ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XXVI, NR. 1, 2019, ISSN 1453 - 7397

# Models for measuring distances used to obtain optimal locations

Olga-Ioana Amariei, Codruta-Oana Hamat, Daniel-Mihai Amariei

*In this paper we propose a protocol for economic decision-making by the management of an industrial enterprise, which wishes to set up a new production unit. The problem is finding the location of the new production unit, which will minimize the total transport cost, consisting of an amount of costs directly proportional to the distances between the new and existing locations. In order to obtain the optimum location, the QSB software package was used, which uses, besides the Manhattan model and the Euclidean and Euclidean quadratic distance measurement model.*

***Keywords****: median method, optimization, Manhattan distance, Euclidean distance, location, total cost.*

## 1. Introduction

Starting from the decision of the management of an industrial enterprise to open a new work point in which to produce some of the representative products of the company, we arrive at solving the problem of locating a new production unit, the objective being to minimize the total transport cost between this and the supply / distribution units. For this type of problem, we first apply the median method. After finding the location of the new production unit, following the application of the median method, the QSB software package is used, because it uses in addition to the rectilinear (Manhattan distance) distance measurement and Euclidean as well as the Euclidean square. Manhattan distance and Euclidean distance are the most common functions of distance in a location analysis [4].

The rectilinear model is used especially when we study the arrangement of production units, American cities, etc. generally when the arrangement is made after a grid of rectangles. For this reason, the distance measurement model is also called the *Manhattan measurement system* [5, 9].

## 2. Materials and Methods

Starting from the fact that we do not yet know where the new production unit will be located, we cannot specify the distance between it and the supply and distribution bases, without a general expression for calculating the distance between two locations. If $(x_i, y_i)$ and $(x_j, y_j)$ represent the coordinates of two locations i and j, then the distance between i and j, depending on the distance measurement model, can be [8, 9]:

- $|x_i - x_j| + |y_i - y_j|$, in the case of the rectilinear model (Manhattan distance);
- $[(x_i - x_j)^2 + (y_i - y_j)^2]^{0,5}$, in the case of the Euclidean model;
- $(x_i - x_j)^2 + (y_i - y_j)^2$, in the case of the Euclidean square model;

### 2.1. Locating the new production unit by the median method

For the new production unit, a location is sought, taking into account the factors influencing the location, namely, those regarding the existence of the supply bases and the customers. This new production unit will be sourced from five suppliers (rated A, B, C, D and E) and will deliver its finished products to two central warehouses (rated F and G). The number of annual transport routes between the new production unit and these units, as well as the locations of these supply and distribution bases are presented in table 1.

**Table 1.** Input data of the problem

| Supply / distribution base locations | | Coordinates [km] | | Nr. transport courses | Unit cost of transport course [u.m./km] |
|---|---|---|---|---|---|
| | | x | y | N | C |
| Suppliers | A | 20 | 80 | 1200 | 1 |
| | B | 100 | 70 | 1000 | 1 |
| | C | 40 | 60 | 900 | 1 |
| | D | 80 | 40 | 800 | 1 |
| | E | 100 | 30 | 1300 | 1 |
| Clients | F | 10 | 20 | 1200 | 1 |
| | G | 70 | 90 | 2600 | 1 |

Starting from the existing data, the input data of the problem, the median value of the total cost per unit of distance is established. Unit transport costs depend on the location to which the transport is made. Being different, instead of using the value specific to the total number of courses as a median value, we will use the median value of the total cost per unit of distance [7]. We proceed as follows:

- for each supply / distribution location, the cost of the location per distance unit is calculated, by multiplying the number of transport course by the unit cost of a course
- the total cost per unit of distance is then calculated, adding up the costs of locations per unit of distance;
- the median value of the total costs is calculated as follows:

$$V_{mc} = (\sum N \cdot C/2)+1, \quad \text{if} \quad \sum N \cdot C \quad \text{odd number} \tag{1}$$

$$V_{mc} = \sum N \cdot C/2, \quad \text{if} \quad \sum N \cdot C \quad \text{even number} \tag{2}$$

$$V_{mc} = \sum N \cdot C/2 = 9000/2 = 4500 \tag{3}$$

Now having the median value, the x and y coordinates of the location of the new production unit are determined. For this we go through the x-axis, then y in the positive direction, starting from the origin and identifying the locations in the order in which they appear. [7].

For the x coordinate, the median value (4500) corresponds to the location G ($x_0 = x_G = 70$), and for the y coordinate, the median value corresponds to the location B ($y_0 = y_B = 70$). Therefore, the optimal location, following the application of the median method, for the production unit has the coordinates (70,70). The total transportation cost is calculated as the sum of costs directly proportional to the distances between the new location and the existing ones [3], and is equal to 445.000 m.u. according to table 2.

**Table 2.** Calculation of the total transport cost

| Location | Coordinates | | Nr. transport courses | Unitary cost per course | Distance from the new production unit to the locations of supply / distribution | | | Total cost per unit distance |
|---|---|---|---|---|---|---|---|---|
| | x | y | N | C | $\lvert 70-x_i \rvert$ | $\lvert 70-y_i \rvert$ | $D_i$ | $N_i*C_i*D_i$ |
| A | 20 | 80 | 1200 | 1 | 50 | 10 | 60 | 72000 |
| B | 10 | 70 | 1000 | 1 | 30 | 0 | 30 | 30000 |
| C | 40 | 60 | 900 | 1 | 30 | 10 | 40 | 36000 |
| D | 80 | 40 | 800 | 1 | 10 | 30 | 40 | 32000 |
| E | 10 | 30 | 1300 | 1 | 30 | 40 | 70 | 91000 |
| F | 10 | 20 | 1200 | 1 | 60 | 50 | 110 | 132000 |
| G | 70 | 90 | 2600 | 1 | 0 | 30 | 30 | 78000 |
| | | | | | | | ∑ | **445.000** |

### 2.2. Optimal location of the production unit using WinQSB

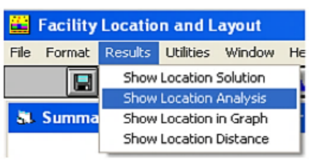
The QSB software package uses three types of distance measurement models, namely [5]: rectilinear, Euclidean and quadratic Euclidean.

After entering the input data of the problem, presented in table 1, after using the rectilinear distance measurement model, we obtain the coordinates (70,70) as the optimal location for the new production unit (fig.1).

| 02-17-2019 | New Facility | X Axis | Y Axis |
|---|---|---|---|
| 1 | UP | 70 | 70 |
| Total | Flow to&from | New Location | = 9000 |
| Total | Cost to&from | New Location | = 445000 |
| (by | Rectilinear | Distance) | |

**Figure 1.** The solution of the problem. Manhattan distance

Also, the value of the total cost per unit of distance is equal to 9000 m.u., and the total cost of transport is 445.000 m.u., resulting values and following the analytical calculation performed in section 2.1.




| 02-17-2019 20:28:39 | Facility Name | X Axis | Y Axis | Flow To All Facilities | Cost To All Facilities |
|---|---|---|---|---|---|
| 1 | A | 20 | 80 | 1200 | 72000 |
| 2 | B | 100 | 70 | 1000 | 30000 |
| 3 | C | 40 | 60 | 900 | 36000 |
| 4 | D | 80 | 40 | 800 | 32000 |
| 5 | E | 100 | 30 | 1300 | 91000 |
| 6 | F | 10 | 20 | 1200 | 132000 |
| 7 | G | 70 | 90 | 2600 | 52000 |
| 8 | UP | 70 | 70 | 0 | 0 |
| | Total | | | 9000 | 445000 |
| | Distance | Measure: | Rectilinear | | |

**Figure 2.** Analysis of the optimal location solution (rectilinear distance)

The graphical representation of the new location of the production unit is shown in Figure 3, and the distances between units in Figure 4.

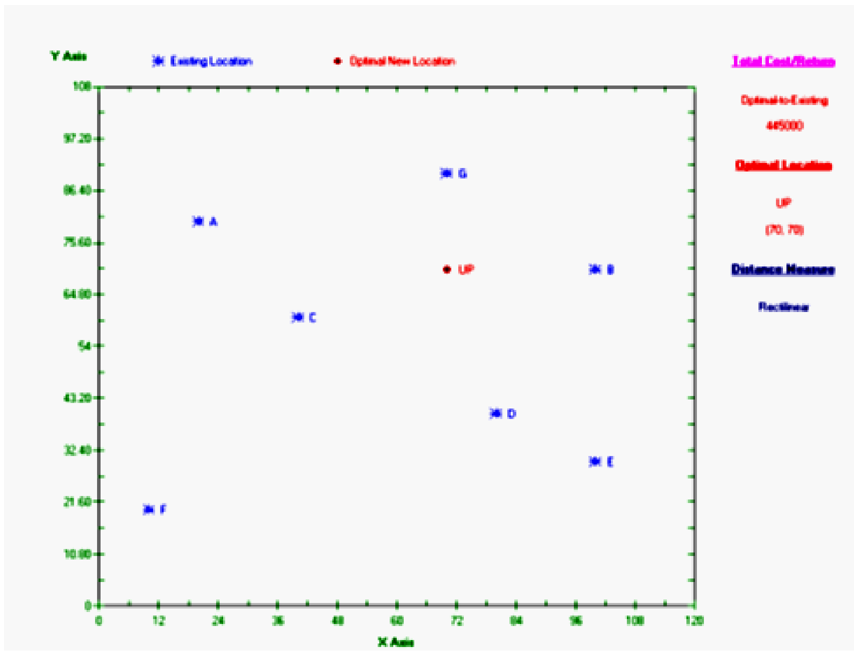
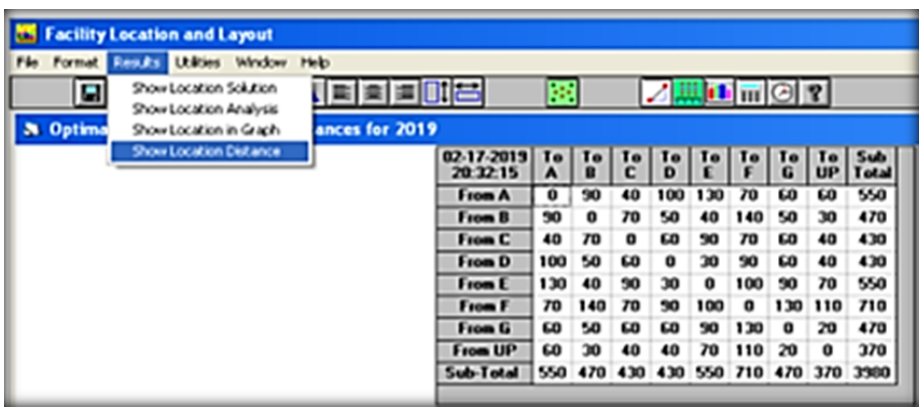


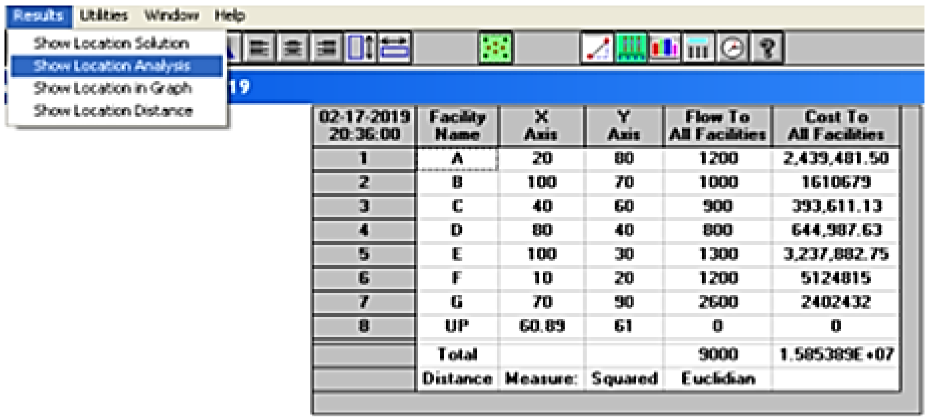

**Figure 3.** Location of the new production unit




| 02-17-2019 20:32:15 | To A | To B | To C | To D | To E | To F | To G | To UP | Sub Total |
|---|---|---|---|---|---|---|---|---|---|
| From A | 0 | 90 | 40 | 100 | 130 | 70 | 60 | 60 | 550 |
| From B | 90 | 0 | 70 | 50 | 40 | 140 | 50 | 30 | 470 |
| From C | 40 | 70 | 0 | 60 | 90 | 70 | 60 | 40 | 430 |
| From D | 100 | 50 | 60 | 0 | 30 | 90 | 60 | 40 | 430 |
| From E | 130 | 40 | 90 | 30 | 0 | 100 | 90 | 70 | 550 |
| From F | 70 | 140 | 70 | 90 | 100 | 0 | 130 | 110 | 710 |
| From G | 60 | 50 | 60 | 60 | 90 | 130 | 0 | 20 | 470 |
| From UP | 60 | 30 | 40 | 40 | 70 | 110 | 20 | 0 | 370 |
| Sub-Total | 550 | 470 | 430 | 430 | 550 | 710 | 470 | 370 | 3980 |

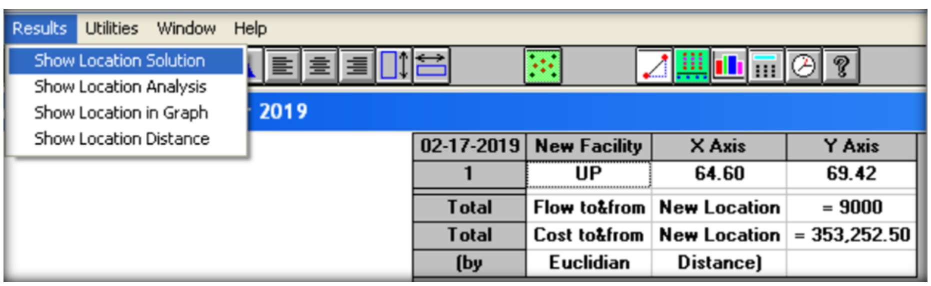
**Figure 4.** The distances between units

The results from using the square of the Euclidean distance (Figure 5) and the Euclidean distance (Figure 6) are presented below.




| 02-17-2019 20:36:00 | Facility Name | X Axis | Y Axis | Flow To All Facilities | Cost To All Facilities |
|---|---|---|---|---|---|
| 1 | A | 20 | 80 | 1200 | 2,439,481.50 |
| 2 | B | 100 | 70 | 1000 | 1610679 |
| 3 | C | 40 | 60 | 900 | 393,611.13 |
| 4 | D | 80 | 40 | 800 | 644,987.63 |
| 5 | E | 100 | 30 | 1300 | 3,237,882.75 |
| 6 | F | 10 | 20 | 1200 | 5124815 |
| 7 | G | 70 | 90 | 2600 | 2402432 |
| 8 | UP | 60.89 | 61 | 0 | 0 |
| | Total | | | 9000 | 1.585389E+07 |
| | Distance | Measure: | Squared | Euclidian | |

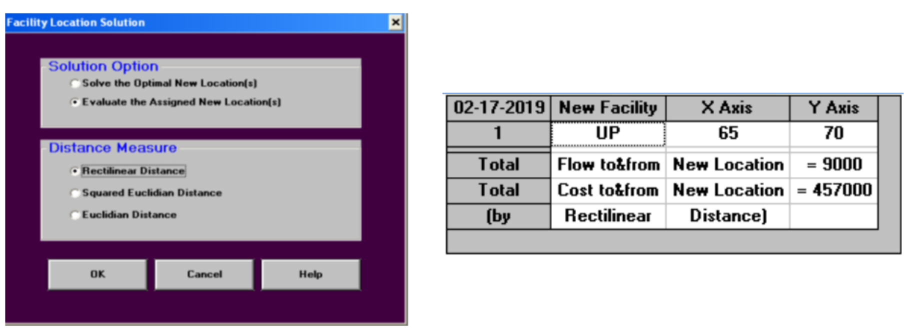
**Figure 5.** The optimal solution (the square of the Euclidean distance)




| 02-17-2019 | New Facility | X Axis | Y Axis |
|---|---|---|---|
| 1 | UP | 64.60 | 69.42 |
| Total | Flow to&from | New Location | = 9000 |
| Total | Cost to&from | New Location | = 353,252.50 |
| (by | Euclidian | Distance) | |

**Figure 6.** The optimal solution for placing in synthetic form (Euclidean distance)

The total transport cost is calculated by entering the coordinates of a certain placement (65,70).




| 02-17-2019 | New Facility | X Axis | Y Axis |
|---|---|---|---|
| 1 | UP | 65 | 70 |
| Total | Flow to&from | New Location | = 9000 |
| Total | Cost to&from | New Location | = 457000 |
| (by | Rectilinear | Distance) | |

**Figure 7**. Rectilinear model. Coordinates (65,70)

It can be observed that by changing the x coordinate of the location of the new production unit, the total transport cost also increased from 445.000 m.u. to 457.000 m.u.

## 3. Conclusion

From the solution provided by the WinQSB software, the coordinates at which the production unit should be located are different depending on the model used to measure distances, as follows:

**Table 3.** The results of the production unit location

| Model | Coordinates | | Total cost |
|---|---|---|---|
| | x | y | |
| The rectilinear distance | 70 | 70 | 445.000 |
| The Euclidean distance | 64,60 | 69,42 | 353.252,50 |
| The square of the Euclidean | 60,89 | 61 | 15.853.890 |

The choice of the location of the new production unit will be made between the solution given by the rectilinear model (calculated and analytical) and the Euclidean one. As the total transport cost is lower in the case of the Euclidean distance than in the case of the Manhattan distance, this is considered the optimal solution. Therefore, the coordinates of the new production unit are (64.60; 69.42), and the total transport cost is 353.252,50 m.u.

Before making the final decision on the location of the production unit, some clarifications must be made, namely:

1. It has been taken into account that any point in the xOy coordinate system is fit for location;
2. The uneven terrain was not excluded from the analysis;
3. The existing roads, energy, population density, etc. are not taken into account.

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*Addresses:*

- Lect. Dr. Eng. Olga-Ioana Amariei, "Eftimie Murgu" University of Reşiţa, Piața Traian Vuia, nr. 1-4, 320085, Reşiţa, o.amariei@uem.ro
- Prof. Dr. Eng. Codruta-Oana Hamat, "Eftimie Murgu" University of Reşiţa, Piața Traian Vuia, nr. 1-4, 320085, Reşiţa, c.hamat@uem.ro
- Prof. Daniel-Mihai Amariei, "Traian Lalescu" National College, B-dul. A.I. Cuza, Nr. 7, 320108. Reşiţa,, amarieidaniel@yahoo.com