Finding a Hamiltonian path in an anisotropic graph of the complete factorial experiment

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Abstract: The article presents a solution of the problem of finding a Hamiltonian path in a fully accessible graph of a multifactorial experiment. The option of problem formulation when the quality of the path is only defined by a number of changes of the factor level is examined. A simple non-search method for the exact solution of the formulated problem is offered.

Key words: multifactorial experiment, fully accessible graph, Hamiltonian path, best-route building technique.

Introduction

The problem considered in the paper is related to W.R. Hamilton's well-known problem about a "round-the-world voyage" (1853). In

this article we inquire into a particular case of finding the shortest route of an orthogonal graph vertices traversal which corresponds to a multifactorial experiment – an open Hamiltonian cycle. This problem arises when examining systems using a complete factor experiment. The corresponding mathematical model is described as follows. Let the system effectiveness depend on factors $F_1, F_2, \dots, F_n$, each of them being able to possess the value at one of two levels (0 or 1). Herewith, the whole set of experiments is specified by a combination of 2^n vertices of a hypercube with numbers starting with zero to 2^n-1. If, for example, $n=3$ the number of vertices is $2^3=8$ and their numbers in decimal and binary representation form the matrix M.

The traversal of tree can be started from any of the vertices and the path appearing at that is an open Hamiltonian path. During the transition from one vertex to another it becomes necessary to change the level for one or several factors. For example, in the path M during the transition from the vertex with number 0 to the vertex with number 1, the factor F_1 level should be changed from 0 to 1, and when passing from Vertex 3 to Vertex 4 we should change the level of all three factors F_1, F_2, F_3. Each change is attended with costs (material or temporal). The task is to find the path that ensures the minimum costs.

Problem formulation

Consider first the simplest of the problems arising here when the costs related to the change of a factor level are equal for all factors and do not depend on the levels the change is made from and to. Then the quality of the route is only defined by the total number of transitions from one level to another for all factors. For example, the path realizing the traversal of vertices in accordance with their number order contains one transition for the factor F_3, three transitions for F_2 and seven transitions for F_1. The total number of transitions equals 11. The optimal index can theoretically be found using simple search. However, the total number of routes for vertices $N=2^n$ equals $R=2^n!$ and grows very rapidly with the increase in the number of factors. In particular, for $n=4$, $N=2^4=16$, $R=16!\approx 2{,}092\cdot 10^{13}$; for $n=6$, $N=2^6=64$, $R=64!=1{,}2\cdot 10^{89}$. Thus, a standard search is impossible for as many as four factors.

The most efficient one from the quasi-search methods of solving the problem – the branch and bound method – [1] is practically limited to a problem with four factors. The known attempts to solve this problem using other approximate methods have not been successful.

Results

Let the number of factors be equal to two and the corresponding number of the hypercube vertices equals $R=2^2=4$ with the coordinates (0,0), (0,1), (1,0), (1,1). Choose an initial vertex in some manner and then define the following vertices so that when passing from one vertex to another the level only for one of the factors would change. It can be easily done and even more than in one way. For example, introduce the paths

$$M_1=\begin{pmatrix}0&0\\0&1\\1&1\\1&0\end{pmatrix};\ M_2=\begin{pmatrix}0&1\\1&1\\1&0\\0&0\end{pmatrix}.$$

In all these cases during the transition from one vertex to another the level of only one factor changes and the total number of level changes equals three. The total number of such paths equals eight. Using any of them, it is easy to build the optimal path for three factors. Choose, for example, the path M_1. For the four rows of the matrix M_1 add four rows more which are mirror images to the existing four, and then add to them in some way another column consisting of zeros and ones but with only one level change.

Possible options:

$$M_{11}=\begin{pmatrix}\mathbf{0}&0&0\\\mathbf{0}&0&1\\\mathbf{0}&1&1\\\mathbf{0}&1&0\\\mathbf{1}&1&0\\\mathbf{1}&1&1\\\mathbf{1}&0&1\\\mathbf{1}&0&0\end{pmatrix};\ M_{12}=\begin{pmatrix}0&\mathbf{0}&0\\0&\mathbf{0}&1\\1&\mathbf{0}&1\\1&\mathbf{0}&0\\1&\mathbf{1}&0\\1&\mathbf{1}&1\\0&\mathbf{1}&1\\0&\mathbf{1}&0\end{pmatrix}.$$

These options correspond to the routes with the numbers of vertices

$$S_{M_{11}}=\{0;1;3;2;6;7;5;4\},\ S_{M_{13}}=\{0;1;5;4;6;7;3;2\}.$$

The added column is typed in bold. In all these options the transition for any pair of vertices is attended with the level change of only one factor and the total number of changes equals to the theoretical minimum which is $2^3-1=7$. The total number for three factors generated by one bifactorial path is six.

The further procedure repeats the one described above. To build the optimal four-factor path, it is sufficient to choose any optimal three-factor path, to add first to the corresponding matrix its mirror image and then to add, in any possible way, a column containing eight zeros and as many ones with a single transition. This additional column can be placed the first on the left, it can be placed between the first and the second column, between the second and the third column and, finally, it can be the last to the right. Besides, the disposition of zeros and ones in this column can vary. Thus, each three-factor route enables to build eight optimal four-factor paths with the minimal number of changes of the factor level equaling to the theoretical minimum $2^4-1=15$.

Let us complicate the task taking into account possible differences in costs associated with changing levels for each of the factors. According to this, introduce s_i – level change costs for i-factor, $i=1,2,\dots,n$. The routes, which are optimal and equal in the number of transitions, will be, herewith, different in total costs.

The order of traversal of vertices for the chosen route is defined by the disposition of rows, where zero-one distribution specifies the binary representation of the number of the corresponding vertices. The vector which defines the order of traversal of vertices for a given routing table is calculated as the scalar product of this matrix with the vector which is composed of numbers representing powers of two and sets the desired order of the columns. Let us illustrate this with the example corresponding to a three-factor problem.

Use M_{11} as the initial matrix. In this matrix the total number of the factor level changes is seven. Herewith, the number of changes of the factor level corresponding to the first column is unity; for the factor corresponding to the second column equals two and for the third factor the number of changes equals four. When making a route and considering differences in the cost, it is necessary to take into account the number of factor level changes and their corresponding cost values. If, for example, the cost ratio in a specific system at the change of the factor level complies with the inequality $S_2>S_3>S_1$, it is advisable to attribute the most costly factor F_2 to the first column, the factor F_3 to the second column and the factor F_1 to the third column with the least number of changes. Define the rational traversal M_{11}, using the correlation

$$\begin{pmatrix}0&0&0\\0&0&1\\0&1&1\\0&1&0\\1&1&0\\1&1&1\\1&0&1\\1&0&0\end{pmatrix}\times\begin{pmatrix}2^1\\2^2\\2^0\end{pmatrix}=\begin{pmatrix}0\\1\\5\\4\\6\\7\\3\\2\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&1\\1&0&1\\1&0&0\\1&1&0\\1&1&1\\0&1&1\\1&1&0\end{pmatrix}.$$

Thus, the rule of finding the required Hamiltonian path ensuring the minimum of costs consists of two points:

· For a given number of n factors we form a required matrix with a minimal total number of factor level changes equaling $2n-1$

· This matrix is multiplied scalarly by the vector composed of powers of two whose sequence order is defined by cost factor values.

Conclusion

This paper presents, thus, the technique of finding a path in the traversal of a set of vertices in the plan of a complete factor experiment for different cost options related to the change of factor level value during the transition from one vertex to another. An easy rule of formation of a plan that minimizes the total costs has been obtained. We have examined the possibility of applying this rule in the case when money and time costs are different during the passage of the route. To solve the problem in this case, the Pareto-set of plans is used.

References

1. Little J.D.C., Murty K.G., Sweeney D.W., and Karel C. An algorithm for the traveling salesman problem. Operations Research. v. 11 (1963), pp. 972-989.

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$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 2^1 \\ 2^2 \\ 2^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 5 \\ 4 \\ 6 \\ 7 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

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References

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The Conceptual Underpinnings Of Digital Transformation

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The Fourth industrial Revolution and the Knowledge Economy

Throughout history knowledge has always been at the centre of economic growth. It is innovation and the ability to create new knowledge, which can be embodied in products, processes and organisations that fuels development. The Concise Oxford Dictionary describes knowledge as "the sum of what is known" while the creation of knowledge itself is described as the process of adding value to previous knowledge through innovation and invention. Schumpeter's as far back as 1931 described "new combinations of knowledge" as the essence of innovation and entrepreneurship and is one of the earliest acknowledgements of the importance of knowledge in the economy.

Professor Klaus Schwab Founder of the World Economic Forum argues that mankind is poised on the brink of technological revolution which has been termed the fourth industrial revolution. It will alter the way we interact with each other across all aspects of our lives. While the first industrial revolution was heavily reliant on information, and on the application and development of knowledge, research and development characterised the second industrial revolution. Additionally the second also used electric power to create mass production, and the third used electronics and information technology to automate production. The Fourth Industrial Revolution fuses technologies and blurs the lines between the physical and the digital. Many labels have been used to define this fundamental shift including Castells concept of the network society and Tapscott's networked intelligence which are increasingly dependent on the Internet and emerging digital platforms.

The increasing reliance on knowledge or networked economy is according to Castell's (a) informational because the ability for organisations and countries to effectively compete is fundamentally dependent on their capacity to create, maintain, process, and apply information, (b) global because the production, consumption and circulation of goods and services are a global effort, and (c) networked because production and competition occurs in a "global network of interaction between business networks."

The emergence paradigm demands that organisations display a degree of flexibility not before seen or even required. It also requires a workforce skilled in the use of emerging technologies and systems of innovation that must be embedded in the DNA of organisations.

Digital Transformation and Digital Business Transformation

While there is no universal definition 'digital transformation' it is still a useful conceptual umbrella under which a variety of organisational functions and processes can be grouped. With respect to business digital transformation refers to the skills and acumen required to fully leverage the opportunities presented by rapidly emerging and digital technologies. MIT Sloan identifies three fundamental of digital transformation namely transforming the customer experience, transforming operational processes and transforming business models. It demands the embedding of digital technologies into all areas of a business. It is also to a large extent an issue of cultural change that must create an enabling environment in which innovation can thrive, which often means embracing risk and the possibility of failure to a greater degree than previously. Many commentators relate business digital transformation to improved performance and an improved customer experience, business models, leadership, and innovation. There are a number of reasons that a business may undergo digital transformation, but by far, the most likely reason is that they have to. It's a survival issue for many. Examples abound of companies who were once deemed to be invincible, succumbing to the lack of innovation and eventually meeting an untimely demise. Notable examples are Kodak and Blackberry. Authors Ismail, Malone and van Geest coined the term Exponential Organizations to describe companies that are structured in such a way that allows them to realize the full potential of the digitized business economy. Examples also exist of companies who have been able to embrace the digital world most notably Uber, AirBnB, RelayRides and TaskRabbit.

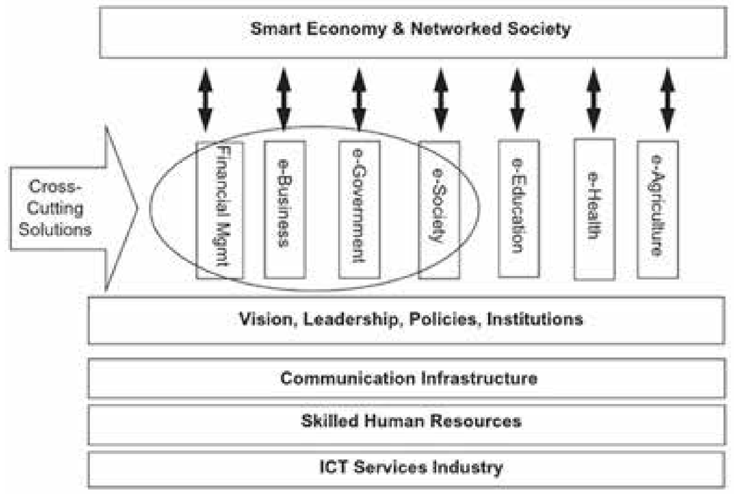
Digital Transformation and the Public Sphere

It would be remiss not to explore digital transformation with respect to the public sphere. The concept of the public sphere is largely associated with the work of Jurgen Habermas who describes it as the space that shapes and holds public opinion and where "access is guaranteed to all citizens". The public sphere thus describes a space that intercedes between the issues that are of concern to the general population and the domain of government and corporate influence. All citizens must be granted an equal opportunity to make contributions and where participation and communication must be free from external and internal coercion. A functioning public sphere, which entails a collection of communication spaces that promotes debate and the circulation of ideas, has always had a relationship and dependence on the mass media included the mass, and more recently, newer emerging technologies are conspicuous. The effect of this has been the transformation of the public sphere into technology-enabled spaces, where the discourse has no geographic boundaries or any constraints of time and political interests.

A Digital Transformation Architecture

Various models have been proposed to define the concept of digital transformation. Theoretical models like the one proposed by Hanna attempts to encapsulates all of the aspects that underpin digital transformation (See Figure 1).

Figure 1: Digital transformation architecture



Other proprietary models include Accenture's Digital Operating Model which identifies seven enablers for digital transformation which includes concepts like 'digital intelligence'.

Conclusion

The role of government is to create an enabling environment across all sectors e.g. health and commerce. This is achieved by policy frameworks, from which emerges legislation and regulation. The same is true for digital transformation. A World Bank report argues that digital revolution needs offline help to realise its potential that include implementing policies that provide an enabling environment for inter alia infrastructure development and the creation of services. Whatever the path that countries and organisations choose towards digital transformation, what is clear is that this is journey that must be embraced sooner rather than later.