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Unsteady flow past on vertical cylinder in the presence of an inclined magnetic field and chemical reaction

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The chemical reaction effect on unsteady flow of a viscous, incompressible and electrically conducting fluid past an oscillating vertical cylinder with exponentially accelerated wall temperature and variable mass diffusion in the presence of inclined magnetic field is considered. The concentration and temperature level near the surface increases linearly and exponentially with time respectably. The dimensionless governing equations for this investigation are solved numerically by using Crank-Nicolson implicit finite difference method. The velocity profile is discussed with the help of graphs drawn for different parameters like chemical reaction parameter, the inclination of magnetic field, oscillating and exponentially accelerated parameter. The numerical values obtained for skin-friction, Nusselt number and Sherwood number have been tabulated.

***Keywords**: MHD flow, chemical reaction, an inclined magnetic field*

1. Introduction

The study of electrically conducting fluids in the presence of magnetic field and chemical reaction is of considerable and phenomenal interest because they are encountered in many processes of engineering and applied sciences. The reason is the chemical reaction effect along with influence of magnetic field on such a flow within porous and non-porous medium has important in engineering applications, such as in advanced types of power plants for nuclear rockets, in the designing of heat exchangers, MHD accelerators, electrostatic filters, cooling reactors, purification of crude oil, fluid droplets, MHD pumps, MHD generator, nuclear reactors, oil exploration, space vehicle propulsion etc. Chemical reaction and radiation effects on unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of viscous dissipation was analyzed by Anitha [1]. Kandasamy et. al. [2] have demonstrated the model on chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Rajput and Kumar [3] studied unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current. Chemical reaction effect on unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current was explained by Rajput and Kumar [4]. Mythreye et al [5] have studied chemical reaction on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Rajput and Kumar [6] have demonstrated chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current. Effects of radiation and chemical reaction on MHD flow past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current was examined by Rajput and Kumar [7]. Paul et. al. [8] have worked on numerical study of chemical reaction effects on unsteady MHD fluid flow past an infinite vertical plate embedded in a porous medium with variable suction. Unstesdy MHD Poiseuille flow between two infinite parallel plate through porous medium in an inclined magnetic field with Heat and mass transfer was examined by Rajput and Kumar [9]. Chemical reaction effect on unsteady MHD flow past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current was presented by Rajput and Kumar [10]. Vishnu et. al. [11] has discussed the hydromagnetic axisymmetric slip flow along a vertical stretching cylinder with convective boundary condition. Yadav and Sharma [12] have worked on effects of porous medium on MHD fluid flow along a stretching cylinder. The effect of inclined magnetic field on unsteady flow past on moving vertical plate with variable temperature was proposed by Singh et. al. [13]. Ziyauddin and Kumar [14] studied MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation. The flow model under consideration analyzes the effect of chemical reaction on MHD flow past a vertical cylinder. The problem is solved numerically using Crank-Nicolson implicit finite difference technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction and local Nusselt number and Sherwood number have been tabulated.

2. Mathematical analysis

Consider an unsteady flow of an incompressible viscous electrically conducting fluid mixture past an impulsively started semi-infinite vertical cylinder of radius r_0 . Here the x - axis is taken along the axis of cylinder in the vertical direction and the radial coordinate r is taken normal to the cylinder. The gravitational acceleration g is acting downward. The magnetic field B0 of uniform strength is applied inclination to the flow. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞ for all t≤0. At time t > 0, the cylinder starts oscillating vertical cylinder in its own plane and temperature of the wall is exponentially accelerated raised to T_w. The concentration C_w near the surface is raised linearly with respect to time. Then under these assumptions and the Boussinesq's approximation, the flow is governed by the following system of equations:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial r^2} + \frac{\upsilon}{r}\frac{\partial u}{\partial r} + g\beta\,(T - T_\infty) + g\beta^*\,(C - C_\infty) - \frac{\sigma B_0^2 u\ Sin\,\alpha}{\rho} \quad (1)$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial r^2} + \frac{\alpha}{r}\frac{\partial T}{\partial r} \quad (2)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial r^2} + \frac{D}{r}\frac{\partial C}{\partial r} - K_C\,(C - C_\infty) \quad (3)$$

The initial and boundary conditions are:

$$\left.\begin{array}{l} t \le 0 : u = 0,\ T = T_\infty,\ C = C_\infty, \text{ for every r} \\ t > 0 : u = u_0 Cos\omega t,\ T = T_\infty + (T_w - T_\infty)e^{bt},\ C = C_\infty + (C_w - C_\infty)\dfrac{t\upsilon}{r_0^2}, \text{ at } r = r_0 \\ u \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } r \to \infty \end{array}\right\} \quad (4)$$

Here u is the velocity of fluid, g-the acceleration due to gravity, β -volumetric coefficient of thermal expansion, t-time, T-temperature of the fluid, β^* -volumetric coefficient of concentration expansion, C- species concentration in the fluid, ν -the kinematic viscosity, ρ- the density, C_p- the specific heat at constant pressure, k-thermal conductivity of the fluid, D-the mass diffusion coefficient, T_w-temperature of the plate at z= 0, c_W -species concentration at the plate z= 0, B_0- the uniform magnetic field, K_c -chemical reaction, σ- electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2) and (3) into dimensionless form:

$$
\left.\begin{array}{l}
R=\frac{r}{r_0},\ \bar{u}=\frac{u}{u_0},\quad \theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},\ S_c=\frac{\upsilon}{D},\ \mu=\rho\upsilon, \\
\bar{C}=\frac{(C-C_\infty)}{(C_w-C_\infty)}, G_r=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0^3},\ M=\frac{\sigma B_0^2 r_0^2}{\upsilon\rho},\ P_r=\frac{\upsilon}{\alpha}, \\
G_m=\frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},\ K_0=\frac{\upsilon K_C}{u_0^2},\quad \bar{t}=\frac{t\upsilon}{r_0^2}.
\end{array}\right\} \tag{5}
$$

where $\bar{u}$ is the dimensionless velocity, $\bar{t}$-dimensionless time, θ-the dimensionless temperature, $\bar{C}$-the dimensionless concentration, Gr - thermal Grashof number, G_m- mass Grashof number, μ - the coefficient of viscosity, K_0 -chemical reaction parameter, P_r- the Prandtl number, S_c - the Schmidt number, M- the magnetic parameter.

The flow model in dimensionless form is:

$$
\frac{\partial \bar{u}}{\partial \bar{t}}=\frac{\partial^2 \bar{u}}{\partial R^2}+\frac{1}{R}\frac{\partial \bar{u}}{\partial R}+G_r\,\theta+G_m\,\bar{C}-M\ Sin\alpha\,\bar{u} \tag{6}
$$

$$
\frac{\partial \theta}{\partial \bar{t}}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial R^2}+\frac{1}{R\,P_r}\frac{\partial \theta}{\partial R} \tag{7}
$$

$$
\frac{\partial \bar{C}}{\partial \bar{t}}=\frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial R^2}+\frac{1}{R\,S_c}\frac{\partial \bar{C}}{\partial R}-K_0\bar{C} \tag{8}
$$

The corresponding boundary conditions (4) become:

$$
\left.\begin{array}{ll}
\bar{t}\le 0:\bar{u}=0,\ \theta=0,\ \bar{C}=0, & \text{for every R} \\
\bar{t}>0:\bar{u}=Cos\bar{\omega}\bar{t}, \theta=e^{\bar{b}\bar{t}},\ \bar{C}=\bar{t}, & \text{at R =0} \\
\bar{u}\to 0,\ \theta\to 0,\ \bar{C}\to 0, & \text{as } R\to\infty
\end{array}\right\} \tag{9}
$$

Dropping bars in the above equations, we get

$$
\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}+G_r\,\theta+G_m C-Mu \tag{10}
$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R\,P_r}\frac{\partial \theta}{\partial R} \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial R^2} + \frac{1}{R\,S_c}\frac{\partial C}{\partial R} - K_0 C \tag{12}$$

The boundary conditions become

$$\left.\begin{array}{ll} t \le 0 : u = 0,\ \theta = 0, C = 0, & \text{for every R} \\ t > 0 : u = Cos\,\omega t, \theta = e^{bt},\ C = t, & \text{at R=0} \\ u \to 0,\ \theta \to 0,\ C \to 0, & \text{as } R \to \infty \end{array}\right\} \tag{13}$$

Method of solution

Equations (10)-(12) are non-linear partial differential equations are solved using boundary and initial conditions (13). These equations are solved by Crank-Nicolson implicit finite difference method for numerical solution. The finite difference equations corresponding to equations (10)-(12) are as follows:

$$\begin{aligned} u_i^{j+1} - u_i^j = & \frac{\Delta t}{2(\Delta R)^2}(u_{i+1}^j - 2u_i^j + u_{i-1}^j + u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}) \\ & + \frac{\Delta t}{4(1+(i-1)\Delta R)(\Delta R)}(u_{i+1}^j - u_{i-1}^j + u_{i+1}^{j+1} - u_{i-1}^{j+1}) \\ & + \frac{\Delta t G_r}{2}((\theta_i^{j+1} + \theta_i^j)) + \frac{\Delta t G_m}{2}(C_i^{j+1} + C_i^j) - \frac{\Delta t M}{2}(u_i^{j+1} + u_i^j). \end{aligned}$$

$$\begin{aligned} \theta_i^{j+1} - \theta_i^j = & \frac{\Delta t}{2\,P_r\,(\Delta R)^2}(\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j + \theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1}) \\ & + \frac{\Delta t}{4\,P_r\,(1+(i-1)\Delta R)(\Delta R)}(\theta_{i+1}^j - \theta_{i-1}^j + \theta_{i+1}^{j+1} - \theta_{i-1}^{j+1}) \end{aligned}$$

$$\begin{aligned} C_i^{j+1} - C_i^j = & \frac{\Delta t}{2\,S_c\,(\Delta R)^2}(C_{i+1}^j - 2C_i^j + C_{i-1}^j + C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}) \\ & + \frac{\Delta t}{4\,S_c\,(1+(i-1)\Delta R)(\Delta R)}(C_{i+1}^j - C_{i-1}^j + C_{i+1}^{j+1} - C_{i-1}^{j+1}) \\ & - \Delta t K_0 (C_i^{j+1} + C_i^j) \end{aligned}$$

Here index i refers to R and j refers to time t, $\Delta t = t_{j+1} - t_j$ and $\Delta R = R_{j+1} - R_j$ .Knowing the values of u, θ and C at time t, we can compute the values at time $t+\Delta t$ as follows: we substitute i = 1, 2, ..., N-1, where N correspond to ∞. The implicit Crank-Nicolson finite difference method is a second order method ($o(\Delta t^2)$) in time and has no restriction on space and time steps, that is, the method is unconditionally stable. The computation is executed for ΔR = 0.1, Δt = 0.002 and procedure is repeated till R = 40.

Now it is important to calculate the physical quantities of primary interest, which are the local shear stress, local surface heat flux and Sherwood number.

2.1. Skin friction

The dimensionless local wall shear stress or skin-friction at the surface is obtained as,

$$\tau = -\left(\frac{\partial u}{\partial R}\right)_{R=0}$$

The numerical values of τ for different parameters are given in table-1.

2.2. Nusselt number

The dimensionless local surface heat flux or Nusselt number at the surface is obtained as

$$Nu = -\left(\frac{\partial \theta}{\partial R}\right)_{R=0}$$

The numerical values of Nu for different parameters are given in table-2.

2.3. Sherwood number

The dimensionless the local Sherwood number at the surface is obtained as

$$Sh = -\left(\frac{\partial C}{\partial R}\right)_{R=0}$$

The numerical values of S_h for different parameters are given in table-3.

3. Results and discussion

In order to explain the significance of the study a representative set of numerical results for different parameters like, inclination of magnetic field (α), exponentially accelerated wall temperature parameter (b), chemical reaction parameter

(K0),oscillating vertical surface parameter (ω) are shown graphically in Figures 1 to 4. In Figure 1, it is observed that the velocity of fluid decrease when the plate angle (α) is increased. This is in agreement with the actual flow, since the velocity of the fluid decreases as we increase the inclination of the magnetic field from the horizontal. If inclination of magnetic field is perpendicular then the effect of magnetic parameter M is decreasing the fluid velocity. It is due to the application of transverse magnetic field that acts as Lorentz's force which retards the flow. It is observed that when acceleration parameter is increased then the fluid velocity is increased (figure 2). This implies that acceleration parameter tends to accelerate MHD fluid velocity throughout the boundary layer region near the surface. If K0 the chemical reaction parameter is increased then the velocity is decreased (figure 3). It is observed from figure 4 that when angular velocity ω increases, the velocity of fluid near the surface is decreased.

Skin friction is given in table 1. The value of Skin friction increases with the increases in the Inclination of the magnetic field parameter, chemical reaction parameter. It decreases with an oscillating vertical cylinder with exponentially accelerated wall temperature. Nusselt number is given in table 2. The value of Nusselt number increases with the increase in the exponentially accelerated wall temperature. Sherwood number is given in table 3. The value of Sherwood number increases with the increase in chemical reaction parameter.

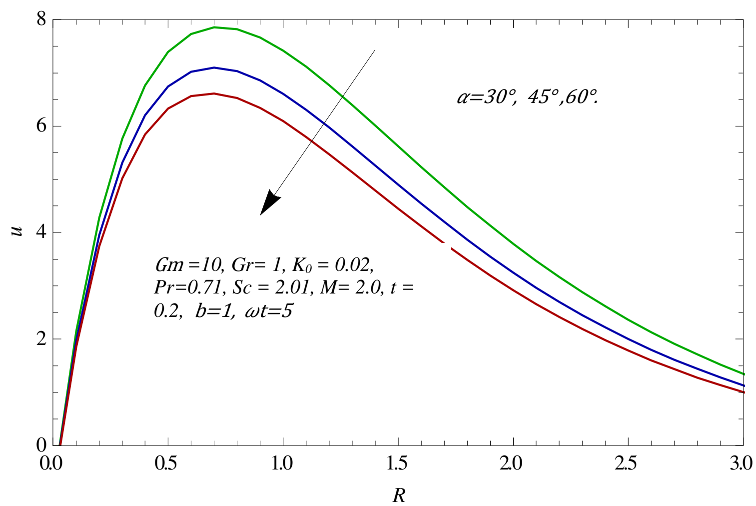

Figure 1. Velocity u for different values of α

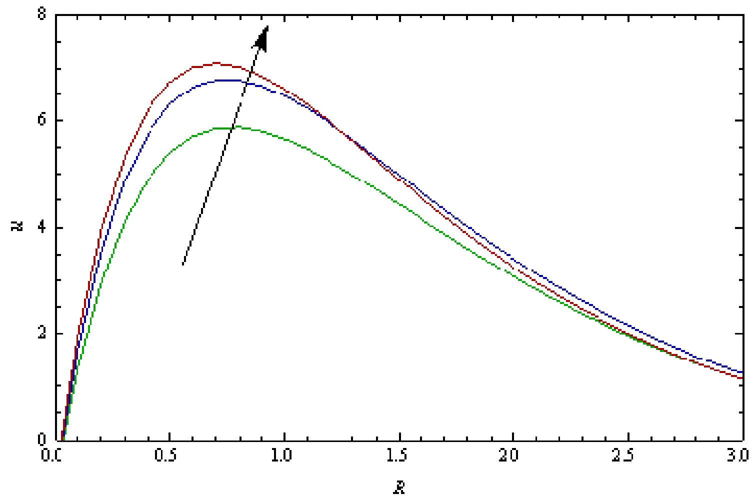

Figure 2. Velocity u for different values of b

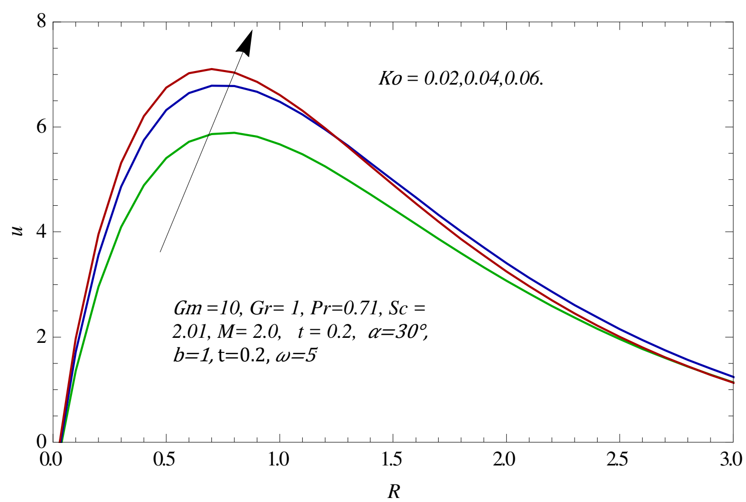

Figure 3. Velocity u for different values of Ko

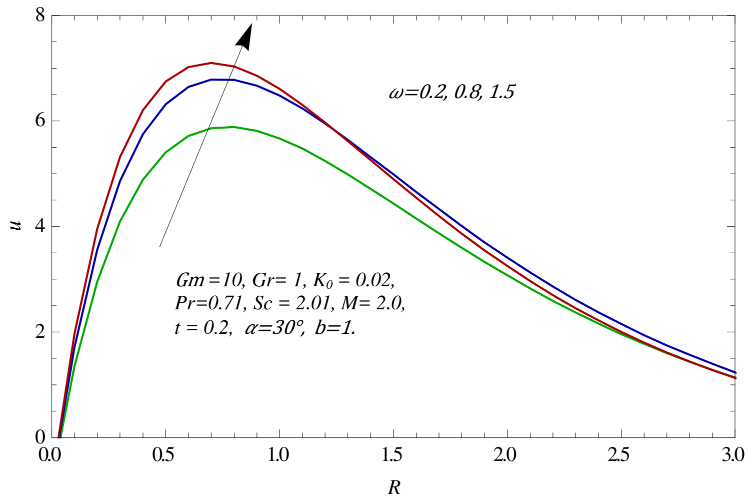

Figure 4. Velocity u for different values of ω

Table 1. Skin friction for different parameters

α	M	Pr	Sc	Gm	Gr	K_0	b	ω	t	τ
30°	2.00	0.71	4.00	10.00	1.00	0.02	1	5	0.02	−29.846402697359594
45°	2.00	0.71	4.00	10.00	1.00	0.02	1	5	0.02	−28.123616308678425
60°	2.00	0.71	4.00	10.00	1.00	0.02	1	5	0.02	−27.00443099809796
45°	2.00	0.71	4.00	10.00	1.00	0.04	1	5	0.02	−29.768873051637605
45°	2.00	0.71	4.00	10.00	1.00	0.06	1	5	0.02	−29.724726046529707
45°	2.00	0.71	4.00	10.00	1.00	0.02	0.8	5	0.02	−21.940869714205213
45°	2.00	0.71	4.00	10.00	1.00	0.02	0.9	5	0.02	−25.551124231554976
45°	2.00	0.71	4.00	10.00	1.00	0.02	1	0.2	0.02	−27.503446193439824
45°	2.00	0.71	4.00	10.00	1.00	0.02	1	0.8	0.02	−29.092996553574885
45°	2.00	0.71	4.00	10.00	1.00	0.02	1	1.5	0.02	−30.320528997728033

Table 2. Nusselt number for different parameters**.**

Pr	*b*	*t*	*nu*
0.71	0.8	0.02	5.626868668112923
0.71	0.9	0.02	7.111572568976792
0.71	01	0.02	8.964730684977287

Table 3. Sherwood number for different parameters**.**

Sc	K_0	*t*	*sw*
4.00	0.02	0.02	6.138969169081858
4.00	0.04	0.02	7.524544746262507
4.00	0.06	0.02	8.521486855166929

4. Conclusions

In this research paper, the numerically study has been done for the chemical reaction effect on unsteady flow of a viscous, incompressible and electrically conducting fluid past an oscillating vertical cylinder with exponentially accelerated wall temperature and variable mass diffusion in the presence of inclined magnetic field. The flow model under consideration by transforming the governing non-linear partial differential equations into non-dimensional form and solved by using Crank-Nicolson implicit finite difference method. To investigate the solutions obtained, standard sets of the values of the parameters have been taken. The numerically result obtained is discussed with the help of graphs and table. We seen that the numerically result obtained is in concurrence with the actual flow behavior of fluid flow.

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