Effect of variable suction on unsteady MHD oscillatory flow of jeffrey fluid in a horizontal channel with heat and mass transfer

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Abstract The effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer has been studied. The temperature prescribed at plates is uniform and asymmetric. A perturbation method is employed to solve the momentum and energy equations. The effects of various dimensionless parameters on velocity and temperature profiles are considered and discussed in details through graphs. It is found that, the velocity $u$ increases with increase in $h_2, \alpha_1, \lambda_1, Gr, Gc, N, Re$ and $Sc$. The velocity also increases with decrease in $h_1, \alpha_2, Ha,$ and $K_c$. It is also observed that the temperature $\theta$ increases with increase in $N$ and $Pe$. Increase in Schmidt number $Sc$ and chemical parameter $K_c$ respectively increase and decrease the species concentration or the concentration boundary layer thickness of the flow field.

Keywords Heat and Mass Transfer, Unsteady, MHD, Jeffrey Fluid, Oscillatory Flow, Variable Suction

1. Introduction

The effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffrey fluid in horizontal media with variable suction are encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched in cooling systems; the fluid mechanical properties of penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymers fluids like polyethylene oxide and polysobutylene solutions in a cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Also, the radiative heat transfer is an important factor of thermodynamics of very high temperature systems such as electric furnaces, solar collectors, storage of nuclear wastes packed bed catalytic reactors, satellites, steel rolling, cryogenic engineering etc. Mass transfer processes are evaporation of water from a pond to the atmosphere, the diffusion of chemical impurities in lakes, rivers and ocean from natural or artificial sources.

The study of such flow under the influence of magnetic field and heat transfer has attracted the interest of many investigators and researchers.

Asadullah et al (2013) consider the MHD flow of a Jeffrey fluid in converging and diverging channels [1]. The flows between non parallel walls have a very significant role in physical and biological sciences. Kavita et al (2012) investigated the influence of heat transfer on MHD oscillatory flow of Jeffery fluid in a Channel [2]. They found out that an analysis of first order homogeneous chemical reaction and heat source on MHD oscillatory flow of viscous–elastic fluid through a channel filled with saturated porous medium are reported by Devika et al (2013) [3].

An oscillatory flow of a Jeffrey Fluid in an elastic tube of variable cross – section has been investigated at low Reynolds number by Badari et al (2012) [4]. Their main concentration is on the excess pressure of the tube. The equation has been solved numerically and investigations are made for different cases on the tube.

Kumari et al (2012) studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel with slip effect at a lower wall where the expressions for the velocity and temperature are obtained analytically.
Unsteady Flow of a Jeffrey Fluid in an elastic tube with stenosis was investigated by Sreenadh et al [5]. The governing equation for the excess pressure is obtained for Jeffrey model. The governing equations was solved numerically and investigations are made for different cases for strength, tapered and constricted tubes. An oscillatory flow of a Jeffrey fluid in an elastic tube of variable cross section at low Reynolds number has been investigated by Badari et al (2012) [4]. The main concentration is on the excess pressure of the tube. The equations have been solved numerically.

Hayat et al (2012) analyzed the effects of Newtonian heating and magnetohydrodynamic (MHD) in a flow of Jeffrey fluid in stagnation point of view over radially stretching surface [6]. The effect of heat transfer through radiation on velocity, magnetic and temperature fields in the case of two dimensional hydromagnetic oscillatory flow of a viscous incompressible and electrically conducting fluid past a porous, a limiting surface, subjected to variable suction and moving impulsively with a constant velocity in the presence of transverse magnetic field has been analyzed.

Idowu et al (2013) studied the effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with chemical reaction. They found that the velocity is more of Jeffrey fluid [7].

The effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffrey fluid with variable viscosity through porous medium was investigated. He found out that the velocity for Jeffrey fluid with variable viscosity is less than velocity of Jeffrey fluid with constant velocity. This result is in agreement with Kavita (2012) [2].

This present work focused on the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer. This study is expected to help in understanding the concept of Jeffrey fluid as a non-Newtonian fluid and the effect of heat and mass on unsteady oscillatory flow of Jeffrey fluid with variable suction and slip flow parameters.

The study have potential applications in oil recovery, filtration systems, geophysical, astrophysical, cosmical studies and in medicine especially in finding remedy for atherosclerosis and several applications.

2. Problem Formulation

We consider the flow of Jeffrey fluid in horizontal infinite parallel plates channel. The channel width is $h$. The constitute equation for $S$ Jeffrey fluid [2] is

$$ S = \mu + \lambda_3 \frac{d\gamma}{dt} + \lambda_4 \frac{d^2\gamma}{dt^2} $$

(2.1)

Where $\mu$ is the dynamic viscosity, $\lambda_3$ is the ratio of relaxation to retardation times, $\lambda_4$ is the retardation time and $d\gamma/dt$ is the shear rate.

Now, let $u'$ be the velocity of the fluid in $x'$ direction taken along the horizontal infinite parallel plate channel under a chemical reaction with species concentration $C'$ and $y' - x'$ axis is taken normal to the direction of flow. The radiative heat term in $x'$ direction is considered negligible in comparison with $y'$ direction. The upper and lower plates are kept at $T' = T'_w$ and $T' = T'_0$, respectively. The Cartesian coordinate system and the flow configuration are shown in figure 1. At a time $t' > 0$, the plate is given an impulsive motion in a horizontal direction with uniform mean velocity $U$. Moreover at this stage an unsteady component $\varepsilon T' e^{iwt}$ where $\varepsilon \ll 1$ is the amplitude of oscillation, is assumed to be superimposed on the temperatures of the plates.

$$ T' = T'_0 + \varepsilon T' e^{iwt} $$

Figure 1: Physical schematic of the flow configuration

A magnetic field of uniform strength $B_0$ is applied normal to the plate along $x'$ direction and the induced magnetic field is assumed negligible. It is also assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The pressure gradient is also assumed negligible. Since the plate
is considered infinite in x’ direction, all the physical quantities will be independent of x’. Under these assumptions, the physical variables are functions of y’ and t’ only. The time dependent suction is
\[ V'(t) = -V_0'(1 + \varepsilon e^{-i\omega t}) \] (2.2)
The basic equations of momentum, energy and species concentration govern such a flow, subject to Boussinesq approximation, are
\[ \frac{\partial u}{\partial t} - V_0'(1 + \varepsilon e^{-i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} + \frac{\rho}{\rho_0} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta_0 - \varepsilon \beta_0}{\rho} + g \beta (T' - T_0') + g \beta' (C' - C_0') \] (2.3)
\[ \frac{\partial T'}{\partial t} - V_0'(1 + \varepsilon e^{-i\omega t}) \frac{\partial T'}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} \] (2.4)
\[ \frac{\partial C'}{\partial t} - V_0'(1 + \varepsilon e^{-i\omega t}) \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} - K' (C' - C_0') \] (2.5)
The \( q \) in (2.4) is called the radiative heat flux. It is given by
\[ \frac{\partial q}{\partial y} = 4\varepsilon \beta (T_0' - T') \] (2.6)
Where, \( u' \) is the axial velocity, \( T' \) is the fluid temperature, \( \rho \) is the fluid density, \( \lambda_1 \) is the ratio of relaxation to retardation times, \( B_0 \) is the magnetic field strength, \( \sigma \) is the conductivity of the fluid, \( g \) is the acceleration due to gravity, \( \beta \) is the coefficient of volume expansion due to temperature, \( \beta' \) the coefficient of volume expansion due to species concentration, \( c_p \) is the specific heat at constant pressure, \( k \) is the thermal conductivity, \( D \) is the mass diffusion coefficient, \( K' \) is the chemical reaction parameter.
The boundary conditions are given by
\[ u' - U_1 = L_1 \frac{\partial u}{\partial y'}, \quad T' = T_{w'}, \quad C' = C_{w'} \quad \text{at} \quad y' = 0, \ t' \leq 0 \]
\[ u' - U_2 = L_2 \frac{\partial u}{\partial y'}, \quad T' = T_{w'}, \quad C' = C_{w'} \quad \text{at} \quad y' = h, \ t' \geq 0 \] (2.7)
In order to write the governing equations and the relevant boundary conditions in non – dimensional form, the following dimensionless quantities are introduced
\[ x = \frac{x}{h}, \quad y = \frac{y}{h}, \quad u = \frac{u}{u_0}, \quad \theta = \frac{T - T_0}{T_{w} - T_0}, \quad t = \frac{\varepsilon y_0}{h}, \quad Ha = \frac{\varepsilon^2 h^2}{\mu}, \quad Gr = \frac{\varepsilon^2 h^2 (T_{w} - T_0)}{\mu y_0}, \]
\[ Sc = \frac{u_0}{h}, \quad Gc = \frac{\varepsilon^2 h^2 (C_{w} - C_0)}{y_0} \] (2.8)
The momentum equation (2.2), the energy equation (2.3) and the species concentration equation (2.4) now become
\[ Re \frac{\partial u}{\partial t} - Re (1 + \varepsilon e^{-i\omega t}) \frac{\partial u}{\partial y} = \frac{1}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} - Ha^2 u + Gr \theta + Gc C \] (2.9)
\[ Pe \frac{\partial T}{\partial t} - Pe (1 + \varepsilon e^{-i\omega t}) \frac{\partial T}{\partial y} = \frac{1}{\varepsilon^2 \lambda_1} \frac{\partial^2 T}{\partial y^2} - N^2 \theta \] (2.10)
\[ Sc \frac{\partial C}{\partial t} - Sc (1 + \varepsilon e^{-i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{Sc \varepsilon^2} \frac{\partial^2 C}{\partial y^2} - K'C \] (2.11)
The boundary conditions in dimensionless form are
\[ u = \alpha_1 + h_1 \frac{\partial u}{\partial y}, \quad \theta = 0, \quad C = 0 \quad \text{at} \quad y = 0, \ t \leq 0 \]
\[ u = \alpha_2 + h_2 \frac{\partial u}{\partial y}, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0, \ t \geq 0 \] (2.12)
3. Solution of the Problem
To solve equations (2.9) – (2.11) subject to the boundary conditions (2.12), we expand the functions \( u(y, t) \)and \( \theta(y, t) \) as a power series in the perturbative parameters. Here we assumed small amplitude of oscillations\((\varepsilon \ll 1)\), thus
\[ u(y, t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t} + o(\varepsilon^2) \] (3.1)
\[ \theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y)e^{i\omega t} + o(\varepsilon^2) \] (3.2)
\[ C(y, t) = C_0(y) + \varepsilon C_1(y)e^{i\omega t} + o(\varepsilon^2) \] (3.3)
The first terms in equations (3.1), (3.2) and (3.3) are called the harmonic terms while the second terms are called the non – harmonic terms.
Substituting equations (3.1), (3.2) and (3.3) into equations (2.9), (2.10) and (2.11), equating the coefficients of the harmonic and non – harmonic terms and neglecting the coefficients of \( \varepsilon^2 \), we get:
\[ au_0''(y) + Reu_0' - Ha^2 u_0(y) = -Gr \theta_0(y) - Gc C_0(y) \] (3.4)
\[ a\theta_1''(y) + Re \theta_1' - b^2 \theta_1(y) = -AReu_0(y) - Gr \theta_1(y) - Gc C_1(y) \] (3.5)
Where, $a = \frac{1}{1+\lambda_1}$ and $b^2 = Ha^2 + i\omega Re$

$$\theta_0(y) + Pe\theta_0(y) + \theta_0(y) = 0 \quad (3.6)$$

$$\theta_1(y) + a_2\theta_1(y) + a_3\theta_1(y) = -APe\theta_0(y) \quad (3.7)$$

where $a_2 = N^2 - i\omega Pe$ and $a_3 = i\omega Pe$

$$C_0(y) + ScC_0(y) - KcScC_0(y) = 0 \quad (3.8)$$

$$C_1(y) + ScC_1(y) - a_2C_1(y) = -AScC_0(y) \quad (3.9)$$

where $a_4 = KcSc + i\omega Sc$

The corresponding boundary conditions become

$u_0 = a_1 + h_1\frac{\partial u_0}{\partial y}, \quad u_1 = h_1\frac{\partial u_1}{\partial y}, \quad \theta_0 = 0, \quad C_0 = 0, \quad \theta_1 = 0, \quad C_1 = 0 \quad \text{at} \quad y = 0, \quad t \leq 0$

$u_0 = a_2 + h_2\frac{\partial u_0}{\partial y}, \quad u_1 = h_2\frac{\partial u_1}{\partial y}, \quad \theta_0 = 1, \quad C_0 = 1, \quad \theta_1 = 0, \quad C_1 = 1 \quad \text{at} \quad y = 1, \quad t \geq 0 \quad (3.10)$

We now solved equations (3.4) – (3.9) under the relevant boundary conditions (3.10) for the mean flow and unsteady flow separately.

The mean flows are governed by the equations (3.4), (3.6) and (3.8) where $u_0, \theta_0, C_0$ are respectively called the mean velocity, mean temperature and mean species concentration. The unsteady flows are governed by equations (3.5), (3.7) and (3.9) where $u_1, \theta_1$ and $C_1$ are the unsteady components.

These equations are solved analytically under the relevant boundary conditions (3.10) as follows:

Solving equations (3.6) and (3.8) subject to the corresponding relevant boundary conditions in (3.10), we obtain the mean temperature as

$$\theta_0(y) = A_1e^{m_1y} + A_2e^{m_2y} \quad (3.11)$$

$$C_0(y) = A_3e^{m_3y} + A_4e^{m_4y} \quad (3.12)$$

Similarly, solving equations (3.7) and (3.9) under the relevant boundary conditions in (3.10), the unsteady temperature becomes

$$\theta_1(y) = A_5e^{m_3y} + A_6e^{m_4y} + K_1e^{m_1y} + K_2e^{m_2y} \quad (3.13)$$

$$C_1(y) = A_7e^{m_7y} + A_8e^{m_8y} + K_9e^{m_5y} + K_4e^{m_6y} \quad (3.14)$$

Putting equations (3.11), (3.12) and equations (3.13) and (3.14) into equations (3.4) and (3.5) respectively and using the corresponding boundary conditions in (3.10), we obtain the mean velocity $u_0(y)$ and the unsteady velocity component $u_1(y)$ as follows;

$$u_0(y) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + K_5e^{m_1y} + K_6e^{m_2y} + K_7e^{m_5y} + K_8e^{m_6y} \quad (3.15)$$

$$u_1(y) = A_{11}e^{m_1y} + A_{12}e^{m_2y} + e[A_2y^{m_3} + A_4y^{m_4y} + K_9e^{m_5y} + K_2e^{m_6y}]e^{i\omega t} \quad (3.16)$$

Therefore, the solutions for the velocity and temperature profiles are

$$u(y,t) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + K_5e^{m_1y} + K_6e^{m_2y} + K_7e^{m_5y} + K_8e^{m_6y} + \varepsilon[A_2y^{m_3} + A_4y^{m_4y} + K_9e^{m_5y} + K_2e^{m_6y}]e^{i\omega t} \quad (3.17)$$

$$\theta(y,t) = A_1e^{m_1y} + A_2e^{m_2y} + e[A_3y^{m_3} + A_4y^{m_4y} + K_9e^{m_5y} + K_2e^{m_6y}]e^{i\omega t} \quad (3.18)$$

$$C(y,t) = A_5e^{m_3y} + A_6e^{m_4y} + e[A_7y^{m_7} + A_8y^{m_8y} + K_9e^{m_5y} + K_4e^{m_6y}]e^{i\omega t} \quad (3.19)$$

where, $m_1 = \frac{-Pe + \sqrt{Pe^2 - 4Kc^2}}{2}, \quad m_2 = \frac{-Pe - \sqrt{Pe^2 - 4Kc^2}}{2}, \quad A_1 = \frac{1}{e^{m_1} - e^{m_2}}$

$$A_2 = -\frac{1}{2}\frac{m_3 - m_4}{e^{m_1} - e^{m_2}}, \quad A_3 = \frac{-a_3 + \sqrt{a_3^2 - 4a_2^2}}{2}, \quad A_4 = \frac{-a_3 - \sqrt{a_3^2 - 4a_2^2}}{2}, \quad m_4 = \frac{1}{2}\frac{m_3 - m_4}{e^{m_1} - e^{m_2}}$$

$$A_{15} = -A_{14}m_1Pe, \quad A_{61} = -A_{2}m_2Pe, \quad K_1 = \frac{A_{51}}{m_1^2 + m_3m_1 + m_2^2}\quad (3.20)$$

$$K_2 = \frac{A_{61}}{m_2^2 + m_3m_2 + m_1^2}, \quad a_5 = -(K_1 + K_2), \quad l_1 = -(K_1e^{m_1} + K_2e^{m_2})$$

$$l_2 = l_1 - a_2e^{m_2}, \quad A_3 = \frac{l_2}{2(m_3 - e^{m_4})}, \quad A_4 = a_5 - A_3, \quad m_5 = \frac{-Sc + \sqrt{Sc^2 + 4KcSc}}{2}\quad (3.21)$$

$$m_6 = \frac{-Sc - \sqrt{Sc^2 + 4KcSc}}{2}, \quad A_5 = \frac{1}{2(m_5 - e^{m_4})}, \quad A_6 = \frac{-Sc + \sqrt{Sc^2 + 4KcSc}}{2}\quad (3.22)$$

$$m_8 = \frac{-Sc + \sqrt{Sc^2 + 4KcSc}}{2}, \quad K_3 = \frac{-A_{2}m_2Sc}{m_2^2 + m_3m_2 - a_4}, \quad K_4 = \frac{-A_{2}m_2Sc}{m_2^2 + m_3m_2 - a_4}\quad (3.23)$$

$$l_23 = -(K_3 + K_4), \quad l_24 = -(K_3e^{m_3} + K_4e^{m_4}), \quad A_8 = \frac{l_24}{2(m_3 - e^{m_4})}\quad (3.24)$$

$$A_7 = l_23 - A_9, \quad m_9 = \frac{-Re + \sqrt{Re^2 + 4a_0h_a^2}}{2a}, \quad m_{10} = \frac{-Re - \sqrt{Re^2 + 4a_0h_a^2}}{2a}\quad (3.25)$$

$$K_5 = \frac{A_{10}Gr}{am_7^{1/2} + Rem_2 - Ha^n}, \quad K_6 = \frac{-A_{10}Gr}{am_7^{1/2} + Rem_2 - Ha^n}, \quad K_7 = \frac{-A_{10}Gr}{am_7^{1/2} + Rem_2 - Ha^n}\quad (3.26)$$

$$K_8 = \frac{-A_{10}Gr}{am_7^{1/2} + Rem_2 - Ha^n}, \quad l_3 = K_5 + K_6 + K_7 + K_8, \quad (3.27)$$

4. Analysis and Discussion of Results

In this section, 2-term perturbation series are employed to evaluate the dimensionless velocity, dimensionless temperature and the dimensionless species concentration profiles.

To study the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer, the velocity \( u \), temperature \( \theta \) and the species concentration \( C \) profiles are depicted graphically against \( y \) for different values of different parameters; Slip flow parameters \( h_1 \) and \( h_2 \) , Suction parameters \( \alpha_1 \) and \( \alpha_2 \), material parameter \( \lambda_1 \), Grashof numbers \( Gr \), Hartman number \( Ha \), Reynolds number \( Re \), radiation parameter \( N \), Schmidt number \( Sc \), chemical reaction parameter \( K_c \) and Peclet number \( Pe \).

The velocity of the flow field varies to a great extent with the variation of the flow parameters. The main factors affecting the velocity are: Slip flow parameters \( h_1 \) and \( h_2 \) , Suction parameters \( \alpha_1 \) and \( \alpha_2 \), material parameter \( \lambda_1 \), Grashof numbers \( Gr \) and \( \lambda_c \), Hartman number \( Ha \), Reynolds number \( Re \), radiation parameter \( N \), Schmidt number \( Sc \) and chemical reaction parameter \( K_c \). The effects of these parameters on the velocity field are analyzed in figures 2 – 13. These effects are discussed quantitatively.

Figures 2 and 3 demonstrate the effect of slip flow parameters \( h_1 \) and \( h_2 \) on velocity \( u \). It is observe in figure 2 that the velocity \( u \) decreases with increase in \( h_1 \) while figure 3 shows that the velocity \( u \) increases with increase in \( h_2 \).

Figures 4 and 5 show the effect of suction parameters \( \alpha_1 \) and \( \alpha_2 \). It can be seen in figure 4 that the velocity \( u \) increases with increase in \( \alpha_1 \). While the velocity \( u \) decreases with increase in \( \alpha_2 \) in figure 5.

Figure 6 shows the effect of Grashof number \( Gr \) on velocity \( u \). It is observed that as \( Gr \) increases, the velocity increases. To this effect, at higher Grashof number \( Gr \) the flow at the boundary is turbulent while at lower \( Gr \) the flow at the boundary is laminar.

Figure 7 depicts the effect of Grashof number \( Gr \) for mass transfer on velocity \( u \). It is observed that the velocity \( u \) increases with increase in \( Gr \).

The effect of Reynolds number \( Re \) on velocity \( u \) is shown in figure 8. It is shown that the velocity \( u \) increases with increasing \( Re \).

Figure 9 depicts the effect of Hartman number \( Ha \) on velocity \( u \). It is shown that the velocity \( u \) increases with decrease in \( Ha \). This shows the effect of magnetic field on the fluid flow and this effect suppresses the turbulence flow of the Jeffrey fluid. Physically, when magnetic field is applied to any fluid, then the apparent viscosity of the fluid increases to the point of becoming visco elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of electromagnet which give rise to many possible control – based applications, including MHD power generation, electromagnetic casting of metals, MHD ion propulsion etc.
Figure 10 demonstrates the effect of material parameter $\lambda_1$ on velocity $u$. It is evident that the velocity $u$ increases with increase in $\lambda_1$. To this effect the ratio of relaxation to retardation enhances the increase flow of velocity when it becomes small.

The effect of the radiation parameter $N$ on velocity $u$ is depicted in figure 11. It is observed that the velocity $u$ increases as the radiation parameter $N$ increases.

Figure 12 illustrates the effect of chemical reaction parameter $K_c$ on velocity $u$. It is observed that as the chemical reaction parameter decreases, the velocity increases.

Figure 13 demonstrate the effect of Schmidt number $Sc$ on velocity $u$. It shows that the velocity increases with increase in $Sc$.

The temperature field suffers a major change in magnitude due to the variation of radiation parameter $N$ and Peclet number $Pe$. The effects of these parameters on the temperature field are discussed in figures 14-15.

Figure 14 depicts the effect of radiation parameter $N$ on temperature $\theta$. It is found out that the temperature $\theta$ increases with increase in $N$.

The effect of Reynolds number $Pe$ on temperature $\theta$ is shown in figure 15. It is observed that the temperature $\theta$ increases with increase in $Pe$.

The presence of foreign mass in the flow field greatly affects the species concentration of the flow field. The factors or parameters responsible for this variations are Schmidt number $Sc$ and chemical reaction parameter $K_c$.

The effect of Schmidt number $Sc$ on species concentration $C$ is shown in figure 16. It is shown that the species concentration $C$ increases with increase in $Sc$.

Figure 17 reveals the effect of chemical reaction $K_c$ on species concentration $C$. It is observed that increase in $K_c$ decreases the species concentration $C$.

**Figure 2:** Effect of slip flow $h_1$ parameter on velocity with $h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $\Delta t = 0.5$.

**Figure 3:** Effect of slip flow $h_2$ parameter on velocity with $h_1 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $\Delta t = 0.5$. 
Figure 4: Effect of slip flow $\alpha_1$ parameter on velocity with $h_1 = 1, h_2 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $t = 0.5$.

Figure 5: Effect of slip flow $\alpha_2$ parameter on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $t = 0.5$.

Figure 6: Effect of Grashof number $Gr$ on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $t = 0.5$. 
Figure 7: Effect of Grashof number $Gc$ due to mass transfer on velocity with $h_1 = 1$, $h_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\lambda_1 = 0.3$, $Gc = 1$, $Ha = 1$, $N = 1$, $\omega = 1$, $Pe = 1$, $Re = 1$, $Sc = 1$, $K_c = 1$, $\varepsilon = 0.02$, $A = 0.5$ and $t = 0.5$.

Figure 8: Effect of Reynolds number $Re$ on velocity with $h_1 = 1$, $h_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\lambda_1 = 0.3$, $Gc = 1$, $Ha = 1$, $N = 1$, $\omega = 1$, $Pe = 1$, $Sc = 1$, $K_c = 1$, $\varepsilon = 0.02$, $A = 0.5$ and $t = 0.5$.

Figure 9: Effect of Hartmann number $Ha$ on velocity with $h_1 = 1$, $h_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\lambda_1 = 0.3$, $Gc = 1$, $Re = 1$, $N = 1$, $\omega = 1$, $Pe = 1$, $Sc = 1$, $K_c = 1$, $\varepsilon = 0.02$, $A = 0.5$ and $t = 0.5$. 
Figure 10: Effect of material parameter $\lambda_1$ on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Figure 11: Effect of Radiation Parameter $N$ on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Re = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Figure 12: Effect of Chemical Parameter $K_c$ on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \lambda_1 = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$. 
Figure 13: Effect of Chemical Parameter $Sc$ on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, \lambda_1 = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Figure 14: Effect of Radiation Parameter $N$ on temperature with $\omega = 1, Pe = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Figure 15: Effect of Peclet number $Pe$ on temperature with $\omega = 1, N = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$. 
Figure 16: Effect of Schmidt number $Sc$ on species concentration with $\omega = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Figure 17: Effect of chemical parameter $K_c$ on species concentration with $\omega = 1, Sc = 1, \varepsilon = 0.02, A = 0.5$ and $dt = 0.5$.

Summary and Conclusion

In this section, we studied the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer. The governing equations, the momentum, energy and species equations have been written in dimensionless form.

A perturbation method has been employed to evaluate and solved the dimensionless equations for velocity $u$, the dimensionless temperature $\theta$ and the dimensionless species concentration $C$.

The main findings are summarized below:

i. Increase in slip flow parameter $h_2$, suction parameter $\alpha_3$, material parameter $\lambda_1$, Grashof number for heat transfer $Gr$, Grashof number for mass transfer $Gc$, Radiation parameter $N$, Reynolds number $Re$ and Schmidt number $Sc$ have accelerating effects on velocity of the flow field.

ii. Decrease in slip flow parameter $h_1$, Hartman number $Ha$ and chemical parameter $K_c$ accelerates the velocity of the flow field.

iii. Increase in the Radiation parameter $N$, Peclet number $Pe$ and accelerate the magnitude of temperature of the flow field.

iv. Increase in Schmidt number $Sc$ and chemical parameter $K_c$ respectively increase and decrease the species concentration or the concentration boundary layer thickness of the flow field.

It is concluded that these results show that the velocity is more of Jeffrey fluid than the Newtonian fluid.
5. References