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The Allowable Time Approach of the Uncertain Task for Three U-shaped Lines with the Minimum Workstations

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Abstract: UL (U-shaped Line) reduces the waste material in a manufacturing process. This paper presents the application of three U-shaped Lines with the minimum workstations which consists of UL-1, UL-2, and UL-3. The uncertain task of three U-shaped Lines determines the time as an interval. Three U-shaped Lines formulate the model with a binary integer linear programming. Each uncertain task reduces the normal time to the optimism time. The normal time reduction relates to the allowable time inversely. This paper indicates the procedure of the normal time reduction that applies MATLAB R2014a. The allowable time of UL-1, UL-2, and UL-3 for each uncertain task is 1.5 times of the optimistic time, 1.2 times of the optimistic time, and 1 time of the most optimistic time, respectively. For the utility of this paper, the uncertain task adjusts the finish time which does not affect the modification of the minimum number of workstations.

Keywords: U-shaped line, Binary integer linear programming, Line balancing.

1. Introduction

A manufacturing process which applies the lean production, the just-in-time fundamental, and the quality development is the UL (U-shaped Line) that is more qualified than the straight line. The UL qualification consists of volume flexibility, operator flexibility, a reduction for the number of workstations, and many decreasing quality problems [1]. Many industries apply the UL instead of the straight line. The UL decreases a set-up time, a throughput time, a lead time, and a work in the process [2-5].

In general, the category of the UL divides into three optimization problems which consist of type-1 problem, type-2 problem, and type-3 problem. The minimum number of workstations for a prescribed cycle time is the objective for type-1 problem. The minimum cycle time for a prescribed number of workstations is the objective for type-2 problem. The maximum line efficiency is the objective for type-3 problem [6-8]. The characteristic of the UL product divides into three types which consist of the single-model UL, the mixed-model UL, and the multi-model UL. The single-model UL manufactures the identical product. The mixed-model UL manufactures several models of the identical product. The multi-model UL manufactures several products in batches [6-7, 9-10]. The time characteristic of the UL task divides into two types which consist of the certain time, and the uncertain time. The certain time is the obvious operation time. The uncertain time is not the obvious operation time. The UL divides the uncertain time into four types which consist of the stochastic time, the fuzzy time, the scenario time, and the interval time. The stochastic time defines the characteristic as the known probabilistic distribution. The fuzzy time defines the characteristic as the fuzzy set. The fuzzy time is not concern the probabilistic distribution but relates to the membership function. The scenario time defines the characteristic as the predetermined plausible value. The scenario time is not concern the probabilistic distribution. The interval time defines the characteristic as the interval between the lower

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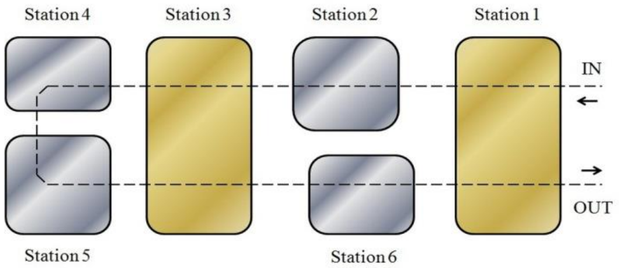

Figure.1 UL configuration

bound and the upper bound. The interval time does not know the probabilistic distribution [11].

The UL implementations for the manufacturing industries are indicated by the conference paper [12]. The benefits of the UL are admirable. They comprise an average of 76% in a productivity improvement, 75% in a relieving lead-time, 86% in a reducible work-in-process, and 83% in reducible faulty rates [4-5]. In an allocation procedure, all tasks assign to the workstation on the UL in both backward and forward path. They accord with the precedence relation. The task allocation for the UL is a pliable manner. For the task assignment of each workstation, the total times are not more than the cycle time. The entryway and the exit of the UL configuration are the same position. The UL eliminates the waste material in the manufacturing process. It approaches the productive quality development [4]. The UL configuration is shown in Fig. 1.

In this study, the UL is a type-1 problem that decreases the number of workstations and manufactures one identical product [13]. The smallest number of workstations is the criterion of the UL. It applies to the UL when a new UL is designed [10]. In literature, the maximum ranked positional weight heuristic that accomplishes the approximate solution solves the UL problem. This heuristic methodology does not achieve the solution optimistically [2,14-15]. For the deterministic UL model, Miltenburg and Wijngaard solved the 7-11 tasks problem with a dynamic programming and the 21-111 tasks problem with the maximum ranked positional weight heuristic [14]. Urban applied an integer linear programming for the UL formulation and solved the solution with general-purpose software CPLEX [2]. Scholl and Klein solved the solution of the UL with the ULINO procedure. This procedure was a code which codified on Borland's Pascal 7 [16]. Aase, Schniederjans, and Olson applied an integer linear programming for the UL formulation and introduced the either-or constraints to this UL model. They solved the solution of the UL with the U-OPT procedure [17]. Gökçen and Ağpak applied a goal programming to the UL model. This UL model tried to accomplish a satisfying solution rather than an optimal solution [18]. Fattahi and Turkay applied the alteration of the mixed integer linear programming for the UL model which consisted of the either-or constraints [19]. In a reality, some task times are not a certainty. They are changeable. The instances of the changeable task time are indicated by the conference paper [12]. For the stochastic UL model, Urban and Chiang applied a chance constraint programming for the UL model and utilized a piecewise linear program for the UL solution. This UL model classified to two categories such as a low variance model, and a high variance model. Two category models were solved by AMPL/CPLEX 6.5. This academic work explored the effect of variation [3]. Ağpak and Gökçen indicated the straight line model and the UL model which formulated their models with a chance constraint programming. They utilized a goal programming to the reliable increment [20]. In [3] and [20], the standard normal distribution with mean and variance is the attribute assumption for the stochastic task time.

On another instance, the stochastic task time may indicate the attribute assumption as a non-standard distribution which characterizes several forms [8]. Gurevsky, Hazır, Battaïa, and Dolgui indicated the straight assembly lines with the interval task times. These lines were a type-1 problem. They formulated these lines as a robust optimization model. They solved this robust model with a breadth-first search procedure. This academic work explored the minimal number of workstations for the straight assembly lines when the proportion of all pessimistic tasks in a workstation was varied [21]. Hazır and Dolgui indicated the straight assembly line for the interval task time which was a type-2 problem. They formulated this line as a robust optimization model and divided this line into two robust models. They suggested the Benders decomposition algorithm to two robust models. For the maximum time deviation of the cycle time constraint, first robust model defined the number of the pessimistic tasks in a workstation, and second robust model defined the proportion of all pessimistic tasks in a workstation. They presented and compared the task allocation for these two robust models [22]. Hazır and Dolgui indicated the UL for the interval task time which was a type-2 problem. They formulated this UL as a robust optimization model and suggested an iterative approximate algorithm to this robust model. For the maximum time deviation of the cycle time constraint, this robust model defined the number of the pessimistic tasks in a workstation. They presented the task allocation for this robust model and explored the optimum cycle time of this UL when the number of the pessimistic tasks was varied [23]. Pereira and Álvarez-Miranda indicated the straight assembly line as a type-1 problem and formulated this line as a robust optimization model. They determined the possible number of workstations for this line which was estimated by the lower bound and the upper bound. The lower bound used three methods which comprised the bin packing bound, the max-flow bound, and the Dantzig-Wolfe decomposition. The upper bound used the Hoffmann heuristic. They solved this robust model with the branch and bound procedure. This procedure comprised the BBR algorithm, the dominance rules, and the preprocessing rules [24]. For the feature of the robust optimization model, it is the two objective problems which comprise the goal of the objective function and the goal of the cycle time constraint. The optimization solver cannot solve the robust model directly. To be able to apply the optimization solver, the robust model must transform the two objective problems into the single objective problem. The robust model needs to apply the decomposition algorithm on the problem solution [23]. According to literature, the robust model is a worst-case problem that conforms to the characteristic of the pessimistic task allocation in a workstation.

On a later stage, Varnasilpin and Masuchun proposed the UL model that consisted of 8 certain tasks and 3 uncertain tasks. This UL model indicated the uncertain time which comprised the pessimistic time, the normal time, and the optimistic time [25]. Next, Varnasilpin and Masuchun proposed the UL model that consisted of 11 uncertain tasks. The uncertain time characteristic of this UL model conformed to the uncertain time characteristic of the [25] paper [26]. In both academic works, they formulated both UL models with a binary integer linear programming. Both UL models were a type-1 problem. For the smallest number of workstations, the workstation quantity of the [25] UL model and the [26] UL model was five and four, respectively. The uncertain time of both UL models were the optimistic time. Afterwards, Varnasilpin and Masuchun proposed and compared two type-1 U-shaped Lines that consisted of 8 certain tasks and 3 uncertain tasks. The uncertain task time for each UL characterized a different variance. First UL indicated the uncertain time which corresponded to the uncertain time of the UL model in the [25] paper. Second UL indicated the uncertain time which comprised the most pessimistic time, the pessimistic time, the normal time, the optimistic time, and the most optimistic time. The uncertain time of first UL was less various than the uncertain time of second UL. The smallest workstation quantity of first UL and second UL was five and four, respectively. The uncertain time of first UL and second UL was the optimistic time and the most optimistic time, respectively. The number of workstations of second UL was less than the number of workstations of first UL [12].

This paper presents and suggests the allowable time of the uncertain task which controls and preserves the minimum number of workstations for three U-shaped Lines. Three U-shaped Lines are a type-1 problem and formulate their UL models as a binary integer linear programming. Three U-shaped Lines comprise UL-1, UL-2, and UL-3. The uncertain tasks of three U-shaped Lines determine the time as an interval. They differ from the uncertain tasks of three previous papers which were the scenario time [12, 25-26]. The uncertain tasks of UL-1 which correspond to the [25] paper deviate from the normal time between -1 and 1. The uncertain tasks of UL-2 which correspond to the [26] paper deviate from the normal time between -1 and 1. The uncertain tasks of UL-3 which correspond to the [12] paper deviate from the normal time between -2 and 2. Three UL models formulate the cycle time constraints which do not concern the parameters approximation of the uncertain tasks. This academic work uses the prototype of Armin Scholl which applies to three UL models [6]. The original characteristic of the prototype is a straight line and all tasks are a certain time. The cycle time of the prototype is 10 minutes/unit that conform to this academic work. Three study models replace the straight line with the UL (U-shaped Line). The uncertain tasks of UL-1 and UL-2 define the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the optimistic time. The uncertain tasks of UL-3 define the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the most optimistic time. The normal time reduction of each uncertain task implies the allowable time expansion which refers to the optimism time. This academic work differs from the robust model which determines the pessimism time of the uncertain task in a workstation. On the robust model, the uncertain task in a workstation is selected by the maximum deviation between the normal time and the pessimistic time. The difference between this academic work and the robust model is the uncertain task that allocates to a workstation and the model transformation. The type of the uncertain task for

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the robust model is the pessimism task. The type of the uncertain task for this academic work is the optimism task. For the model transformation, the robust problem needs to convert the model for the solution with the decomposition algorithm. This academic work does not convert the model and applies the model for the solution directly. The purpose of this academic work investigates the allowable time of the application of three U-shaped Lines which influences the preservation of the minimum workstations.

2. Methodology

2.1 Problem details

This subsection details the precedence relation and task time of three U-shaped Lines. UL-1 and UL-3 consist of 8 certain tasks and 3 uncertain tasks. UL-2 consists of 11 uncertain tasks. Table 1 shows the precedence relation and task time of three U-shaped Lines. On UL-1 and UL-3, task B, H, and I are an uncertain task. They are a different interval time. The interval of UL-1 is ±1. The interval of UL-3 is ±2. All tasks of UL-2 are uncertain. The interval of the uncertain time of UL-2 is ±1. The uncertain tasks of UL-3 are more various than the uncertain tasks of UL-1 and UL-2. Fig. 2 indicates the precedence diagram for UL-1, UL-2, and UL-3. On Fig. 2, each node exhibits the task. The numbers in the bracket which locate over the node exhibit the task time for three U-shaped Lines. The line which links between two nodes exhibits the task relation.

Table 1. Precedence relation and task time for three U-shaped Lines

Task	Precedence relation	Task time (minutes)		
		UL-1	UL-2	UL-3
A	-	5	5 ± 1	5
B	-	3 ± 1	3 ± 1	3 ± 2
C	A	5	5 ± 1	5
D	A,B	3	3 ± 1	3
E	B	3	3 ± 1	3
F	C	5	5 ± 1	5
G	C,D	5	5 ± 1	5
H	F	4 ± 1	4 ± 1	4 ± 2
I	G	6 ± 1	6 ± 1	6 ± 2
J	I	5	5 ± 1	5
K	E,H,J	1	2 ± 1	1

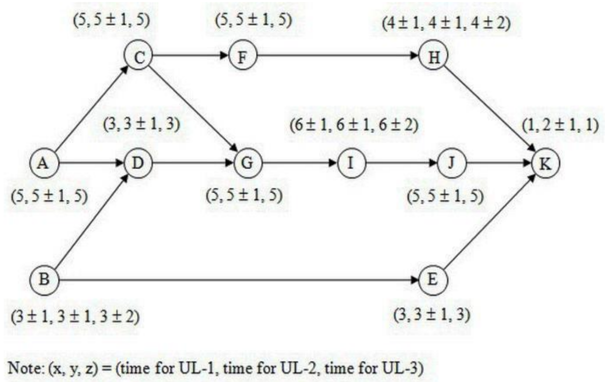

Figure.2 Precedence diagram for UL-1, UL-2, and UL-3

2.2 Problem methods

This subsection details the procedure for the smallest number of workstations which solves three U-shaped Lines. The procedure determines the latest station index and the earliest station index for each task. These two station indexes indicate the station interval of each task. The station interval which eliminates the inessential index presents the lower index and the upper index of each task. The earliest station index, the latest station index, and the station interval are based on Eqs. (1) - (3), respectively [2,6,18].

$$E_j = min\{E_a, E_b\}, \tag{1}$$

$$E_a = \left\lceil \frac{\left(t_j + \sum_{h \in p} t_h\right)}{C} \right\rceil^+, E_b = \left\lceil \frac{\left(t_j + \sum_{h \in f} t_h\right)}{C} \right\rceil^+.$$

$$L_j = \left\lceil \frac{\left(\sum_{all\ j} t_j\right)}{C} \right\rceil^+. \tag{2}$$

$$SI_j = \{E_j, L_j\}. \tag{3}$$

Eqs. (1) – (3) details the notation as follows:

- E_a the earliest station index for the task in forward path
- E_b the earliest station index for the task in backward path
- E_j the earliest station index for task j
- L_j the latest station index for task j
- t_j the operation time for task j
- t_h the operation time for task h

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C	the cycle time
p	the predecessor task for task j
f	the successor task for task j
SI_j	the station interval for task j
$[x]^+$	the minimal positive integer that is not less than x
min	the minimal value

On the station interval, UL-1 and UL-2 comprise the station index of the uncertain task that corresponds to the time between the pessimistic time and the optimistic time. UL-3 comprises the station index of the uncertain task that corresponds to the time between the most pessimistic time and the most optimistic time. The station index of the certain task for UL-1 and UL-3 does accord with the normal time. The most pessimistic time and the pessimistic time represent the first latest finish duration and the second latest finish duration, respectively. The most optimistic time and the optimistic time represent the first earliest finish duration and the second earliest finish duration, respectively. The normal time represents the mean finish duration. The station interval, the earliest station index, and the latest station index for three U-shaped Lines show the detail in the [25] paper for UL-1, the [26] paper for UL-2, and the [12] paper for UL-3.

Three U-shaped Lines apply a binary integer linear programming to the numerical model. UL-1 and UL-3 models comprise two variable types that consist of the uncertain decision variables, and the certain decision variables. UL-2 model comprises the only uncertain decision variables. The operation time of the uncertain decision variables in three UL models is adjustable. On UL-1 and UL-2, the operation time reduces the normal time to the optimistic time. The uncertain tasks of these models define the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the optimistic time. On UL-3, the operation time reduces the normal time to the most optimistic time. The uncertain tasks of this model define the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the most optimistic time. The procedure of the normal time reduction for UL-1 and UL-3 shows as the flow diagram in Fig. 3. On Fig. 4, it is the flow diagram for UL-2 which indicates the procedure of the normal time reduction. The characteristic of three UL models selects all decision variables which relate to the station index for the minimum objective function [27]. UL-1 and UL-3, and UL-2 indicate the mathematical model in Subsection 2.4, and Subsection 2.5, respectively.

2.3 Notation for three UL models

Three U-shaped Lines detail the notation for the mathematical model as follows:

X_{pq}	1, if task p allocates to station q in forward path or 0, otherwise
Y_{pq}	1, if task p allocates to station q in backward path or 0, otherwise
U_{sq}	1, if task s allocates to station q in forward path or 0, otherwise
V_{sq}	1, if task s allocates to station q in backward path or 0, otherwise
t_p	the operation time for certain task p
t_s	the operation time for uncertain task s
d_s	the interval time for uncertain task s
θ	the adjustment for the uncertain task, θ = {0.0, 0.1, 0.2, 0.3, …, 0.9, 1.0}
p	the set of the certain task, p = {A,C,D,E,F,G,J,K} for UL-1and UL-3
s	the set of the uncertain task, s = {B,H,I} for UL-1and UL-3, s = {A,B,C,D,E,F,G,H,I,J,K} for UL-2
q	the set of the station index, q = {1,2,3,4,5} for UL-1, q = {1,2,3,4,5,6} for UL-2 and UL-3
B_q	the set of the task that assigns to station q
L_h	the latest station index for task h that is a predecessor task
L_k	the latest station index for task k that is a successor task
SI_p	the station interval for certain task p
SI_s	the station interval for uncertain task s
SI_h	the station interval for the predecessor task
SI_k	the station interval for the successor task

2.4 Mathematical model for UL-1 and UL-3

UL-1 and UL-3 indicate the mathematical model as follows:

$$Min\ Z = \sum_{p \in B_q} \sum_{q \in SI_p} qX_{pq} + \sum_{p \in B_q} \sum_{q \in SI_p} qY_{pq}, \tag{4}$$
$$+ \sum_{s \in B_q} \sum_{q \in SI_s} qU_{sq} + \sum_{s \in B_q} \sum_{q \in SI_s} qV_{sq}.$$

subject to

$$\sum_{q \in SI_p} X_{pq} + \sum_{q \in SI_p} Y_{pq} = 1, \ for\ all\ p. \tag{5}$$

$$\sum_{q \in SI_s} U_{sq} + \sum_{q \in SI_s} V_{sq} = 1, \ for\ all\ s. \tag{6}$$

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$$\sum_{p \in B_q} t_p X_{pq} + \sum_{p \in B_q} t_p Y_{pq} + \sum_{s \in B_q} (t_s - \theta d_s) U_{sq}, \quad (7)$$
$$+ \sum_{s \in B_q} (t_s - \theta d_s) V_{sq} \leq 10, \; for \; all \; q.$$

$$\sum_{q \in SI_k} (L_k - q + 1) X_{kq} + \sum_{q \in SI_k} (L_k - q + 1) U_{kq}, \quad (8)$$
$$- \sum_{q \in SI_h} (L_h - q + 1) X_{hq} - \sum_{q \in SI_h} (L_h - q + 1) U_{hq} \leq 0, \; for \; h < k.$$

$$\sum_{q \in SI_h} (L_h - q + 1) Y_{hq} + \sum_{q \in SI_h} (L_h - q + 1) V_{hq}, \quad (9)$$
$$- \sum_{q \in SI_k} (L_k - q + 1) Y_{kq} - \sum_{q \in SI_k} (L_k - q + 1) V_{kq} \leq 0, \; for \; h < k.$$

$$X_{pq}, Y_{pq}, U_{sq}, V_{sq} = \{0,1\}, \; for \; all \; p, q, r, s. \quad (10)$$

In this mathematical model, Eq. (4) is the objective function that indicates the minimum station index. For the summation of the station index in Eq. (4), it implies the smallest number of workstations. Eqs. (5) and (6) are the condition of the certain task assignment and the condition of the uncertain task assignment, respectively. These two conditions imply the allocation of each task to the only one workstation. Eq. (7) is the cycle time condition that both certain tasks and uncertain tasks assign to each workstation. Eq. (8) is the precedence condition that assigns the task in the forward path. Eq. (9) is the precedence condition that assigns the task in the backward path. Eq. (10) is the zero-one condition for the decision variables.

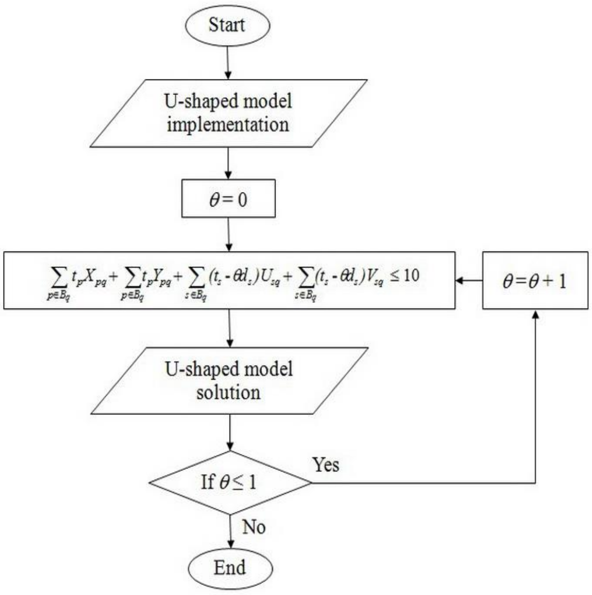

Figure.3 Flow diagram for UL-1 and UL-3

2.5 Mathematical model for UL-2

UL-2 indicates the mathematical model as follows:

$$Min \; Z = \sum_{s \in B_q} \sum_{q \in SI_s} q U_{sq} + \sum_{s \in B_q} \sum_{q \in SI_s} q V_{sq}. \quad (11)$$

subject to

$$\sum_{q \in SI_s} U_{sq} + \sum_{q \in SI_s} V_{sq} = 1, \; for \; all \; s. \quad (12)$$

$$\sum_{s \in B_q} (t_s - \theta d_s) U_{sq}, \quad (13)$$
$$+ \sum_{s \in B_q} (t_s - \theta d_s) V_{sq} \leq 10, \; for \; all \; q.$$

$$\sum_{q \in SI_k} (L_k - q + 1) U_{kq}, \quad (14)$$
$$- \sum_{q \in SI_h} (L_h - q + 1) U_{hq} \leq 0, \; for \; h < k.$$

$$\sum_{q \in SI_h} (L_h - q + 1) V_{hq}, \quad (15)$$
$$- \sum_{q \in SI_k} (L_k - q + 1) V_{kq} \leq 0, \; for \; h < k.$$

$$U_{sq}, V_{sq} = \{0, 1\}, \; for \; all \; q, s. \quad (16)$$

In this mathematical model, Eq. (11) corresponds to Eq. (4) in Subsection 2.4. For the summation of the station index in Eq. (11), it implies the smallest number of workstations. Eq. (13) corresponds to Eq. (7) in Subsection 2.4. It is the cycle time condition that the uncertain tasks assign to each workstation. According to Subsection 2.4, Eq. (12) corresponds to Eq. (6) which concerns the uncertain decision variables. Eqs. (14) – (16) correspond to Eqs. (8) – (10) which exclude the certain decision variables.

3. Results

Three U-shaped Lines apply a binary integer linear programming for the numerical model formulation and solve the results with the code program. The number of decision variables and the number of constraints for UL-1, UL-2, and UL-3 illustrate the detail in Table 2. All decision variables for three UL models are 0 or 1. The code program utilizes MATLAB R2014a for the solution. It

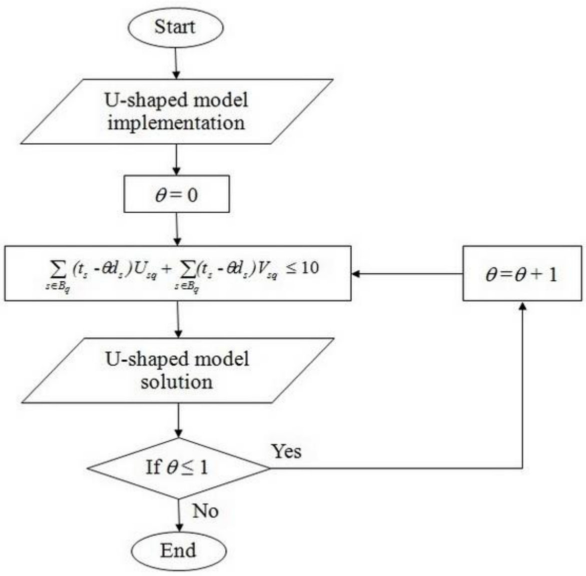

Figure.4 Flow diagram for UL-2

Table 2. Decision variable and constraints for three UL models

Description	UL-1	UL-2	UL-3
Number of decision variables	106	130	130
Number of constraints	42	43	43

applies the INTLINPROG toolbox for the optimum result. The memory 2 GB DDR3 and CPU Intel Core 2 Duo 2.2 GHz on the notebook computer assists and accomplishes the solution effectively. The algorithm for the INTLINPROG toolbox which solves the solution consists of a branch and bound and a cut generation.

On the cycle time constraints, three UL models define the operation time of each uncertain task that is adjustable. On UL-1 and UL-2, the operation time of each uncertain task reduces the normal time to the optimistic time. On UL-3, the operation time of each uncertain task reduces the normal time to the most optimistic time. Each uncertain task of UL-1 and UL-2 defines the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the optimistic time. Each uncertain task of UL-3 defines the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the most optimistic time. The cycle time constraints of three U-shaped Lines differ from the cycle time constraints in three conference papers [12,25-26]. Three previous papers utilize the minimum number of workstations to this academic work. In the [25] paper for UL-1, the minimum number of workstations is five. In the [26] paper for UL-2 and the [12] paper for UL-3, the minimum number of workstations is four. The normal time reduction for three U-shaped Lines illustrates the task allocation and the total operation time for each workstation in Table 3.

Table 3. Normal time reduction for three U-shaped Lines

θ	Type	Station				
		1	2	3	4	5
0.0	UL-1	A,B,K (9)	D,E,H (10)	C,G (10)	F,J (10)	I (6)
	UL-2	A,E,K (10)	B,D,H (10)	C,F (10)	G,J (10)	I (6)
	UL-3	A,H,K (10)	B,D,E (9)	C,G (10)	F,J (10)	I (6)
0.1	UL-1	A,B,K (8.9)	D,E,H (9.9)	F,J (10)	C,G (10)	I (5.9)
	UL-2	A,E,K (9.7)	B,D,H (9.7)	C,G (9.8)	F,J (9.8)	I (5.9)
	UL-3	A,H,K (9.8)	B,D,E (8.8)	C,F (10)	G,J (10)	I (5.8)
0.2	UL-1	A,E,K (9)	B,D,H (9.6)	C,G (10)	F,J (10)	I (5.8)
	UL-2	A,B,K (9.4)	D,E,H (9.4)	C,F (9.6)	G,J (9.6)	I (5.8)
	UL-3	A,H,K (9.6)	B,D,E (8.6)	C,G (10)	F,J (10)	I (5.6)
0.3	UL-1	A,H,K (9.7)	B,D,E (8.7)	C,G (10)	F,J (10)	I (5.7)
	UL-2	A,B,K (9.1)	D,E,H (9.1)	C,G (9.4)	F,J (9.4)	I (5.7)
	UL-3	B,E,H,K (9.8)	A,D (8)	C,J (10)	G,F (10)	I (5.4)
0.4	UL-1	A,H,K (9.6)	B,D,E (8.6)	C,F (10)	G,J (10)	I (5.6)
	UL-2	B,H,K (7.8)	A,D,E (9.8)	C,G (9.2)	F,J (9.2)	I (5.6)
	UL-3	B,E,H,K (9.4)	A,F (10)	C,J (10)	D,I (8.2)	G (5)
0.5	UL-1	B,E,H,K (10)	A,D (8)	C,J (10)	G,F (10)	I (5.5)
	UL-2	B,E,H,K (10)	A,D (7)	I,J (10)	C,F (9)	G (4.5)
	UL-3	B,E,H,K (9)	A,C (10)	D,F (8)	G,J (10)	I (5)

Note: (x) Total operation time for any workstations in minutes

Table 3. (Continued)

θ	Type	Station 1	2	3	4	5
0.6	UL-1	B,E,H,K (9.8)	F,J (10)	A,D (8)	C,G (10)	I (5.4)
	UL-2	B,E,H,K (9.6)	A,C (8.8)	D,G (6.8)	I,J (9.8)	F (4.4)
	UL-3	B,E,H,K (8.6)	A,C (10)	I,J (9.8)	D,F (8)	G (5)
0.7	UL-1	B,E,H,K (9.6)	A,J (10)	D,I (8.3)	C,F (10)	G (5)
	UL-2	A,B,E (8.9)	C,J,K (9.9)	D,F,H (9.9)	G,I (9.6)	-
	UL-3	B,E,H,K (8.2)	A,J (10)	C,F (10)	D,I (7.6)	G (5)
0.8	UL-1	B,E,H,K (9.4)	A,C (10)	F,J (10)	D,G (8)	I (5.2)
	UL-2	A,B,E,K (9.8)	D,H,J (9.6)	F,I (9.4)	C,G (8.4)	-
	UL-3	A,B,H,K (9.8)	D,F (8)	C,E (8)	G,I (9.4)	J (5)
0.9	UL-1	B,E,H,K (9.2)	A,J (10)	D,I (8.1)	F,G (10)	C (5)
	UL-2	A,B,E,K (9.4)	C,D,H (9.3)	F,J (8.2)	G,I (9.2)	-
	UL-3	B,E,H,K (7.4)	A,D (8)	I,J (9.2)	C,F (10)	G (5)
1.0	UL-1	B,E,H,K (9)	A,D (8)	F,J (10)	C,G (10)	I (5)
	UL-2	B,H,J,K (10)	A,D,F (10)	C,E,G (10)	I (5)	-
	UL-3	A,B,D,K (10)	E,J,H (10)	F,I (9)	C,G (10)	-

Note: (x) Total operation time for any workstations in minutes

This table indicates the adjustment (θ) in first column which is the mechanism for the normal time reduction. The task allocation and the total operation time for each workstation illustrate the detail between third column and seventh column. On first row (θ is 0.0), the uncertain task for three U-shaped Lines which assigns to each workstation is the normal time. The minimum number of workstations for three U-shaped Lines is five. On last row (θ is 1.0), the uncertain task for UL-1 and UL-2 which assigns to each workstation is the optimistic time. The uncertain task for UL-3 which assigns to each workstation is the most optimistic time. The minimum number of workstations for three U-shaped Lines is same as three previous papers.

On each adjustment, Table 3 indicates the task allocation and the number of workstations for three U-shaped Lines. According to Fig. 3 and Fig. 4, each adjustment for UL-1 and UL-2 represents the sum of the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the optimistic time. Each adjustment for UL-3 represents the sum of the normal time reduction for any iteration that decreases 10% of the interval between the normal time and the most optimistic time. On the uncertain task, the normal time reduction relates to the allowable time that controls and preserves the minimum number of workstations. The control and preservation mechanism depends upon the number of workstations on the UL and the task assignment.

Table 4 indicates the task allocation and the number of workstations for UL-3. The adjustment of this table is 0.95. This additional adjustment that decreases 5% of the interval between the normal time and the most optimistic time correlates with 90% of the normal time reduction of UL-3 in Table 3.

This work utilizes the minimum number of workstations of three previous papers for the reference. The normal time reduction which substitutes for the allowable time expansion depends upon the minimum number of workstations and the configuration of the task assignment. The relation between the normal time reduction and the allowable time expansion illustrates the detail in Fig. 5. In this figure, the finish time of the uncertain task for three U-shaped Lines is adjusted by the normal time reduction. The normal time reduction shortens the normal time to the finish time. It implies the allowable time expansion that lengthens the optimism time to the finish time.

The normal time reduction of UL-1 (θ is 0.5), UL-2 (θ is 0.8), and UL-3 (θ is 1.0) for each uncertain task decreases 50% of the interval between the normal time and the optimistic time, 80% of the interval between the normal time and the optimistic time, and 100% of the interval between the normal time and the most optimistic time, respectively. The normal time reduction relates to the allowable time inversely. Therefore, the allowable time of UL-1, UL-2, and UL-3 which controls and preserves the minimum workstations is 50%, 20%, and 0%, respectively. For this reason, the allowable time of UL-1, UL-2, and UL-3 for

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Table 4. Normal time reduction for UL-3 ($\theta = 0.95$)

θ	Station				
	1	2	3	4	5
0.95	A,B,H,K (9.2)	C,J (10)	D,E (6)	F,I (9.1)	G (5)

Note: (x) Total operation time for any workstations in minutes

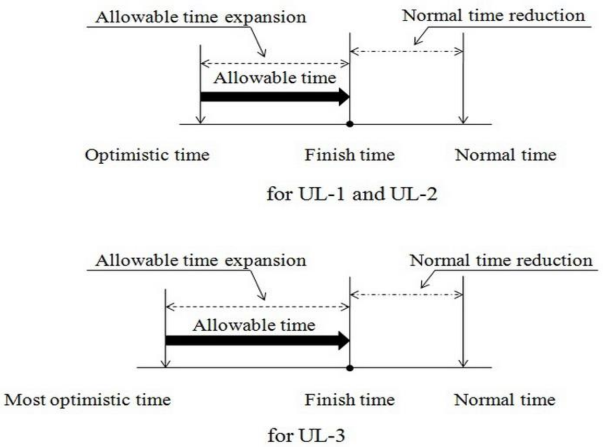

Figure.5 Relation between the normal time reduction and the allowable time expansion

each uncertain task is 1.5 times of the optimistic time, 1.2 times of the optimistic time, and 1 time of the most optimistic time, respectively.

4. Discussion

Three U-shaped Lines apply a binary integer linear programming to the numerical model. They solve the solution with the code program on MATLAB R2014a. The code program applies the INTLINPROG toolbox for the optimum result. The uncertain tasks for three U-shaped Lines determine the time as the interval. They differ from the task time of the stochastic UL model which is the standard normal distribution with mean and variance [3, 20]. According to the cycle time constraints, the parameters of the stochastic UL model are approximate. The stochastic UL model converts the parameters with the linear transformation. Initially, the stochastic UL model prescribes the form of the cycle time as the chance constraint. The chance constraint correlates with the time that is the standard normal distribution and the probability that accomplishes the tasks within the cycle time. For three study models, the cycle time constraints which do not concern the parameters approximation differ from the stochastic UL model. These constraints are shown in Eqs. (7) and (13).

In this work, three UL models determine the uncertain decision variables and the certain decision variables which correspond to the minimum objective function. The uncertain decision variable and the certain decision variable which are the component of three UL models represent the uncertain task and the certain task, respectively. The normal time of each uncertain task for UL-1 and UL-2 models decreases to the optimistic time. The normal time of each uncertain task for UL-3 model decreases to the most optimistic time. For each uncertain task, the allowable time of UL-1, UL-2, and UL-3 is 1.5 times of the optimistic time, 1.2 times of the optimistic time, and 1 time of the most optimistic time, respectively.

According to this result, the uncertain tasks of UL-1 and UL-2 deviate from the normal time between -1 and 1. The uncertain tasks of UL-3 deviate from the normal time between -2 and 2. On the lower bound of the interval, the allowable time of UL-1 is more than the allowable time of UL-3. This cause is due to the dissimilar interval of the uncertain task. The uncertain task time of UL-3 is more various than the uncertain task time of UL-1. On the similar interval, the allowable time of UL-2 is less than the allowable time of UL-1. This cause is due to the different number of the uncertain tasks in the UL. The number of the uncertain tasks for UL-2 is more than the number of the uncertain tasks for UL-1. The characteristic of each UL which depends upon the individual interval time and the individual number of the uncertain tasks affects the allowable time of the uncertain task.

5. Conclusion

Three U-shaped Lines apply a binary integer linear programming to the numerical model. Three UL models formulate the cycle time constraints which do not concern the parameters approximation of the uncertain tasks. The uncertain tasks for three UL models determine the time as the interval. This academic work applies the INTLINPROG toolbox on MATLAB R2014a which solves the optimum result of three UL models. On the lower bound of the interval, the allowable time of UL-1, UL-2, and UL-3 for each uncertain task is 1.5 times of the optimistic time, 1.2 times of the optimistic time, and 1 time of the most optimistic time, respectively. For the uncertain task of each UL, the allowable time which controls and preserves the minimum workstations depends upon the UL characteristic such as the individual interval time, and the individual number of the uncertain tasks.

This paper presents and simulates the application of three U-shaped Lines which controls and preserves the minimum workstations. Each UL application which indicates the precedence relation and the task time simulates the normal time reduction of the uncertain task. This paper suggests the methodology of the normal time reduction. This methodology exhibits the procedure that describes as the flow diagram in Fig. 3 and Fig. 4. It is the intelligent procedure that implies and determines the allowable time. This procedure adjusts the finish time of the uncertain task. The benefit of this paper utilizes the allowable time of the uncertain task for the UL planning. This benefit reduces the uncertain task time and the number of workstations in the UL. The allowable time of the uncertain task can preserve the minimum number of workstations and the configuration of the analogous task allocation in the UL. On the lower bound of the interval, the uncertain task adjusts the finish time which does not affect the modification of the minimum number of workstations. This paper is helpful for the finish time of the uncertain task. The finish time of the uncertain task is flexible. Each uncertain task in the UL adjusts the finish time which lengthens the optimism time and preserves the minimum number of workstations.

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