a basic algorithm of SSA. Section 4 presents the proposed method, which is the stock price forecast using the SSA method through the Hadamard transform. Section 5 presents the experimental setup of this study, including the dataset preparation and performance evaluation. The experimental results and discussion are presented in Section 6. Finally, the conclusions and future works of this study are summarized in Section 7.

2. Related works

There are many utilized methods based on SSA for stock price forecasting. Wen at al. [18] proposed the SSA to analyze the stock price of a trend, market fluctuation, and noise. They also introduced the SVM for price prediction. Their combined method of SSA and SVM is then compared to the ensemble empirical mode decomposition (EEMD) based on SSA, i.e. EEMD-SSA. Based on the mean squared error (MSE), the SSA-SVM performed better than the EEMD-SSA at the stock price predictive results. The other SSA-SVM approach by Xiao et al. [19] using three economic-financial components as additional features, which are long term trends, effects of significant events, and short term fluctuations.

Suksiri et al. [21] proposed the kernel-based extreme learning machine (KELM) as a pre-processing tool before applied the SSA for stock price prediction. Their purposed study is not only to improve the prediction accuracy but also to increase the speed of time training of prediction. Compared to the other SSA-SVM based methods, the SSA-KELM has better performance in root mean squared error (RMSE). Ogrosky et al. [22] modified the basic SSA using conditional predictions (SSA-CP) to improve the time-lag embedding in SSA.

Lahmiri [23] also utilized the SSA using SVR coupled with particle swarm optimization (PSO) to improve the performance of time series based on SSA. The used of PSO in his method able to present the hidden information in time series, such as trend and harmonic components. The experiment result showed that the coupled method SSA-PSO-SVR has better performance compared to the non-SSA based time series analysis. Leles et al. [24] proposed the new technical trading rules (TTR) for the oscillator type to reveal the market movements. Their proposed method, i.e. SSA-TTR outperformed some popular technical oscillators.

The improved SSA based methods are also proposed in the other financial time series. Suhartono et al. [14] proposed the hybrid SSA and Neural Networks (SSA-NN) to forecast currency circulation. Abreau et al. [3] compared the SSA and ARIMA-GARCH methods to evaluate the model of currencies exchange rates. Their experiments show that the SSA forecasts are more appropriate than the ARIMA-GARCH. The used of SSA in currency exchange rates is also studied by Hassani et al. [25], using both univariate and multivariate SSA.

Regarding the use of SSA, there are basically three main problems in the choice of parameters in SSA, that are: (1) features of interest, (2) type of residuals, and (3) choice of the window length. The features of interest are related to the parameter estimation of damping factors in noise [17]. It is also affecting to choose the number of eigentriples that are used for time series reconstruction [11, 17]. The type of residuals is related to the behavior of estimation error that is caused by either deterministic or stochastic residuals. The choice of the window length is related to the size of the lag vectors that will be used for time series embedding. This study is proposed to address the alternative choice of window length in time series embedding.

Generally, in the standard or basic SSA, almost all of the current studies suggested the window length should be larger but not greater than half of the time series length to achieve better separately of trend and residual. This means that if the length of the time series is N, then the window length L for the lagged vectors in time series embedding is about $2 \leq L \leq N/2$ [13, 25, 26, 31, 36]. As the window length is an integer, thus there are many satisfied values of L as the tuning parameter that can be used to cover the time series embedding. It can be set manually using several L values depending on the number of time series periods and type of signal (such as $N/3$, $N/4$, $N/2$, 60, 120, etc.) and then select the best window length to embed the time series.

One of the current studies to choose the window length in the standard SSA is proposed by Khan and Poskitt [37] using the minimum description length (MDL) criterion to identify the dimension of the signal from the time series data. The window length of the time series with a length of N in their method is obtained as $log(N)^c, c \in (1.5, 2.5)$. This technique also needs to pay attention to choose the positive number of c in the range of 1.5 to 2.5. Based on the techniques to choose the window length in the standard SSA above, this work is proposed to determine the static window length without tuning the window length parameter by utilizing the spectrum of time series through the Hadamard transform, which is presented in the proposed method section.

3. Singular spectrum Analysis (SSA)

In time series analysis, SSA is basically a technique of extracting information from time-series data through its trajectories structure, and reconstruct the prediction or forecasting model using the additive components of time series. The basic SSA is presented into the two main stages that are decomposition and reconstruction [12, 13, 16, 25, 26].

3.1 Decomposition

The decomposition stage is a process to decompose the time series data through its trajectory matrix into the explainable components or spectral groups of data series, such as trend, oscillation or cycle, and noise [12, 19, 20, 23, and 27], which are directly difficult to be visualized and analyzed [27]. These are the following steps to perform the time series decomposition, as follows.

3.1.1. Time series embedding

Embedding of time series is a step to construct a trajectory matrix using a sub-series or element dimension of time series, which is usually called as window length. Given a one-dimensional time series data $X_t = (x_1, x_2, \dots, x_N)$ with $1 \leq t \leq N$ of length N. Let $X_i = (x_i, x_{i+1}, \dots, x_{i+L-1})^T$ is a sub-series vector of X_t with $1 < L < N, 1 \leq i \leq K$, and $K = N - L + 1$ [25, 31]. The window length L is an integer between 1 and N. However, the L value should be proportional to the time series length and large to obtain sufficiently separated components, but not greater than $N/2$ [13, 16, 20].

Supposed X_t consists of unknown components such as trend, cycles, or noise, which will be represented as lagged vectors X_i [14, 24, 27, and 28]. If the lagged vector is shifted along with the time series, then the time series X_t can be mapped into the sequence multi-dimensional lagged vectors of X_i, i.e. $X = (X_1, X_2, \dots, X_N)$ of size $L \times K$, as shown in

Eq. (1), and it is called the trajectory matrix of time series X_t[13,16,17,19,25,29], as follows:

$$\boldsymbol{X} = \begin{pmatrix} x_1 & x_2 & \cdots & x_K \\ x_2 & x_3 & \cdots & x_{K+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_L & x_{L+1} & \cdots & x_N \end{pmatrix}. \quad (1)$$

The trajectory matrix $\boldsymbol{X}$ is a Hankel matrix [12, 17, and 29] because the anti-diagonals elements from bottom-left to top-right are equal. The vector columns of the matrix $\boldsymbol{X}$ are sub-series of X_t with a length of L.

3.1.2. Time series decomposition

Time series decomposition is a step to get the elementary components from the trajectory matrix $\boldsymbol{X}$ that are used for recognizing the pattern of time series. The time series decomposition can be performed using singular value decomposition (SVD), as shown in Eq. (2) [18, 19, and 28], as follows:

$$\boldsymbol{X} = \boldsymbol{U\Sigma V^T}. \quad (2)$$

The trajectory matrix $\boldsymbol{X}$ can be formulated as three components, which are $\boldsymbol{U}$, $\boldsymbol{\Sigma}$, and $\boldsymbol{V^T}$ matrices, respectively. The $\boldsymbol{U}$ is a $L \times L$ unitary matrix of the left singular vector of $\boldsymbol{X}$. The $\boldsymbol{\Sigma}$ is a $L \times K$ rectangular diagonal matrix of singular values of$\boldsymbol{X}$. The $\boldsymbol{V^T}$ is a $K \times K$ unitary matrix of the right singular vector of $\boldsymbol{X}$. The eigenvalues decomposition works on a diagonal matrix, thus it needs to build the diagonalizable matrix for first. Multiplying $\boldsymbol{X^T}$ to the $\boldsymbol{X}$ in Eq. (2) will form the diagonal matrix $\boldsymbol{XX^T}$ that is eligible for eigenvalues decomposition [28]. The $\boldsymbol{XX^T}$ matrix, which is usually called the Gram matrix [30], is symmetric and positive semi-definite, and it is computed using Eq. (3). The singular values $\sigma_1, \sigma_2, \dots, \sigma_L$ of matrix $\boldsymbol{X}$ are the positive square roots, $\sigma_i = \sqrt{\lambda_i} > 0$, of the nonzero eigenvalues of the Gram matrix $\boldsymbol{XX^T}$ [30]. Whilst the corresponding eigenvectors of $\boldsymbol{XX^T}$ are known as the singular vectors of $\boldsymbol{X}$.

$$\begin{aligned} \boldsymbol{XX^T} &= \boldsymbol{U\Sigma V^T X^T} \\ &= \boldsymbol{U\Sigma V^T V \Sigma^T U^T} \\ &= \boldsymbol{U\Sigma\Sigma^T U^T} \\ &= \boldsymbol{U\Sigma^2 U^T}. \end{aligned} \quad (3)$$

Multiplying Eq. (3) with $\boldsymbol{U}$ on the right-hand side gives:

$$(\boldsymbol{XX^T})\boldsymbol{U} = \boldsymbol{U\Sigma^2}. \quad (4)$$

It shows that $\sigma_1, \sigma_2, \dots, \sigma_L$ are the singular values of $\boldsymbol{XX^T}$, which are the diagonal elements of the matrix $\boldsymbol{\Sigma}^2$. On the other hand, $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_L}$ with satisfy $\sqrt{\lambda_1} \geq \sqrt{\lambda_2} \geq \cdots \geq \sqrt{\lambda_L} \geq 0$ are the decreasing order non-negative eigenvalues of $\boldsymbol{XX^T}$ [17, 19, 25, 26, 31]. Besides, Eq. (4) also shows that the matrix $\boldsymbol{U}$ is singular vectors of $\boldsymbol{XX^T}$. Thus, if $U_1, U_2, \dots, U_L$ are the orthonormal eigenvector of $\boldsymbol{XX^T}$ correspond to singular values $\sigma_1, \sigma_2, \dots, \sigma_L$ with $U_i \cdot U_j = 0$ for $i \neq j$, then Eq. (4) shows that the columns of $\boldsymbol{U}$ are eigenvectors of $\boldsymbol{XX^T}$ with eigenvalues $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_L}$. Similarly, if the diagonalizable matrix is constructed using the Gram matrix $\boldsymbol{X^T X}$ then it will show that $V_1, V_2, \dots, V_L$ are eigenvectors of $\boldsymbol{X^T X}$ with the same eigenvalues $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_L}$ [30].

Suppose σ_i is the i^{th} singular value of $\boldsymbol{X}$, d is the rank of trajectory matrix $\boldsymbol{X}$ with $d \leq L$, U_i represents the i^{th} columns of $\boldsymbol{U}$, and let $V_i = X^T U_i / \sigma_i$ the right eigenvectors that are called as the principal components of trajectory matrix $\boldsymbol{X}$ [12, 13, and 17]. The SVD of the trajectory matrix $\boldsymbol{X}$ in Eq. (1) can be rewritten as the sum of rank-one bi-orthogonal elementary matrices [17], as follows:

$$\begin{aligned} X &= \sum_{i=1}^{d} \sqrt{\lambda_i} U_i V_i^T \\ &= \sqrt{\lambda_1} U_1 V_1^T + \sqrt{\lambda_2} U_2 V_2^T + \cdots + \sqrt{\lambda_d} U_d V_d^T \\ &= X_1 + X_2 + \cdots + X_d, \\ &= \sum_{i=1}^{d} X_i \end{aligned} \quad (5)$$

with $X_i = \sqrt{\lambda_i} U_i V_i$ for $i = 1, 2, \dots, d$. The matrices X_i are elementary matrices of $\boldsymbol{X}$ that have rank 1 [14]. In terms of SSA, the set $\{\sqrt{\lambda_i}, U_i, V_i^T\}$ is often called as the i^{th} eigentriple of $\boldsymbol{X}$ [12, 17, 20, and 29]. The SVD in Eq. (5) is optimal, which means that for all matrices $\boldsymbol{X^{(r)}}$ with $r < d$, the matrix $\sum_{i=1}^{r} X_i$ will give the best approximation of the trajectory matrix $\boldsymbol{X}$ with $\|\boldsymbol{X} - \boldsymbol{X^{(r)}}\|$ is minimum [12, 13, 28]. Through the Eq. (5), the Frobenius norm of $\boldsymbol{X}$ will result in the $\|\boldsymbol{X_i}\|_F = \lambda_i$ and $\|\boldsymbol{X_i}\|_F^2 = \sum_{i=1}^{d} \lambda_i$ for $i = 1, 2, \dots, d$. Thus, the ratio $\lambda_i / \sum_{i=1}^{d} \lambda_i$ can be considered as the characteristic of matrix $\boldsymbol{X}$, and the sum of the first of r ratios $\sum_{i=1}^{r} \lambda_i / \sum_{i=1}^{d} \lambda_i$ can be considered as the

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characteristic of the optimal approximation of trajectory matrix $\boldsymbol{X}$ [12,13].

3.2 Reconstruction

The reconstruction stage is performed to reconstruct the prediction or forecasting model using the independent components of the time series, which are extracted from the SVD of the trajectory matrix. This reconstruction stage is divided into two steps, which are grouping and diagonal averaging.

3.2.1. Eigentriple grouping

The eigentriple grouping is a step to split the elementary matrices X_i into m disjoint subsets $I_1, I_2, \dots, I_m$. Suppose $I = \{i_1, i_2, \dots, i_m\}$. The matrix $\boldsymbol{X_I}$ corresponds to the group I is defined as $\boldsymbol{X_I} = \boldsymbol{X_{i_1}} + \boldsymbol{X_{i_2}} + \cdots + \boldsymbol{X_{i_p}}$. Hence, computing for all $I_1, I_2, \dots, I_m$ to the matrix $\boldsymbol{X_I}$ will form the $\boldsymbol{X_I}$ as a sum of the groups I [12,17,29] as shown in Eq. (6):

$$\boldsymbol{X_I} = \boldsymbol{X_{I_1}} + \boldsymbol{X_{I_2}} + \cdots + \boldsymbol{X_{I_m}}, \tag{6}$$

with elements $\boldsymbol{X_I} = \sum_{i \in I}^{m} \sqrt{\lambda_i} U_i V_i^T$ is the resultant matrix corresponds to the group I with the eigentriple $\{\sqrt{\lambda_i}, U_i, V_i^T\}$.

3.2.2. Diagonal averaging

The diagonal averaging is a step to transform the grouped resultant matrix $\boldsymbol{X}$ in Eq. (6) into a new time series with a length of N. Let $\boldsymbol{Y}$ is a matrix of length $L \times K, Y^* = \min(L, K), K^* = \max(L, K)$, and $N = L + K - 1$. If $y_{ij} \in \boldsymbol{Y}$ with $1 \leq i \leq L$, $1 \leq j \leq K$ and $i + j = k + 1$, the matrix $\boldsymbol{Y}$ can be transformed into the series $(y_1, y_2, \dots, y_N)$ of length N using the diagonal averaging criteria in Eq. (7), which is also called as the Hankelization procedure [13-15,19,21,29]:

$$y_k = \begin{cases} \frac{1}{k} \sum_{m=1}^{k} y^*_{m,k-m+1}, & 1 \leq k \leq L^* \\ \frac{1}{L^*} \sum_{m=1}^{L^*} y^*_{m,k-m+1}, & L^* + 1 \leq k \leq K^* \\ \frac{1}{N-k+1} \sum_{m=k-K^*+1}^{N-K^*+1} y^*_{m,k-m+1}, & K^* + 1 \leq k \leq N \end{cases} \tag{7}$$

The reconstruction time series will be obtained after the diagonal averaging applied to the resultant matrix $\boldsymbol{X_{I_k}}$ that is $\tilde{x}^{(k)} = \tilde{x}_1^{(k)} + \tilde{x}_2^{(k)} + \cdots + \tilde{x}_N^{(k)}$ [12-14, 16] and it can be written in Eq. (8) as the sum of the resultants of $\boldsymbol{X_{I_k}}$ as follows:

$$x_n = \sum_{k=1}^{m} \tilde{x}_n^{(k)}. \tag{8}$$

3.3 Forecasting

The SSA forecasting of the time series can be applied through the Linear Recurrent Formulae (LRF) [13,17,20,25] after the SSA decomposition and reconstruction applied to the original time series. The SSA forecasting algorithm is described as follows. Set the last component of the vector U_i for $i = 1, 2, \dots, L$ as $v^2 = \pi_1^2 + \pi_2^2 + \cdots + \pi_L^2$ with $v^2 < 1$. Let the vector $U^\nabla \in \boldsymbol{R}^{L-1}$ is any vector $U \in \boldsymbol{R}^L$ that consists of the first $L-1$ vector components of the vector U. There is a vector $A = (\alpha_1, \alpha_2, \dots, \alpha_{L-1})$ such that satisfy $A = \frac{1}{1-v^2} \sum_{i=1}^{L} \pi_i U_i^\nabla$.

The last component x_L of any vector $X = (x_1, x_2, \dots, x_L)^T$ can be written as the linear combination of the first $L-1$ vector components $(x_1, x_2, \dots, x_{L-1})$ with vector $A = (\alpha_1, \alpha_2, \dots, \alpha_{L-1})$, as follows [13, 20, 25]:

$$x_L = \alpha_1 x_{L-1} + \alpha_2 x_{L-2} + \ldots + \alpha_{L-1} x_1. \tag{9}$$

Finally, the forecasting of individual time series for h terms after the N terms or $X_{N+h} = (x_1, x_2, \dots, x_{N+h})$, is determined using the following formula:

$$x_i = \begin{cases} \tilde{x}_i, & 1 \leq i \leq N \\ \sum_{j=1}^{L-1} \alpha_j x_{i-j}, & N + 1 \leq i \leq N + h \end{cases} \tag{10}$$

with $\tilde{x}_i$ is the reconstructed components series for $i = 1, 2, \dots, N$. Algorithm 1 presents the brief of the standard of SSA is summarized in Algorithm 1, as follows.

Algorithm 1: Standard Singular Spectrum Analysis

Input: Time series X_t of length N and window length L.
Output: Forecasting h series of x_i, $N + 1 \leq i \leq N + h$.

Step 1. Construct the trajectory matrix $\boldsymbol{X}$ using the lagged vector X_i with a window length of L.
Step 2. Construct the Gram matrix $\boldsymbol{XX^T}$.
Step 3. Decompose matrix $\boldsymbol{XX^T}$ using SVD to get the eigentriple $\{\sqrt{\lambda_i}, U_i, V_i^T\}$.
Step 4. Perform eigentriple grouping $\boldsymbol{X_I}$.
Step 5. Reconstruct the time series x_n as the sum of the resultant of matrix $\boldsymbol{X_{I_k}}$.
Step 6. Perform forecasting x_i using the reconstructed time series x_n.

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4. Proposed method

Based on the basic algorithm of SSA that is presented in the previous section, the first stage to implement the SSA is decomposing the time series spectrum through the trajectory matrix. Theoretically, the larger window length is better to construct the trajectory matrix of time series. However, there is no information on what is the best size for window length. In this study, the size of window length is proposed through the Hadamard transform (also known as Walsh Hadamard transform, Walsh transform, or Walsh Fourier transform) [34].

The Hadamard transform is represented by the Hadamard matrix, which is orthogonal and symmetric on $2^n \times 2^n$ squared matrix of integer number n. The Hadamard matrix is constructed by Kronecker product that is defined as [32-34]:

$$\begin{aligned} \boldsymbol{H_N} &= \boldsymbol{H_1} \otimes \boldsymbol{H_{N-1}} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \boldsymbol{H_{N-1}} & \boldsymbol{H_{N-1}} \\ \boldsymbol{H_{N-1}} & -\boldsymbol{H_{N-1}} \end{pmatrix}, \end{aligned} \tag{11}$$

with $\boldsymbol{H_1}$ is the smallest Hadamard matrix and $N = 2^n$. The Hadamard matrix can be computed efficiently in $n \log n$ time complexity using fast Walsh Hadamard transform (FWHT) [34]. Suppose x is a signal vector, X is a signal spectrum, and $\boldsymbol{H_h}$ is the Hadamard matrix. The FWHT and its inverse (IFWHT) is defined as:

$$\begin{aligned} FWHT(x) &= X = \boldsymbol{H_w} x \\ IFWHT(X) &= x = \boldsymbol{H_w} X, \end{aligned} \tag{12}$$

with $\boldsymbol{H_w}$ is the Walsh Hadamard matrix, which can be obtained by reordered the rows of Hadamard matrix $\boldsymbol{H_h}$. The x and X in Eq. (12) can also be written as a linear combination of a set of square waves of different sequences as follows:

$$\begin{aligned} X(k) &= \sum_{n=1}^{N} x(n) \prod_{i=1}^{M} (-1)^{n_i k_{M-1-i}}, k = 0,1,\dots,N \\ x(n) &= \frac{1}{N} \sum_{n=1}^{N} X(k) \prod_{i=1}^{M} (-1)^{n_i k_{M-1-i}}, n = 0,1,\dots,N \end{aligned} \tag{13}$$

with $N = 2^n, M = \log_2 N$, and n_i is the i^{th} of a binary representation of n.

In order to the SSA implementation, the time series embedding and the window length construction are performed as follows. Suppose the time series $X_t = (x_1, x_2, \dots, x_N)$ with $1 \le t \le N$ of length N and $\boldsymbol{X_t^h} = (x_1^h, x_2^h, \dots, x_N^h)$ is the FWHT of X_t using Hadamard matrix $\boldsymbol{H_h}$. The trajectory matrix in Eq. (1) will be constructed using the Hadamard matrix. Let N^* is a length of sub-matrix of $2^h (h = 1, 2, \dots, N)$ such that $N^* < N$. Because of the window length in SSA should large but smaller than the length of time series, the window length of N^* can be selected from the maximum order of Hadamard transform h such as satisfying $2^h < N$.

The window length L^* can be determined as $L^* = 2^{h-1}$ and the sub-matrix X_i with $X_i' = (x_i, x_{i+1}, \dots, x_{i+L^*+1})^T$ can be constructed as the window component with a length of L^* for $1 \le i \le K^*$, and $K^* = N - L^* + 1$. Furthermore, the trajectory matrix $\boldsymbol{X_h}$ of $\boldsymbol{X_t^h}$ can be formulated using Hadamard matrix $\boldsymbol{H_h}$, as follows:

$$\boldsymbol{X_h} = \begin{pmatrix} x_1' & x_1' & \cdots & x_{K^*}' \\ x_2' & x_3' & & x_{K^*+1}' \\ \vdots & \vdots & \ddots & \vdots \\ x_{L^*}' & x_{L^*+1}' & \cdots & x_N' \end{pmatrix}. \tag{14}$$

Practically, the used of window length L^* in this proposed method is summarized in Table 1. For example, suppose there is a time series with 252 periods (or $N = 252$). The sub-series of this time series can be embedded into the trajectory matrix using the window length $L^*$with some length options, which are 1, 2, 4, 8, 16, 32, and 64. In this case, the maximum of L^* is 64 or $L^*=2^6$.

The next steps of SSA are then performed using the same procedures of SSA that have described in the previous section to complete the decomposition and reconstructions stages. The inverse of the Hadamard transform, i.e. FWHT, should be applied to get the time series reconstruction using Eq. (12). The brief of the proposed method is summarized in Algorithm 2. As shown in this algorithm, the Hadamard transform is applied in Step 1 and Step 2 to obtain the static window length L^* and to construct the trajectory matrix $\boldsymbol{X_h}$. It is also applied in Step 7 for time series reconstruction.

Table 1. The used of Hadamard matrix and window length (L^*) based on various size of time series (N)

h	$L^* = 2^{h-1}$	N	$\boldsymbol{H_h}$
1	1	$2 \le N < 4$	$\boldsymbol{H_2}$
2	2	$4 \le N < 8$	$\boldsymbol{H_4}$
3	4	$8 \le N < 16$	$\boldsymbol{H_8}$
4	8	$16 \le N < 32$	$\boldsymbol{H_{16}}$
5	16	$32 \le N < 64$	$\boldsymbol{H_{32}}$
6	32	$64 \le N < 128$	$\boldsymbol{H_{64}}$
7	64	$128 \le N < 256$	$\boldsymbol{H_{128}}$
8	128	$256 \le N < 512$	$\boldsymbol{H_{256}}$
9	256	$512 \le N < 1024$	$\boldsymbol{H_{512}}$

Algorithm 2: Singular Spectrum Analysis with Hadamard Transform

Input: Time series X_t of length N.
Output: Forecasting h series of x_i' , $N+1 \leq i \leq N+h$.

Step 1. Obtain the maximum order of Hadamard transform j such as satisfying $2^h < N$ and let $L^* = 2^{h-1}$.
Step 2. Perform the FWHT on X_t using the Hadamard transform H_h, that is H_t^h.
Step 3. Construct the trajectory matrix $\boldsymbol{X_h}$ using the lagged vector X_i' with a window length of L^*.
Step 4. Construct the Gram matrix $\boldsymbol{X_h X_h^T}$.
Step 5. Decompose matrix $\boldsymbol{X_h X_h^T}$ using SVD to get the triple eigentriple $\{\sqrt{\lambda_i'}, U_i', V_i'^T\}$.
Step 6. Perform eigentriple grouping $\boldsymbol{X_I'}$.
Step 7. Perform the inverse of FWHT on $\boldsymbol{X_I'}$.
Step 8. Reconstruct the time series x_n' as the sum of the resultant of matrix $\boldsymbol{X_{I_k}'}$.
Step 9. Perform forecasting x_i' using the reconstructed time series x_n'.

5. Experimental setup

This study uses a daily stock price dataset in EUR (Euro currency) of SMS2.SG from the STU stock market. This dataset is an individual company stock price that can be downloaded from Yahoo! Finance website. The SMS2.SG dataset contains 210 price records with open, high, low, and close prices. However, the closing stock price is chosen as a single time series attribute for simulation because this study is applied to the univariate SSA.

The time series model development uses the daily stock price from 01/01/2019 to 10/31/2019. This study only uses a day ahead forecasting from 11/01/2019 to 11/30/2019 as the forecasting period. There is no specific data preprocessing method applied to the dataset. Hence, any incomplete or missing data records are removed as-is from the dataset. The accuracy performance of the proposed method is evaluated using integrated square error (ISE), one of the most popular error measurements for non-parametric estimation, which is formulated in Eq. (15) [35], as follows:

$$ISE = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 \quad (15)$$

with N is the length of time series, Y_t is the actual stock price at period t, $\hat{Y}_t$ is the forecasting stock price at period t. The smaller ISE indicates better accuracy compared to the other model.

For experiment validation, the performance of the proposed method will be compared to the performance of the standard SSA algorithm. The objective of this experiment is to determine the static window length parameter of the lagged vectors to construct the trajectory matrix of time series in the standard SSA algorithm, rather than the tuning window length parameter. The proposed method is also compared with the other similar window length selection method, which is the SSA with the MDL model proposed by Khan and Poskitt [37]. In other words, the baseline methods of this experiment are the standard SSA without Hadamard transform and the SSA with the MDL model, respectively, whilst the proposed method is the standard SSA with Hadamard transform.

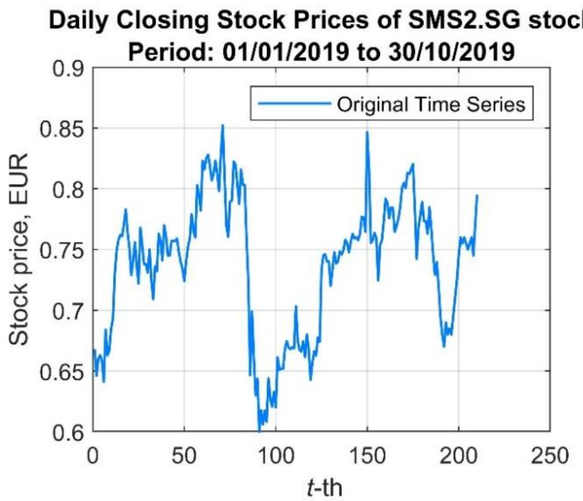

Figure. 1 Initial time series data of SMS2.SG stock

6. Results and discussion

This experiment is mainly conducted to evaluate the use of Hadamard transform on SSA to address the time series embedding issue. Thus, this experiment is designed into two works, which are the SSA without Hadamard transform as the baseline method and the SSA through Hadamard transform as the proposed method. It means that the performance measurements of both methods will be compared to evaluate any performance improvement of the proposed method.

The initial time series data of SMS2.SG in this study is illustrated in Fig. 1 with the length of time series is $N = 210$. Based on the Hadamard matrix properties and the length of time series, the maximum size of sub-series is $N^* = 2^7 = 128$. Thus, the maximum size of time series embedding, in this case, is $L^* = 2^6 = 64$ and the Hadamard matrix $\boldsymbol{H_7}$ is used to convert the initial time series to get the transformed matrix $\boldsymbol{X_t^7}$.

The trajectory matrix is then constructed using sub-series of $\boldsymbol{X_t^7}$ with length of L^*. The trajectory of

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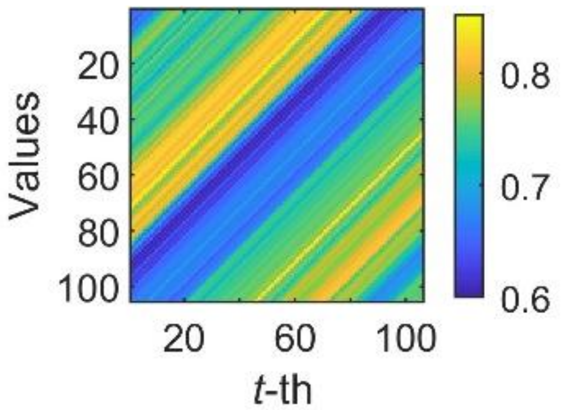
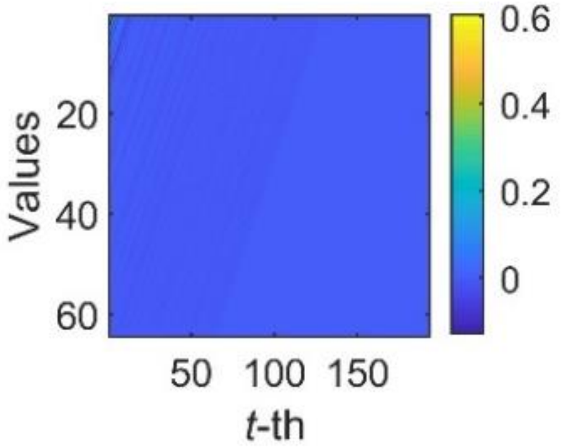
the original and the transformed time series are illustrated in Fig. 2. Fig. 2 (a) shows the trajectory matrix of $\boldsymbol{X}$ in Eq. (1) that is constructed using window element with $L = N/2 = 105$ of length from the original time series without Hadamard transform. While Fig. 2 (b) shows the trajectory matrix $\boldsymbol{X_h}$ in Eq. (14) that is constructed using window element with $L^* = 2^{7-1} = 64$ of length from the transformed time series using Hadamard transform.

The next steps of SSA then performed on the transformed trajectory matrix $\boldsymbol{X_h}$. For time series decomposition, the multiplied matrices of $\boldsymbol{XX^T}$ on the matrix $\boldsymbol{X_h}$ is constructed. The logarithmic scale eigenvalues of this multiplied matrix are illustrated in Fig. 3(a). There are 64 eigenvalues in the decreasing order. Whilst the first eight eigenvectors correspond to the first four eigenvalues of the multiplied matrix are illustrated in Fig. 3(b). The eigenvalues and eigenvectors from this covariance decomposition will be used to perform the principle components of the time series as the orthogonal linear transformation.

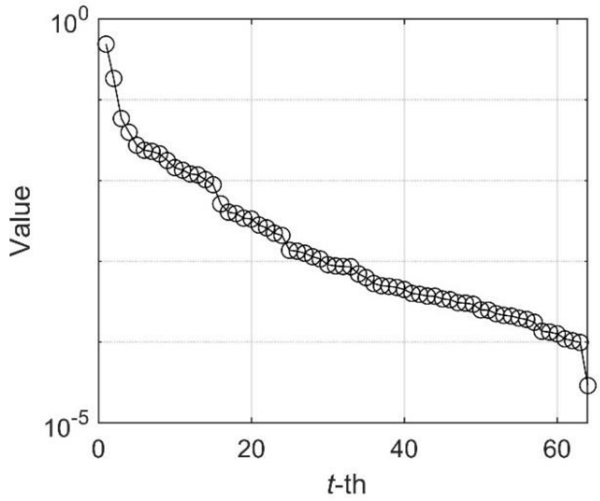


(a)

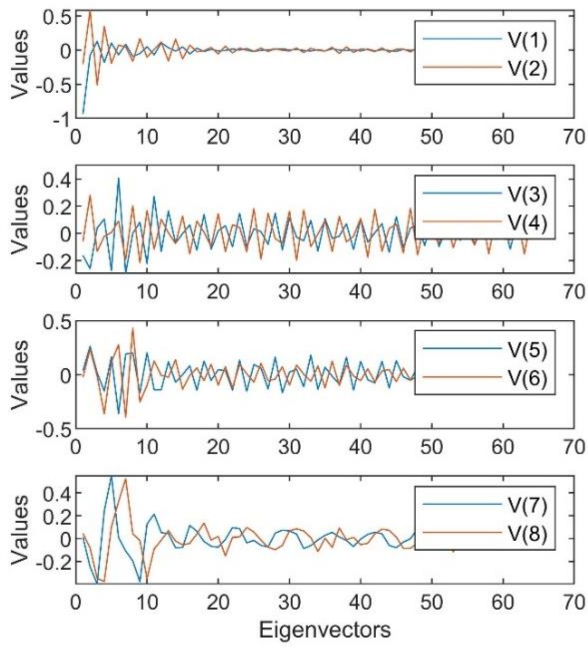


(b)

Figure. 2 Trajectory matrices of time series: (a) without Hadamard transform and (b) with Hadamard transform



(a)



(b)

Figure. 3 The eigendecomposition of multiplied matrix $\boldsymbol{XX^T}$ using $\boldsymbol{X_h}$: (a) eigenvalues and (b) first eight eigenvectors

Through the eigenvectors, the trajectory matrix is projected into the principal components of the time series. The first four of these principal components are illustrated in Fig. 4. As shown in Eq. (4) and Eq. (5), the principal components are basically computed by the trajectory matrix for each eigenvector. It means that the principal components correspond to the projection of time series onto the temporal empirical orthogonal functions. The pair of eigenvalues and associated principal components represent the oscillations of time series. The invert of principal components and eigenvectors are then needed for component reconstruction corresponds to the matrix $\boldsymbol{X_I}$ that is formulated in Eq. (6).

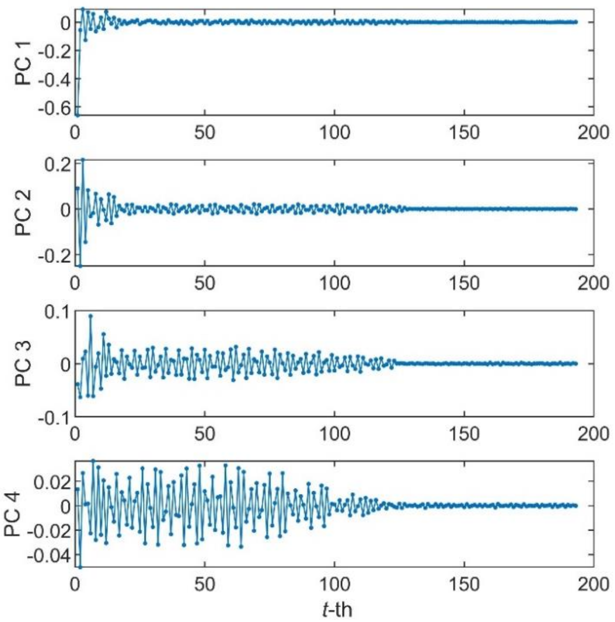


Figure. 4 The first four principal components of the time series

Furthermore, averaging diagonals on this matrix will produce the reconstructed components of time series. This step corresponds to the Hankelization in Eq. (7). The first four of the reconstructed components are illustrated in Fig. 5. It shows that the reconstructed components represent the trend, periodically components, and the noise of time series.

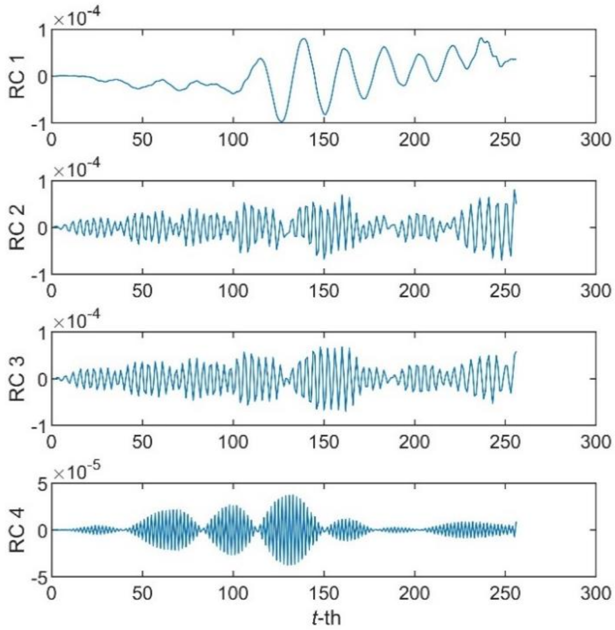


Figure. 5 The first four reconstructed components of time series

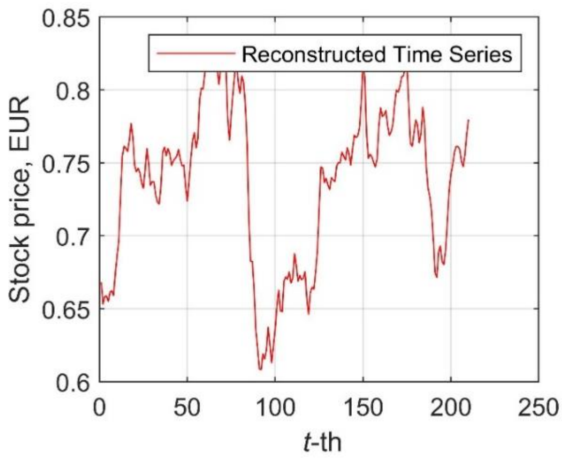


Figure. 6 Reconstructed time series

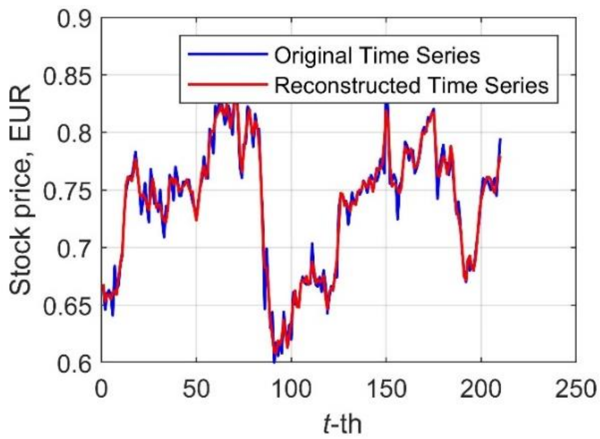


Figure. 7 Comparison between original and reconstructed time series

The sum of N reconstructed components of the time series is illustrated in Fig. 6. It shows that the total reconstructed components nearly close to the original time series. As formulated in Eq. (8), these total reconstructed components correspond to the sum of the resultant matrix $\boldsymbol{X_I}$ after applying the diagonal averaging in Eq. (7). The comparison between the original time series and constructed time series is illustrated in Fig. 7.

The performance between the reconstructed time series using SSA with Hadamard transform and the original time series using standard SSA is measured by ISE, which is formulated in Eq. (15). The experiment result using the same dataset shows that the ISE of SSA without Hadamard transform, SSA with MDL model, and SSA with Hadamard transform are 0.1441, 0.0194, and 0.0088, respectively. The comparison errors in the initial time series length period are illustrated in Fig. 8. It shows that the SSA with Hadamard transform can reduce the errors of the standard SSA (without Hadamard transform) and also the SSA with the MDL model.

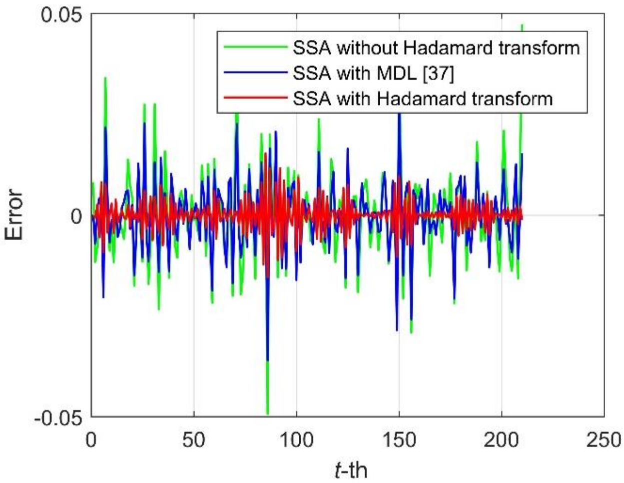


Figure. 8 Error rates comparison between the SSA: without Hadamard transform, with MDL model, and with Hadamard transform

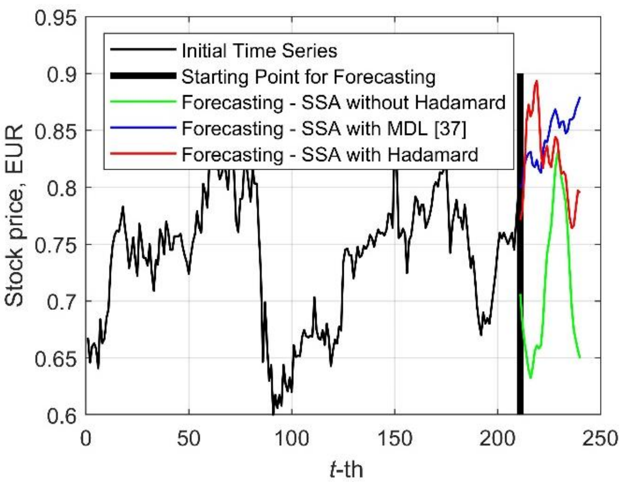


Figure. 9 Initial and forecasting time series using SSA: without Hadamard transform, with MDL model, and with Hadamard transform

The next step is performing the stock price forecasting using the reconstructed time series. In this study, the forecasting time period is 30 days from 11/01/2019 to 11/30/2019. The forecasting time series is formulated in Eq. (10) using $h = 30$ and reconstructed time series in Eq. (8) with the linear combination of the reconstructed components in Eq. (9).

The forecasting result of the SMS2.SG stock price for the next 30 days ahead from the initial time series period is illustrated in Fig. 9. The starting point for forecasting is the end of the initial time series point. The green and the red plots are forecasting results using the SSA without and with Hadamard transform, respectively. While the blue plot is forecasting results using the SSA with the MDL model for determining the window length, which is proposed by Khan and Poskitt [37]. The details of predicted stock prices between the SSA without and with Hadamard transform are presented in Table 2.

Table 2. Forecasting results for SMS2.SG stock price from 11/01/2019 to 11/30/2019

No.	Date	Predicted Stock Price (EUR)		
		$\hat{Y}_1$	$\hat{Y}_2$	$\hat{Y}_3$
1.	11/01/2019	0.7061	0.8000	0.7714
2.	11/02/2019	0.6810	0.8028	0.7796
3.	11/03/2019	0.6643	0.8195	0.8125
4.	11/04/2019	0.6528	0.8259	0.8572
5.	11/05/2019	0.6399	0.8298	0.8725
6.	11/06/2019	0.6324	0.8311	0.8624
7.	11/07/2019	0.6413	0.8197	0.8676
8.	11/08/2019	0.6560	0.8177	0.8884
9.	11/09/2019	0.6609	0.8240	0.8936
10.	11/10/2019	0.6583	0.8162	0.8694
11.	11/11/2019	0.6610	0.8128	0.8322
12.	11/12/2019	0.6813	0.8318	0.8168
13.	11/13/2019	0.7152	0.8411	0.8306
14.	11/14/2019	0.7417	0.8396	0.8363
15.	11/15/2019	0.7533	0.8482	0.8217
16.	11/16/2019	0.7663	0.8583	0.8177
17.	11/17/2019	0.7901	0.8637	0.8320
18.	11/18/2019	0.8162	0.8684	0.8442
19.	11/19/2019	0.8298	0.8621	0.8415
20.	11/20/2019	0.8229	0.8525	0.8248
21.	11/21/2019	0.8075	0.8575	0.8119
22.	11/22/2019	0.7978	0.8577	0.8135
23.	11/23/2019	0.7871	0.8474	0.8104
24.	11/24/2019	0.7623	0.8496	0.7914
25.	11/25/2019	0.7270	0.8593	0.7731
26.	11/26/2019	0.6942	0.8595	0.7642
27.	11/27/2019	0.6743	0.8617	0.7664
28.	11/28/2019	0.6652	0.8694	0.7826
29.	11/29/2019	0.6564	0.8740	0.7971
30.	11/30/2019	0.6498	0.8796	0.7951

$\hat{Y}_1$ is the predicted stock price using standard SSA without Hadamard transform.
$\hat{Y}_2$ is the predicted stock price using SSA with the MDL model [37].
$\hat{Y}_3$ is the predicted stock price using SSA with Hadamard transform.

7. Conclusions and future works

The Hadamard spectrum through the Hadamard transform has been applied successfully on the time series analysis using the SSA algorithm. In this proposed method, the window length can be determined directly at the maximum of the Hadamard order corresponds to the original time series and its length. It means that the embedding time series in SSA does not only consider the length of time series, but it also considers the spectrum of the time series. Based on the error rates using ISE values, the reconstructed time series using SSA with Hadamard transform outperforms the standard SSA without Hadamard transform and also the SSA with the MDL model. The ISE values of the standard SSA without Hadamard transform, the SSA with the MDL model, and the SSA with Hadamard transform are 0.1441, 0.0194, and 0.0088, respectively. Hence, the use of the Hadamard spectrum is strongly suggested as an alternative method to improve the time series analysis based on the SSA, especially to choose the better window length in the time series embedding process.

This paper just presents the use of the Hadamard spectrum based on the simulation test on the SSA algorithm without specifically considering any signal characteristics, such as type of trend signal, seasonal aspects, noise, and residual components. These limitations can be used as a background for future work in order to improve the proposed method. Combining the Hadamard transform based on SSA with the other forecasting techniques is also enough interesting as advance research to achieve better performance of the SSA algorithm. Besides, the time series analysis in this paper is only proposed in the univariate SSA framework. There is a high motivation to evaluate the proposed method in the multivariate SSA framework.

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