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DYNAMIC IDENTIFICATION BY ENUMERATION FOR CO-OPERATIVE KNOWLEDGE DISCOVERY

Oksana Arnold[1], Sebastian Drefahl[2], Jun Fujima[3], Klaus P. Jantke[2]
and Christoph Vogler[2]
[1]Erfurt University of Applied Sciences. Altonaer Str. 25, 99085 Erfurt, Germany
[2]ADICOM Software. Frauentorstr. 11, 99423 Weimar, Germany
[3]National Inst. for Materials Science. 1-2-1 Sengen, Tsukuba, Ibaraki 305-0047, Japan

ABSTRACT

In contemporary information and communication technologies, there is an urgent need for transforming tools into assistant systems. Humans do not need more digital tools that require learning how to wield them, but digital assistants guiding them to unforeseeably valuable results – an effect named serendipity. This applies particularly when dealing with wicked problems which change over time when being tackled. Data analysis, visualization, and exploration is a characteristic domain of this type, particularly when open data are in focus, because the analysts have no background knowledge about the origin of these open data. The paper demonstrates the transformation of a tool for data analysis into an intelligent adaptive assistant. The transformation is based on the exploitation of concepts, methods, and technologies from disciplines such as meme media technology, natural language processing, and theory of mind modeling and induction. In comparison to earlier approaches to computational theory of mind induction, the present one relies on dynamically generated spaces of hypotheses. A rigorous mathematical proof demonstrates the superiority of the novel reasoning technology. A case study in business intelligence serves as proof of concept.

KEYWORDS

Data Analysis, Data Visualization, Data Exploration, Assistant Systems, Intelligent System Assistance, Knowledge Discovery, Meme Media, Natural Language Processing, Inductive Inference, Theory of Mind, Theory Induction

1. MOTIVATION

No doubt, digitalization pervades nearly every sphere of life. Humans are facing more and more digital systems at their workplaces, in everyday education, and in their spare time. Frequently, the process of digitalization is placing on them the burden of learning about new digital systems and how to use them appropriately. Contemporary technology-enhanced education in school, e.g., means in the first place that teachers need to become acquainted with new software tools. In other words, this way of digitalization begins with extra workload without any valuable compensation in near future. No wonder to see so many digitalization projects that fail.

The current digitalization process in industry, business, services, education, health care, tourism, and the like, as Arnold et al. (2017) put it, "bears abundant evidence of the need for a paradigmatic shift from digital tools to intelligent assistant systems" (p. 28). This does apply to the growing internet of things and to the recent development of industry 4.0 and business 4.0, in particular.

This paper is intended to contribute to such a fundamental transformation by novel ideas, methodologies, and techniques applicable to a wide spectrum of domains. It expands on the authors' earlier work (Arnold et al. 2017) presented on the e-Society Conference 2017. Beyond the introduction, explanation, and investigation of a few novelties, there is the particular – possibly a bit unusual and ambitious – goal of the authors to provide a *stringent formal proof* of the superiority of one of these novel concepts over any other related approach in use so far.

Before we are able to state clearly the central problems addressed in the present contribution, we need the following three chapters to relate ideas, concepts, and approaches.

2. ASPECTS OF TRANSDISCIPLINARITY

This paper reports about a project that is highly transdisciplinary. It relies on prerequisites from largely varying involved research areas such as *meme media*, *natural languages processing*, *theory of mind modeling and induction*, *mathematical logic* including *Prolog programming*, and *recursion-theoretic inductive inference*, and it relates to research, development and application fields such as *big data*, *knowledge discovery*, and *machine learning*. All this is complemented by a scientific perspective at so-called *wicked problems* and by the transdisciplinary field of *digital assistant systems* which largely overlaps with *artificial intelligence*.

Research and development toward the paradigmatic shift from conventional software tools to intelligent assistant systems shall address a wide audience. Not to miss the wood for the trees, the authors will tailor their presentation to the needs of an audience that can't be expected to be familiar with all the above-mentioned disciplines. Therefore, first, the underlying disciplinary technicalities are suppressed as much as possible and, second, the present paper concentrates on a few selected results, only.

As in (Arnold et al., 2017), the selected problems in focus will be (i) in conceptualization, (ii) in algorithmics, (iii) in business applications, and beyond the limits of (Arnold et al., 2017), (iv) in recursion-theoretic inductive inference. The authors' novel solutions will be explained theoretically and will be exemplified practically. The thematic frame of the interdisciplinary endeavor is *meme media technology* and its applications (see (Tanaka, 2003) for the origins and both (Dawkins, 1976) and (Blackmore, 1999) for the underlying philosophy). Technologically, the present authors' novel business application assistant is an own meme media implementation.

3. ASSISTANCE, LEARNING, AND SERENDIPITY

One of the key insights underlying the present work is that digital systems able to assist human beings according to the humans' intentions and goal, needs and desires, and further peculiarities need to be *learning systems*. Jantke, Grieser and Lange (2003) put it as follows: "Erst wenn der Benutzer aufgrund seines dynamischen Verhalten 'verstanden' wird, kann ein Computersystem zum adaptiven Assistenten dieses Lernenden werden, der ihn berät und substantiell unterstützt." Roughly translated, the key to adaptivity is the computer's ability to 'understand' its user based on the user's behavior. *Understanding means learning*. Planning adaptive system behavior to meet a human user's needs is learning as well – *planning is learning* (Arnold & Jantke, 1996).

The contemporary digitalization of our society (Krotz, 2007) leaves no room for doubt: There is the urgent need for a paradigmatic shift from digital tools to digital assistant systems (see (Kaschek, 2007), (Kreutzberger et al., 2011) and the contributions therein).

Digital tools are useful when humans know exactly what to do and how to do it and, additionally, if they learned how to wield the tools.

For the majority of wicked problems, digital tools are quite inappropriate (Kaschek, 2006). A problem is called a wicked one if it has the peculiarity to change over time when being tackled.

Problems of data analysis, visualization and exploration form a particularly interesting class of wicked problems. In cases that are scientifically and/or economically relevant, the problem does not only change over time – it really evolves. Even more intriguingly, it emerges over time. The problem that is finally solved has not been there in the beginning.

Digital assistant system that learn about their users pave the way for unexpected results emerging from human-computer interaction or, better to say, human-computer co-operation. The effect is called serendipity (Schubert, 2013), (Jantke & Fujima, 2015).

For already more than a century, serendipity is understood as "the faculty of making happy and unexpected discoveries by accident" (The Oxford English Dictionary, 1912-13). Columbus' discovery of America, Fleming's discovery of penicillin, and Nobel's discovery of dynamite are prominent illustrations of serendipity.

There is a recent trend toward digital humanities (Carter, 2013) which is bringing with it a strong desire to use information and communication technologies in innovative ways. Digital humanities mean much more than providing every scholar and student with database access. New technologies allow for new research. Particularly important is the high potential of computerization to make unexpected findings more likely. Generating hypotheses and forming scientific theories is not what contemporary computers are expected to do. Scientific work is performed by the scientists of a discipline and, sometimes, by teams of scientists working interdisciplinary. The higher expectations of the digital humanities is not to replace scientists by computers, but to provide technologies and assistants that allow for or even provoke new forms of research and experiment. Co-occurrence search as discussed in (Schubert, 2013) is an illustrative example. It may reveal semantic relations which are neither obvious nor familiar. Conventional search tools and strategies do usually not arrive at those results (ibid., p. 182). From the perspective of the present paper, assistant systems are the key to provoke serendipity.

Beyond conventional technology and innovative approaches such as, e.g., co-occurrence search, there is a desire to introduce more flexible technologies which allow for unforeseen data manipulation and investigation toward unsought syntactic results provoking unexpected semantic interpretations. Jantke and Fujima (2015) relate *serendipity, meme media technology, system assistance, systems that learn*, and *data analysis, visualization, and exploration*.

4. DATA ANALYSIS, VISUALIZATION, AND EXPLORATION

This third preparatory section expands on the before-mentioned paper (Jantke and Fujima, 2015) and, even more specifically, on a conference paper by Jantke and Fujima (2016) introducing *scenarios of data analysis, visualization and exploration* (DAVE scenarios, for short).

Figure 1 displays a rather simple interaction state of the authors' underlying DAVE tool. This tool has been transformed into an intelligent assistant system named ADiDAVE which is subsequently demonstrated in its version 0.1. It runs in a browser and, thus, may be deployed as a local software installation as well as a Web service. The principles and technologies of this transformation are in focus of the present contribution (sections 6, 7, and 8) with particular emphasis on the reach of the novel algorithmic ideas under the hood, so to speak (section 9).

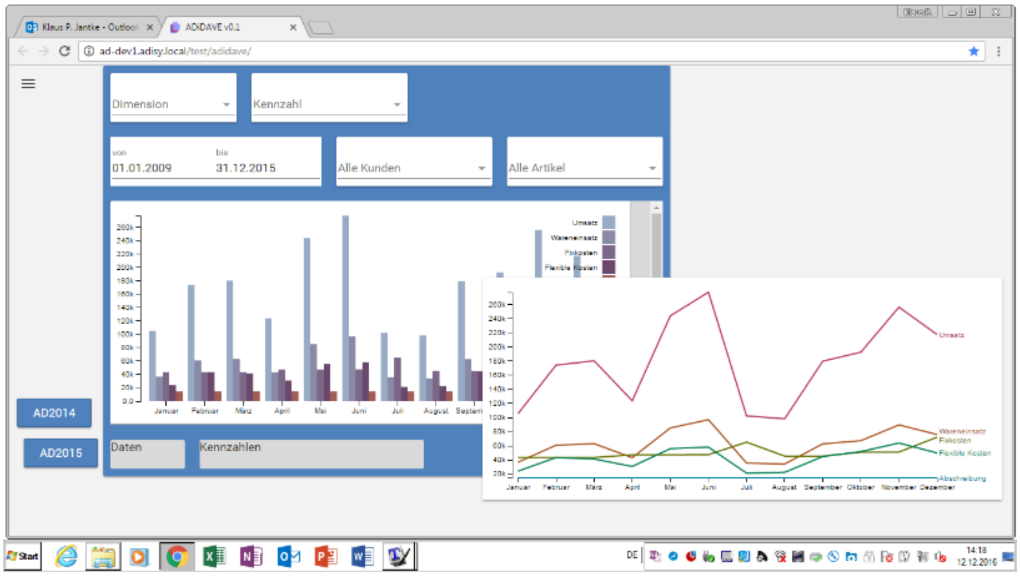

Figure 1. ADICOM's Data Analysis, Visualization and Exploration Tool ADiDAVE, Version 0.1

When human users interact with the system, they perform usually long-lasting sequences of (inter-)actions. There are elementary actions such as loading a database, selecting a certain type of visualization (multiple-bar charts or line charts as on display in figure 1), querying the data base, filtering, changing the rendering (possibly back and forth), and the like.

To be successful, users need to know how to wield the tool. Even if they do not know how to do so, they perform sequences of (inter-)actions. There are algorithmic concepts that allow for an automatic assessment of the human user's competence. In the special application area of game-based learning, there is proof of concept for the concepts' appropriateness (Jantke, 2012).

Some of the human user's activities are memetic in spirit (see section 7) and may be used to determine significance. To sum up this section, there is a tool suite, but the present authors want to abandon it or, better to say, want to advance it *toward an intelligent assistant system*. The key to system assistance is the computer's ability to learn about its current human user.

5. UNIQUE SELLING PROPOSITION

Based on the three preceding preparatory sections, the present brief section is dedicated to a survey of the present paper's contribution to science and technology. In a business context, one might call this the paper's USP.

The research and development is presented in the sequel. The sections 6, 7, and 8 expand on (Arnold et al., 2017), whereas section 9 presents the authors' most recent and most formal result. This result about the reach of *dynamic identification by enumeration* based on *dynamically generated spaces of hypotheses* characterizes the quality of the authors' overall approach.

As discussed in the sections above, the authors' aim is to contribute to a paradigmatic shift and to a practical transformation from software tools to digital assistant systems. There is a quite large number of related research questions. Four of them shall be discussed in some more detail. When discussing the following research questions, it may be helpful to recall the authors' tool ADiDAVE sketched in the preceding section 4 and pondering ways of this tool's transformation.

Question 1, being more conceptual in spirit: Human-technology interactions may be diverse. How to determine significance of (inter-)actions in such a way that observed sequences of significant activities are likely to reveal a human user's intentions, goals, desires, and the like?

Question 2, being more algorithmic in spirit: Under the assumption of an appropriate concept of significance, how to process a growing sequence of observations about significant activities toward an understanding of the human user?

Question 3, being more application oriented: How to dovetail a human user's creativity and a digital system's syntactic learning power toward a high probability of unforeseeable insights? In particular, how to do this in applications of data analysis, visualization, and exploration?

For investigating the forth completely new question, it may be helpful to relate the authors' present approach to a certain preceding project on player modeling.

GORGE is the name of a digital game primarily developed as a research tool by Jantke (2010). A subsequent qualitative study has revealed the game's appropriateness to game-based learning about Artificial Intelligence (Jantke et al., 2010). The intuitiveness of the game's interface and of the rules of play have encouraged Jantke, Schmidt & Schnappauf (2016) to take the game as an experimental basis for a novel approach to user modeling. In the particular setting of playing GORGE, this means player modeling or learner modeling, resp. The underlying key approach is, so to speak, bio-inspired. The authors adopted and adapted so-called *theories of mind* (see, e.g., (Carruthers and Smith, 1996) and (Goldman, 2012)) for describing a human player's intentions.

Schmidt (2014) has developed and implemented this novel way of modeling a human user. The digital system's key component is an inductive learning algorithm (Jain et al., 1999) which is implemented in Prolog (Clocksin & Mellish, 1981). Stripped to its essentials, this algorithm performs *identification by enumeration* (Gold, 1967) on an enumeration of logical formulas generated a priory in dependence on the underlying application system. There is a technical report demonstrating in much detail how the learning proceeds and the way it works in practice (Jantke, 2016). The technicalities mentioned in this paragraph will be detailed in section 9.

However successful, the authors found the approach too restrictive when dealing data analysis, visualization, and exploration. In response, the case study of (Arnold et al., 2017) introduces *dynamically generated spaces of hypotheses*, i.e. enumerations of logical theories.

Question 4, being more focused on the richness of innovation: Is really learning about the system's user based on dynamically generated spaces of hypotheses more expressive than conventional modeling? If so, how much more can be learned with the authors' novel approach?

6. THEORY OF MIND MODELING AND INDUCTION

By the end of the preceding section 5, logical formulas and logical theories come into play. This results from the authors' original approach to invoke theories of mind for user modeling (Jantke et al., 2016).

User modeling is a central field of research, development, and application, in general, with much emphasis on hypermedia systems and technology enhanced learning, in particular (Brusilowsky & Millán, 2007).

The interest in the area is enormous, due to its relevance to a digital system's adaptivity and, in this way, to its intelligent appearance (see (Houben et al., 2009), (De Bra et al., 2010), (Konstan, 2011), (Masthoff et al., 2012), (Carberry et al., 2013), (Dimitrova et al., 2014), (Ricci et al., 2015), (Vassileva et al., 2016), (Bielikova, 2017) and the references therein).

Roughly speaking, conventional approaches to human user modeling determine a more or less high-dimensional space in which human users–be they users, in general, or learners or players, or data analysts, in particular–are modeled by assigning values to them for every coordinate of the space. Conceptually, this approach dates far back to Carl Gustav Jung's work (Jung, 1921) almost a century ago.

So far, research in animal behavior has not attracted much attention from the user modeling community. There is much evidence that certain animals reflect about intentions and behaviors of other animals (Carruthers & Smith, 1996). Birds of the species Aphelocoma californica are food-caching. They do not only cache food, but also colorful objects such as plastic toys. In case such a bird, let's name it A, is caching food or other treasures and if it is watched by another bird of its species, we name it B, then A returns shortly after to unearth the treasures cached before (Emery et al, 2004) (Emery and Clayton, 2009). The interpretation is, loosely speaking, that bird A thinks about the possibly malicious thoughts of bird B.

The present authors' novel approach (see Jantke et al., 2016), for first applications) consists in the design and implementation–including all the necessary conceptualization and basic investigation–of a computer program A being able to understand the intentions of its user B, notwithstanding that these intentions are malicious or not.

To represent on a computer a human user's intentions is called *theory of mind modeling* and the construction of such a human user's profile according to this user modeling concept is called *theory of mind induction*.

As sketched in (Arnold et al., 2017), section 4, and discussed in much detail in the report (Jantke, 2016), the authors' game GORGE serves as proof of concept. A Prolog program (Clocksin & Mellish, 1981) is able to learn a human player's intentions, preferences, altruism, vengefulness, and the like. Whatever the human player does is seen as a significant action. Such a computer program in the background is learning from significant actions and is able "to understand" a player's intentions (Jantke et al., 2016). Technically, the computer program generates a set of logical formulas explaining the system's observations of human behavior. Conceptually, such a set is a logical theory–a theory of mind.

The key reasoning mechanism (see section 9 for details) is *identification by enumeration* as introduced by Gold (1967), widely used in learning theory (Jain et al., 1999), and discussed in depth in work ranging from (Wiehagen, 1991) to (Kötzing, 2014). The mechanism assumes a computable enumeration. When observations are made, the system searches for the first object in the enumeration sound with all the observation data available. In this way, theory induction is implemented.

7. MEME MEDIA CONCEPTS AND TECHNOLOGIES

As said above, when playing GORGE, every player's (inter-)action establishes a significant observation. The theory induction computer program gets the observations fed in and returns logical theories of mind. In DAVE scenarios (Jantke & Fujima, 2016), in contrast, there exist by far too many opportunities of interaction. To overcome this difficulty, Fujima et al. (2017) introduce an approach to separate the wheat from the chaff based on meme media technologies.

Richard Dawkins [9] introduced the term *Meme* to denote units of non-biological evolution, a perspective that allows for the interpretation, the understanding and even the forecasting of phenomena in varying fields such as fashion, architecture and technology (Blackmore, 1999). Yuzuru Tanaka took up the challenge to carry over Dawkins' ideas toward the development of software components that may be seen as encapsulated knowledge units and that may be subject to a kind of evolution through replication, mutation, and cross-over including criteria of fitness. Tanaka's efforts resulted in a wide spectrum of implementations as reported in (Tanaka & Imataki, 1989), (Tanaka & Sugibuchi, 2001), (Okada, 2005), (Ito et al., 2006), (Kuwahara & Tanaka, 2010), and (Fujima, 2013), to mention a few. All of them are obeying to the standards summarized by Tanaka in (Tanaka, 2003). For an overview, see also (Arnold et al., 3013).

The tool introduced and illustrated in section 4 is implemented in a particular meme media dialect which goes slightly beyond the original approach by Tanaka (2003). The intelligent assistant system ADiDAVE has a meme media architecture similar to the predecessor tool (see figure 2). The difference to the tool remains under the hood, so to speak.

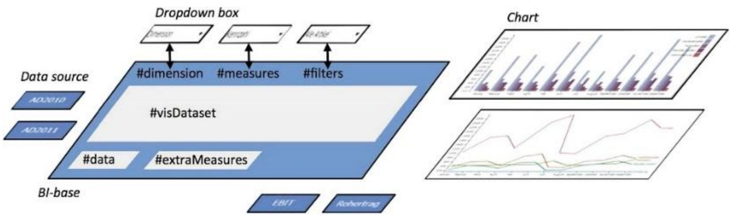

Figure 2. Compound Object Architecture of the Interface of ADiDAVE, Version 0.1

Seen from the perspective of knowledge evolution[1], human users together with the digital assistant system form a biotope in which knowledge may evolve.

So-called memetic steps of knowledge evolution, especially replication, mutation and cross-over, appear when humans manipulate meme media objects. The technology supports automatic connections of objects by drag & drop (including direct execution (Fujima & Jantke, 2012)) and peeling-off, i.e. extracting components of a compound object, perhaps for later use, perhaps for inspection and/or comparison. Numerous illustrations are on display in section 8 below.

The assistant system's findings are encapsulated in meme media objects and offered to the human user on the screen.

If the user considers the objects–more precisely: their content or semantics, resp.– valuable, she may peel off the objects of interest to use them in whatever way. By way of illustration, users may combine system-generated objects with each other or with already existing objects. Alternatively, one may put objects aside to use them later in a possibly unforeseen way.

Compound objects are representing novel insights which emerged from the human-computer co-operation. This comprehensive knowledge media approach is underlying the work and results reported subsequently.

8. TRANSFORMATION OF A TOOL TO AN ASSISTANT

The sections 8 and 9 form the main part of the present publication. In this section, the authors describe the way from their original DAVE tool to the novel assistant system ADiDAVE currently available in version 0.1. The aim is to provide answers to the first three research questions. The subsequent section is intended to answer the fourth research question by means of a rigorous formal modeling and a related stringent formal reasoning.

To understand the transformation as a whole, to understand the advantage of the assistant system over the tool, and to understand the practical advantage of co-operating with the assistant, it is not sufficient to know the details of the internal algorithmics of natural language processing and of logical reasoning. The latter will be investigated formally in section 9. To address a wide audience, the authors decided to suppress the space-consuming details. Instead, they will exemplify and illustrate the results and focus (a) on a few essential notions and notations and (b) on three characteristic solutions announced at the outset of the paper.

For every assistant system, there is a possibly large set of (inter-)actions that may be performed by one or the other agent including the system itself. Let us denote the set of all actions by A, where $A^* \subset A$ is the subset of those actions that may be performed by the human being observed. A^+ denotes the set of all non-empty finite strings over A.

The study of so-called DAVE scenarios is of a particular interest (Jantke & Fujima, 2016). Let us denote all the sequences of actions that may be performed subsequently from the very beginning to any time point by $\Pi \subseteq A^+$, a notation which goes back to digital games research as in (Jantke et al., 2016), where the letter Π is intended to resemble the word *play*.

When a user and an assistant system interact with each other, the system is observing the human behavior with the goal to draw conclusions.

An action of game play $\alpha \in A^*$ is called *significant*, if it bears the potential to learn from. When playing GORGE, a player action is significant exactly if the user had some choice. By way of illustration, the technical report (Jantke, 2016) summarizes a fully documented game play of about 45 Minutes duration. The player has acted 44 times. 35 of his actions have been significant. In processing these observations, the digital assistant has been able to learn what the player had in mind. Assume that $\alpha_1\ \alpha_2 \ldots\ \alpha_{n-1}\ \alpha_n \in \Pi$ is some game play that took place. The action α_n is significant, if $\alpha_n \in A^*$ and there exists some alternative action β_n such that $\alpha_1\ \alpha_2 \ldots\ \alpha_{n-1}\ \beta_n \in \Pi$ holds as well. Losely speaking, the player had the choice between α_n and β_n, at least.

Let us assume that $\pi \in \Pi$ is some human-assistant interaction in which exactly r significant human actions $\sigma_1, \sigma_2, \ldots, \sigma_{r-1}, \sigma_r \in A^*$ occur. In this case, π may be written as a finite sequence $\pi = \pi_1\sigma_1\pi_2\sigma_2 \ldots \pi_r\sigma_r\pi_{r+1} \in \Pi$.

Every pair $\omega_\rho = (\pi_\rho,\sigma_\rho)$ with $1 \leq \rho \leq r$ is called an *observation*. An intelligent assistant system learns user profiles from sequences of observations ω_1 , ω_2 , ω_3 , ω_4 , ω_5 , ω_6 , …

inductively (Jain et al., 1999). When processing a sequence $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \ldots$, the assistant hypothesizes logical theories $T_1, T_2, T_3, T_4, T_5, T_6, \ldots$ such the every theory of mind T_n is sound with the observations $\omega_1, \ldots, \omega_n$ made so far. Note that there is no need to update any theory T_n as long as it is sound with subsequently made observations $\omega_{n+1}, \omega_{n+2}, \omega_{n+3}, \ldots$

In the digital game case study of (Jantke, 2016), there are 35 significant observations resulting in 10 subsequently generated theories of mind $T_0, T_1, T_2, \ldots, T_9$. Indeed, the final theory of mind T_9 correctly describes what the human user had in mind.

8.1 The Significance Problem

The *significance problem* is the selected issue (i) of conceptualization announced in section 2. In harsh contrast to playing games like GORGE, in DAVE interaction scenarios, for literally every human action there does exist an alternative one. Hence, it does not make sense to carry over the significance concept from before. We are facing the problem to find a more appropriate concept of significance for DAVE.

Following (Tanaka, 2003), the concept should refer to human actions that are memetic in spirit (section 7).

What are the actions by means of which the user delivers the most explicit contributions to the interactive process of knowledge discovery? According to the currently available meme media implementation of ADiDAVE, version 0.1, these actions are annotations ultimately attached to a visualization object. Subsequences of interactions $\pi \in \Pi$ are of particular interest, in case the following human user activities take place,

i. mandatory: *first*, opening an annotation object
ii. mandatory: writing some text
iii. optionally: formatting the object frame
iv. optionally: coloring the annotation object
v. mandatory: connecting to a visualization object
vi. mandatory: *finally*, saving the compound object

where writing, formatting, scaling, and coloring may be interchanged and may be repeated several times. Connecting the annotation object to the visualization object may occur at any time between opening and saving which puts an end to the current annotation.

This defines a pattern (see the spectrum from (Alexander, 1979) to (Angluin, 1980)) that may be represented by some regular expression which is dropped here due to its length. Alternatively and equivalently, one may represent the pattern by some non-deterministic finite state acceptor able to accept exactly all its instances. The acceptor is on display in figure 3, where the roman numbers in parentheses represent classes of actions according to the above list.

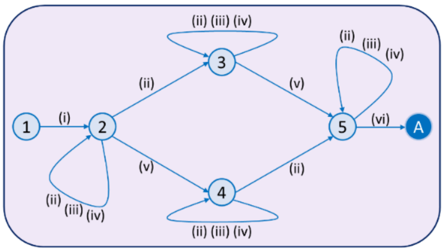

Figure 3. Non-Deterministic Finite State Acceptor for Instances of the Pattern of Significance

Because the pattern is regular, its instances form a regular language. The occurrence of instances in a string can be detected automatically. Every possibly occurring interaction sequence π may be represented as $\pi = \pi_1\sigma_1\pi_2\sigma_2 \ldots \pi_r\sigma_r\pi_{r+1} \in \Pi$, where all $\sigma_\rho \in$ A* are accepted by the finite state machine on display in the figure 3 above, i.e. there is a substring ending with σ_ρ that leads to state A of the acceptor. Learning from the observations $\omega_\rho = (\pi_\rho, \sigma_\rho)$ is key to the assistant system's contribution to the emergence of knowledge. The significance lies in the annotation provided by the human user, a memetic aspect to be exploited in the sequel.

8.2 Generating Spaces of Hypotheses Conventionally

Theory induction is key to an assistant system's internal intelligence. The successful learning of a player's intentions sketched at the end of the introductory part to section 8 relies on a single computable enumeration of logical formulas. For the sake of sufficient precision, a few notions and notations are introduced. We assume a computer program *g* that generates logical theories step by step; in the digital game application reported in (Jantke et al., 2016) and (Jantke, 2016), this is a Python program generating Horn clauses for further processing by means of Prolog. Theories enumerated are briefly denoted by *g(0), g(1), g(2), g(4), …* The learning procedure is identification by enumeration. To describe it concisely, we use the minimum operator μ from recursion theory (Rogers, 1967) and the consequence operator $\models$ which in our applications usually relies on modal logics (Blackburn et al., 2001). In formal terms, learning by means of identification by enumeration on an enumeration *g* is defined as

$$L_g^{IdbyEn}(\Omega_\rho) \ = \ g(\ \mu n[\ (\ g(n) \cup \Omega_\rho \ \not\models \square\ )\ ]\ ),$$

where $\Omega_\rho = \omega_1 , \ldots , \omega_\rho$ abbreviates a finite set of observations. We have chosen a notation which explicitly points to the refutation potentials of logic programming. Informally speaking, this learning algorithm searches for the first index–this is the meaning of $\mu n[\ldots]$ –such that the *n*-th theory enumerated by *g* does not allow for the derivation of a contradiction–this is the meaning of $\ldots \not\models \square$. It returns the corresponding hypothetical theory of mind *g(n)*.

The *generation of spaces of hypotheses* is the selected problem (ii) of algorithmics announced in section 2. Throughout the authors' attempts to transform a tool into an assistant for data analysis, visualization, and exploration, the enumeration of potential theories of mind turned out to be particularly hard. What may be appropriate in a context of interaction depends on the human user's unforeseeable behavior. (Arnold et al., 2017) contains a detailed discussion.

Recall that an observation is considered significant, if the current sequence of interactions ends with an instance of the pattern of significance, a phenomenon detectable by the finite state acceptor on display in figure 3. It is a heuristic assumption that the human user's annotations may carry some information of relevance. Based on this assumption, the digital assistant system generates for every significant observation a particular enumeration of potential hypotheses in dependence on the human user's utterance. (Arnold et al., 2017) contains illustrative cases.

In contrast to conventional approaches to modeling human users, to theory induction, and to hypothesis generation by means of identification by enumeration on a single enumeration g, the present authors' novel approach consists in dynamically generating varying enumerations in response to the human user's behavior. Building hypothetical theories of mind by means of identification by enumeration on dynamically generated spaces of hypotheses is briefly called *dynamic identification by enumeration*.

The assistant getting significant inputs $\omega_\rho = (\pi_\rho,\sigma_\rho)$ incl. annotations needs "to think about" appropriate responses. For this purpose, *indexed families of logical formulas* (Jantke et al., 2016) are generated dynamically. In doing so, the assistant system is continuously broadening its horizon and expanding the world–concepts, relations, implications, and the like–it is able to reason about. The human user triggers the evolution of the assistant's language for reasoning.

A set $\Psi = \{\ \psi_n\ \}_{n=1,2,\dots}$ of logical formulas is called an indexed family, if it is effectively enumerable (Rogers, 1967), possibly finite, and for any two different formulas ψ_i and ψ_j in the enumeration with $i < j$ it never holds that ψ_j logically implies ψ_i (for first applications, see (Jantke et al., 2016) and (Arnold et al. 2017)).

8.3 Performing Identification by Enumeration Dynamically

This subsection deals with the generalization of conventional learning by means of identification by enumeration toward dynamic identification by enumeration. The subsequent subsection 8.4 deals with an application case adopted from (Arnold et al., 2017) and the final section 9 aims at a proof of the novel technology's superiority.

The standard learning algorithm L_g^{IdbyEn} as introduced above searches a unique enumeration. Even recent theoretical research on identification by enumeration such as (Kötzing, 2014) has always uniquely fixed enumerations in focus.

Beyond the expressive limits of earlier approaches, the authors investigate generators of spaces of hypotheses. A generator is an operator γ that constructs enumerations in dependence on significant observations. For every sequence of observations up to some point $\omega_\rho = (\pi_\rho,\sigma_\rho)$, $\gamma(\Omega_\rho)$ is a certain enumeration. Its construction takes the annotation residing in σ_ρ into account.

Arnold et al. (2017) discuss a case where the annotation "(2015.Q4.U < 2015.Q3.U)" triggers the generation of a sequence $F = \{\ f_n\ \}_{n=1,2,\dots}$ of factors upward converging to *1*. Based on *F* as an index set, one gets the indexed family $\Psi = \{\ \psi_f\ \}_{f\in F}$, where every individual formula ψ_f is *(2015.Q4.U < f∗2015.Q3.U)*. On given data, identification by enumeration finds a usually stronger statement than the user's utterance. More precisely, it finds the strongest one valid in the given database. In the case study of (Arnold et al., 2017), the assistant system finds *(2015.Q4.U < 0,75∗2015.Q3.U)*.

Interpreted in everyday language, the user states that there is a decline of turnover from the third quarter of 2015 to the fourth quarter of the year. The system, in response, reports to the user that in 2015 the turnover of the last quarter is even more than 25% smaller than in the quarter before. This is the strongest statement valid over the data given. Stronger invalid statements have been refuted in the process of identification by enumeration on $\Psi = \{\ \psi_f\ \}_{f\in F}$.

Intuitively, the assistant system "did not know" that the user wants to investigate issues of business development and in particular of the quarterly turnover. The human user's utterance triggers the system's extension of its language and of its range of reasoning.

Now, the stage is set for the announced investigation of a problem of algorithmics indicated as problem (ii) in section 2. We introduce dynamic identification by enumeration more formally. As said informally in subsection 8.2, γ shall be a generator of indexed families of formulas. Generators of spaces of hypotheses need to obey the following three requirements.

I. **Operational Appropriateness:** For every sequence of significant observations Ω_ρ, the space of hypotheses $\gamma(\Omega_\rho)$ is an indexed family of logical formulas that contains at least one formula sound with these observations.

II. **Conversational Appropriateness:** For every sequence of significant observations ω_1, ω_2, ω_3, …, there exists a time point τ such that $\gamma(\Omega_\rho)$ does not change after τ.

III. **Semantic Appropriateness:** After the time point τ, the generated indexed family does contain at least one hypothesis sound with all observations in the sequence.

Dynamic identification by enumeration is a learning process which, in response to a sequence of observations Ω_ρ, (i) generates an individual space of hypotheses $\gamma(\Omega_\rho)$ on which (ii) it performs conventional identification by enumeration, i.e. $L_{\gamma}^{IdbyEn}(\Omega_\rho) = L_{\gamma(\Omega\rho)}^{IdbyEn}(\Omega_\rho)$.

When a space of hypotheses is changed, it brings in new terminology and related knowledge. The requirement of *conversational appropriateness* prevents the user from an unbounded Babylonian confusion.

8.4 Intelligent System Assistance for Business Data Analysis

This subsection sketches the business application which is the selected problem (iii) announced in the introductory section 2. For more details, readers are directed to (Arnold et al. 2017).

All figures in the present subsection are screenshots from an interaction session of the fourth author with the assistant system ADiDAVE, version 0.1, on December 13, 2016. The DAVE session will be told like a story that really began with the inspection of business data from 2015 as on display in figure 4 on the left.

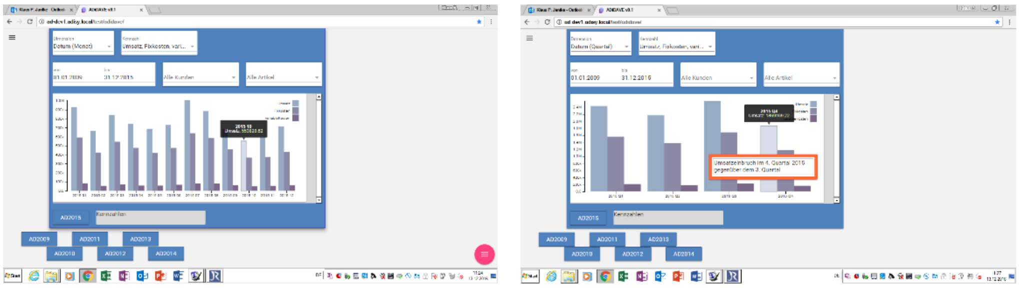

Figure 4. Business Data from 2015 by the Month (left) and by the Quarter (right)

In figure 4, the three bars represent, from left to right, turnover (in German: Umsatz), fixed costs (Fixkosten), and variable costs (variable Kosten). The user’s vague impression induced by the visualization on the left is that business data are declining toward the end of the year 2015. This does not become thoroughly clear, as the data of November and December are up again in comparison to October (see the tooltip for inspection). On the right are the same data as on the left, but quarterly. The slum in quarter 4/2015 compared to the quarter before becomes obvious.

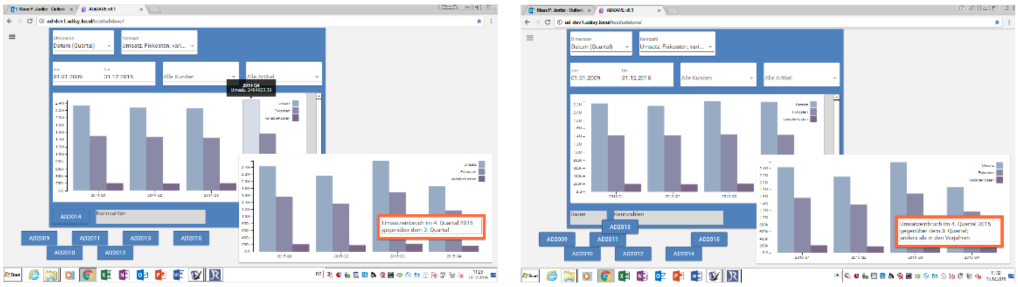

Figure 5. Comparison of Data from 2014 and 2013 with Short (left) and Extended Annotation (right)

The user attaches an annotation (framed box) saying “Umsatzeinbruch im 4. Quartal 2015 gegenüber dem 3. Quartal”. The approximate translation is “Slump of the turnover in the 4. quarter of 2015 compared to the 3. quarter”. Natural language processing transforms this annotation into the formula (2015.Q4.U < 2015.Q3.U) from which there is internally generated the more expressive formula (2015.Q4.U < 0.75·2015.Q3.U).

As can be seen in on the left, the user has peeled off the annotated chart–in figure 4 on the right it is still sitting in the blue frame–to keep it separately for future use. Meanwhile, the user has been inspecting the business data of 2014 and turned now to 2013. The reader can easily see that the data in 2013 look different from the data in 2015 which are still sitting on the screen.

That the financial data from 2014 and from 2013 show a remarkable difference to the data from 2015 motivates the user to extend his annotation from before. The third line says “anders als in den Vorjahren” (German) which means approximately “different from the prior years”.

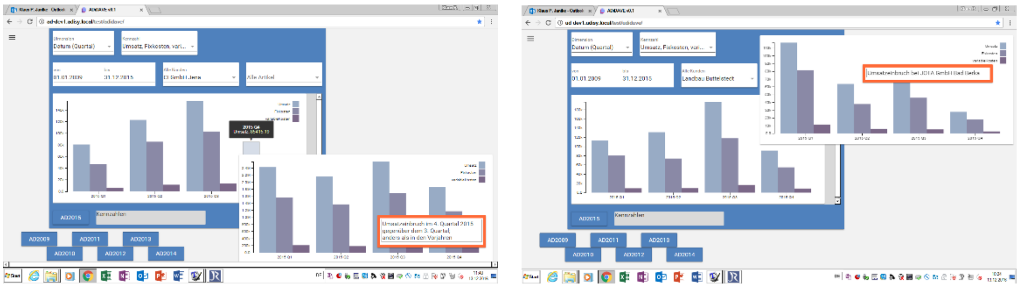

Figure 6. Drilling Down to Check one (left) and Possibly More Critical Customers (right)

Figure 6, left, shows the dataset of 2015 reloaded (lower left corner of the blue window). The financial data of the business with one particular customer–the CI GmbH Jena–are selected and visualized. A further inspection, right, seems to suggest some more customers blamable for the problems in 2015; see the decline of business data from the third to the fourth quarter.

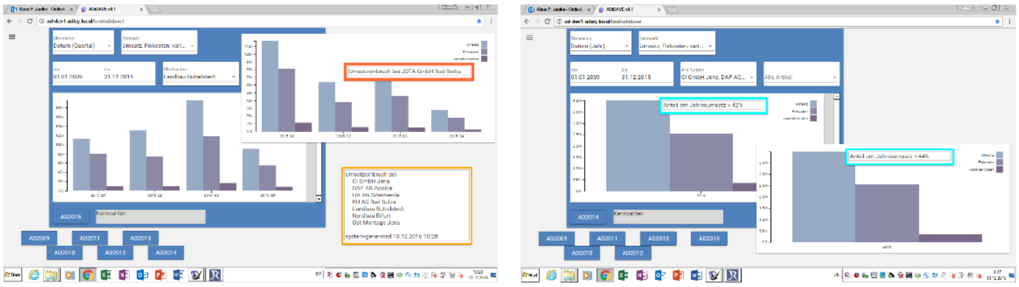

Figure 7. Response about Critical Customers (left) and User Interest in their Share in Turnover (right)

In response to the user's interest in certain critical customers' share in turnover (above right), the language of interaction is extended by new terms such as share in turnover and by new types of formulas such as expressing percentage of one figure in another.

Let us have a closer look at the present business case study from the viewpoint of dynamic identification by enumeration. The first annotation (figure 4, on the right, figure 5 on the left) triggers the generation of an enumeration of inequalities comparing values of quarterly turnover. The extended annotation triggers the generation of formulas in which values from different years are compared to each other. Third, the inspection of single customers triggers the generation of formulas in which individual customers occur. Next, the user's annotations (figure 7, right) about costumers and their share in turnover triggers the generation of formulas about certain customers' share in turnover. In the very end–see (Arnold et al., 2017), discussion on page 35, especially figure 14)–the user and the system arrive co-operatively at a valuable novel insight: *The critical customers are not at all critical, but performed extremely well in the 3rd Quarter of the year 2015 having a share in turnover of more than 51%.* This is a case of *serendipity*.

The crux is that during interactions as illustrated, *the language of discourse evolves* in an unforeseeable way and *knowledge can emerge* that has been completely out of scope initially.

9. THE MODELING AND INDUCTIVE LEARNING POWER OF DYNAMIC IDENTIFICATION BY ENUMERATION

Classification and appraisement of a novel algorithmic idea that is implemented in a complex environment using different programming paradigms and languages is an involved endeavor.

The authors' analysis departs from the concept of interaction scenarios as introduced by Jantke and Fujima (2015). Human-computer interaction unfolds as sequence (section 8) in which certain substrings are significant (section 8.1). The assistant system is forced to learn from those sequences that inevitably represent the user's aims, and the like only incompletely. This scenario perfectly resembles the overall ideas of *inductive inference* (Jain et al., 1999).

9.1 Notions and Notations

The fundamentals of recursive function theory are adopted from (Rogers, 1967). Only the necessary essentials are explicitly introduced. $\mathbb{N}$ denotes the set of natural numbers. $\mathbb{P}$ is the set of all partial recursive functions over $\mathbb{N}$ and $\mathbb{R}$ is the subset of total recursive functions. In case the arity of functions is restricted to n, the corresponding sets of functions are $\mathbb{P}^n$ and $\mathbb{R}^n$.

For any $h \in \mathbb{P}^1$, $h(n)\downarrow$ indicates that the value is defined. Every function $f \in \mathbb{R}^1$ may be identified with its graph $f(0), f(1), f(2), f(3), f(4), f(5), \dots$ In recursion-theoretic inductive inference, the key question is how to learn functions from only finite subsets of their corresponding graphs. Learning means to find a function's description within any acceptable numbering φ. Slightly generalized, one may assume any sequence $\chi = x_0, x_1, x_2, \dots$ of natural numbers containing every element of $\mathbb{N}$ at least once. X denotes the set of all those sequences. For every total recursive function $f \in \mathbb{R}^1$, for every sequence $\chi \in X$ and for every index $n \in \mathbb{N}$, the term $f_\chi[n]$ denotes the values of f on the initial segment of χ until point x_n, i.e. $f_\chi[n] = f(x_0), \dots, f(x_n)$. Systems that learn build hypotheses based on pieces of information such as $f_\chi[n]$. If arguments occur in the standard ordering $0,1,2,3,\dots$, the notation is simplified to $f[n]$. The goal is to find an index of f within the acceptable numbering φ. Given a class of functions $C \subseteq \mathbb{R}^1$, the question is whether or not there exists any uniform learning algorithms that works for every function in C.

For $g \in \mathbb{R}^1$, C_g abbreviates $\{ \varphi_{g(0)}, \varphi_{g(1)}, \varphi_{g(2)}, \varphi_{g(3)}, \dots \}$. For $\varphi_i = g$, one may write C_i for C_g. A class $C \subseteq \mathbb{R}^1$ is effectively enumerable, if and only if there exists a total recursive enumeration $g \in \mathbb{R}^1$ with $C \subseteq C_g \subseteq \mathbb{R}^1$. *NUM* denotes the family of all effectively enumerable classes of total recursive functions. A class C in *NUM* is said to be exactly enumerable, if it holds $C = C_g \subseteq \mathbb{R}^1$. *NUM!* denotes the corresponding family of function classes. Trivially, it holds $NUM! \subset NUM$. On all these function classes, identification by enumeration is a correct learning principle. Formally, one may express it in the form $L_g^{IdbyEn}(f[k]) = g(\,\mu n[\,(\,\varphi_{g(n)}[k] = f[k]\,)\,]\,)$ (see the analogy to subsection 8.2); the formalism for any $\chi \in X$ is similar using $f_\chi[k]$ instead of $f[k]$. By way of illustration, for $h \in \mathbb{R}^1$, the class of all its finite variations $Var(h)$ belongs to *NUM!*.

EX denotes the family of all classes $C \subseteq \mathbb{R}^1$ such that there exists a learning device $L \in \mathbb{P}^1$ meeting for every function $f \in C$ the requirements (i) that $L(f[n])$ is defined for all $n \in \mathbb{N}$, (ii) that there is some time point m such that the hypotheses stabilize, i.e. $L(f[m]) = L(f[n])$ for any $m \leq n$, and (iii) that the limiting hypothesis $L(f[m])$ is correct for f, i.e. $\varphi_{L(f[m])} = f$. In the general case that information may be presented in an arbitrary ordering $\chi \in X$, $L(f_\chi[n])$ and $L(f_\chi[m])$, resp., occur in the definition above and the family of function classes is denoted by EX^{arb}.

9.2 Basic Results

For classes C in *NUM*, L_g^{IdbyEn} is such a learning device in $\mathbb{P}^1$ according to both *EX* and EX^{arb}.
For $f_\chi[n]$, searching a class C terminates, if $f_\chi[n]$ occurs in a function of C denoted by $f_\chi[n] \propto C$.

In the field of inductive inference, the relations $NUM! \subset NUM \subset EX = EX^{arb}$ are folklore. The proper inclusion of *NUM* in *EX* indicates that there may be other learning principles than identification by enumeration. Wiehagen (1991) has conjectured that a generalized principle of identification by enumeration might work for every class in *EX*. Kötzing (2014) has provided solutions to Wiehagen's thesis. However, both Wiehagen and Kötzing consider just one fixed enumeration and add several technicalities such as complexity and time bounds, and the like. This is different from the present authors' formal approach introduced below and called *NUM**.

Research on learning from incomplete information leads to hundreds of interesting results (Jain et al., 1999) of which only a few are cited to relate the authors' present achievements.

Concepts are transcendent, as Lakoff (1987) put it. Relevant concepts such as learning principles occur in a wide spectrum of domains. This applies to identification by enumeration as well. According to (Lakoff, 1987), research in the theory of recursion-theoretic inductive inference may be seen as *benchmarking approaches to computerized learning*.

The approach to *EX*-type learning may be refined by varying postulates of naturalness (Jantke and Beick, 1981). One seemingly natural requirement is *consistency*. A learning device $L \in \mathbb{P}^1$ is said to learn consistently, if every generated hypothesis $L(f\chi[n])$ reflects the data it is built upon, i.e. $\varphi_{L(f\chi[n])}[n] = f[n]$. The family of function classes being consistently learnable is denoted by $CONS^{arb}$ and, if information is only considered in the standard ordering, by *CONS*. Related folklore results are $NUM \subset CONS^{arb} \subset CONS \subset EX$ saying that (a) the principle of identification by enumeration is a consistent one, but (b) consistent learning has limitations.

In case a learning device can be total recursive, i.e. $L \in \mathbb{R}^1$ is, this is indicated by a prefix *R* leading to notations such as $R\text{-}CONS^{arb}$, *R-CONS*, and *R-EX*. It holds $R\text{-}CONS^{arb} \subset R\text{-}CONS$ and $R\text{-}EX = EX$. Further results can be seen in subsection 9.4 below. Iteratively learning means that instead of the whole sequence of information $f\chi[n]$ or $f[n]$, only the recent observation $f(x_n)$ or $f(n)$, resp., is processed. This leads to the two learning types IT^{arb} and *IT*, respectively. When postulates of naturalness are combined, the corresponding notations are combined accordingly.

From the approaches investigated in (Jantke and Beick, 1981), the present authors select only a few. A final concept adopted is denoted by *TOTAL*. A class *C* of total recursive function is learnable in the sense of *TOTAL*, exactly if there is a learning device *L* obeying the defining requirements of *EX* such that all its hypotheses generated belong to $\mathbb{R}^1$, i.e. are total recursive.

9.3 Dynamic Identification by Enumeration

For constructing a benchmark inductive inference concept formally representing the idea of dynamic identification by enumeration, it is necessary to reflect (I.) operational appropriateness, (II.) conversational appropriateness, and (III.) semantic appropriateness recursion-theoretically.

A class of functions $C \subseteq \mathbb{R}^1$ belongs to NUM^*, if and only if there exists a generator function $\gamma \in \mathbb{P}^1$ such that for all $f \in C$ and for all $\chi \in X$ it holds (I.) for all $n \in \mathbb{N}$ that $\gamma(f_\chi[n])\downarrow$, $\varphi_{\gamma(f\chi[n])} \in \mathbb{R}^1$, $C_{\gamma(f\chi[n])} \subseteq \mathbb{R}^1$, and $f_\chi[n] \propto C_{\gamma(f\chi[n])}$, (II.) there is a critical point $m \in \mathbb{N}$ such that for all $n \in \mathbb{N}$ larger than m it holds $\gamma(f_\chi[m]) = \gamma(f_\chi[n])$, and (III.) $f \in C_{\gamma(f\chi[m])}$. The novel learning type NUM^* formalizes dynamic identification by enumeration in recursion-theoretic inductive inference.

9.4 Main Result

This subsection presents the most theoretical part of the authors' contribution demonstrating by means of formal methods the superiority of dynamic identification by enumeration over the conventional approach.

Proposition 1: $TOTAL \subseteq NUM^*$.

<u>*Proof*</u>: Assume $L \in \mathbb{P}^1$ to be a device learning all elements of some class *C* according to the definition of *TOTAL*. For $f \in C$, $\chi \in X$, and $n \in \mathbb{N}$, *L* returns an index $L(f_\chi[n])$ of a total recursive function. Given *f* and χ, for brevity, we denote it by h^n. From every $f_\chi[n]$ one may effectively and uniformly construct an index i^n of an enumeration of $Var(\varphi_h{}^n)$, i.e., $C_i^n \in$

NUM!. In this way, $\gamma(f_\chi[n]) = i^n$ meets all requirements and stabilizes on some enumeration which contains *f*.

Proposition 2: $NUM^* \subseteq TOTAL$.

<u>*Proof*</u>: Given a generator of enumerations γ, the task is to transform it into a learning device *L* according to *TOTAL*. For $f \in C$, $\chi \in X$, and $n \in \mathbb{N}$, $\gamma(f_\chi[n])$ is an index (for brevity i^n) of a class C_i^n in *NUM* with $f_\chi[n] \propto C_i^n$. With $g = \varphi_i^n$ we define $L(f_\chi[n]) = L_g^{IdbyEn}$. This returns a total hypothesis. As γ stabilizes on some $\gamma(f_\chi[n])$, $L(f_\chi[n])$ stabilizes as well on a correct result. □

As a consequence of the above propositions, the new concept NUM^* coincides with *TOTAL*.

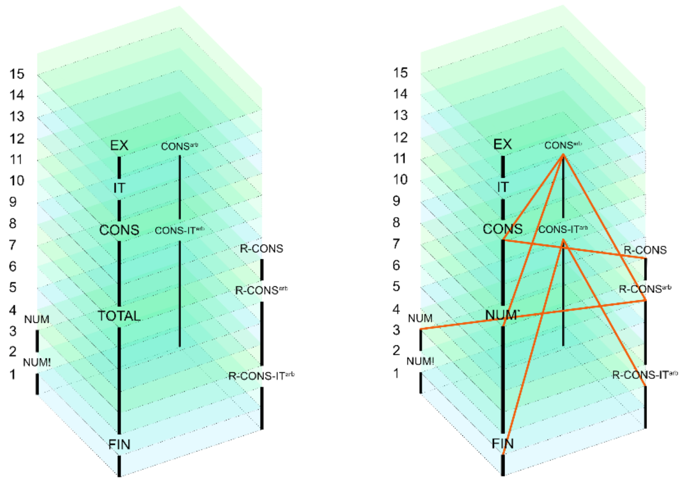

Figure 8. Hierarchies of Learning Concepts and Embedding of Dynamic Identification by Enumeration

In figure 8 above, there is an auxiliary enumeration of levels on the left which is intended to support the inspection of the diagrams. If two concepts are connected by a line, this means that the one on the lower level is properly contained in the other one on the higher level. These line connections are transitive. Concepts not connected in this way are incomparable to each other. The diagram on the right represents 132 relations between the 12 learning concepts on display. By way of illustration, it shows that *NUM* is properly contained in $R\text{-}CONS^{arb}$ and *R-CONS*, whereas NUM^* is not.

The authors' key result is that their original concept NUM^*–the analogue to their practical conceptualization–is considerably above *NUM* in the hierarchies demonstrating that the novel concept of dynamic identification by enumeration is superior to the preceding technology.

10. CONCLUSION

Most importantly, there is a systematic way to transform digital tools into assistant systems. This is the most general message of (Arnold et al., 2017) and still is a key contribution of the present paper.

In the field of data analysis, visualization and exploration (DAVE), this is particularly desirable, as the problems investigated are often wicked. In co-operating with a digital assistant system, the human users may arrive at unforeseeable results they have not been looking for. The assistant system's ability to learn is decisive. The way in which the digital system represents knowledge about the human user is theory of mind modeling. And the way in which the system learns is theory of mind induction. The key algorithmic concept underlying theory induction is identification by enumeration.

The authors have generalized conventional identification by enumeration toward a method named dynamic identification by enumeration. There is a rigorous formalization and a stringent mathematical proof demonstrating the superiority of the innovation over conventional ideas.

The induction of theories of mind takes place as search in effectively enumerable spaces of hypotheses. Formulas are adopted hypothetically, if they are not refutable. As illustrated, the system may generate its spaces of hypotheses in response to prior human utterances. In this way, users are unconsciously guiding the reasoning of the assistant system to unprecedented insights.

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