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Simple and Multi-collision of an Ellipsoid with Planar Surfaces. Part II: Example

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This paper is a complementary one for our previous paper [25]. The collision of the ellipsoid is considered to take place with the main planes of the reference system. In our simulation we obtain the following results: no collision, collision with a single plane, simultaneous collision with two planes, simultaneous collision with the all three planes, and collision for a longer time with one or more planes.

***Keywords**: simultaneous collision, simulation, planar surfaces, ellipsoid*

1. Introduction

This paper contains an example of the vibration of an ellipsoid which collides with one or more planar surfaces. The formulae used in this paper are those obtained by the authors in their previous one [25].

We consider an ellipsoid of mass $m = 60\,\mathrm{kg}$ and the semi-axes equal to $a = 0.1\,\mathrm{m}$, $b = 0.1\,\mathrm{m}$, and $c = 0.2\,\mathrm{m}$. The equation of the ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \tag{1}$$

It results the principal central moments of inertia of the ellipsoid

$$J_x = \frac{m\left(b^2 + c^2\right)}{5} = 0.6\,\mathrm{kgm}^2,\ J_y = \frac{m\left(c^2 + a^2\right)}{5} = 0.6\,\mathrm{kgm}^2,$$
$$J_z = \frac{m\left(a^2 + b^2\right)}{5} = 0.24\,\mathrm{kgm}^2. \tag{2}$$

Because of the equality of the semi-axes a and b, the ellipsoid is a rotational one.

At the point A of coordinates (in the local reference frame $Oxyz$) $x_A = 0$, $y_A = 0$, $z_A = c$, the ellipsoid is acted by a spring of stiffness k. The other end of the spring is linked at the point B (Fig. 1).

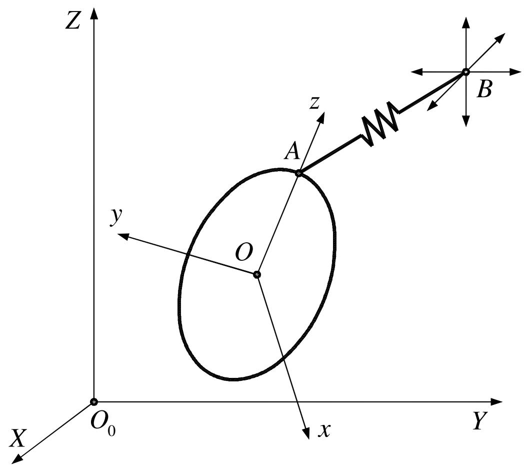


Figure 1. The motion of the ellipsoid.

The point B has an oscillatory motion of equations

$$X_B = (X_B)_0 + A_{XB}\cos(\omega_X t),\ Y_B = (Y_B)_0 + A_{YB}\cos(\omega_Y t),$$
$$Z_B = (Z_B)_0 + A_{ZB}\cos(\omega_Z t) \tag{3}$$

where $(X_B)_0$, $(Y_B)_0$, and $(Z_B)_0$ are the initial coordinates of the point B relative to the fixed reference system O_0XYZ, A_{XB}, A_{YB}, and A_{ZB} are the amplitudes of the vibration on the three directions O_0X, O_0Y, and O_0Z, t is the time, while ω_X, ω_Y, and ω_Z are the pulsations of the vibration on the same main axes.

The ellipsoid is acted only by its own weight mg on the vertical direction and by the elastic force in the spring.

Denoting by X_A, Y_A, Z_A and X_B, Y_B, Z_B, the coordinates of the points A, and B, respectively, by l_0 the non-deformed length of the spring, by l the length of the spring at one given moment, by $\mathbf{i}_0$, $\mathbf{j}_0$, and $\mathbf{k}_0$, the unit vectors of the axes O_0X, O_0Y, and O_0Z, respectively, by $\mathbf{F}_{el}$, and F_{el}, the elastic force in the spring in the fixed reference system O_0XYZ, and its magnitude, respectively, one may write the following relations

$$l = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2 + (Z_A - Z_B)^2}, \tag{4}$$

$$F_{el} = k|l - l_0|, \tag{5}$$

$$\mathbf{F}_{el} = k(l - l_0)\frac{\mathbf{AB}}{AB} = k(l - l_0)\frac{\mathbf{AB}}{l}$$

$$= k(l - l_0)\frac{(X_B - X_A)\mathbf{i}_0 + (Y_B - Y_A)\mathbf{j}_0 + (Z_B - Z_A)\mathbf{k}_0}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}}. \tag{6}$$

If we denote by $[\mathbf{A}]$ the matrix of rotation, by $\{\mathbf{R}_P\}$, and $\{\mathbf{r}_P\}$ the column matrices of coordinates of a generic point P with respect to the systems O_0XYZ, and $Oxyz$, respectively, one has

$$\{\mathbf{R}_P\} = [X_P \;\; Y_P \;\; Z_P]^{\mathrm{T}},\ \{\mathbf{r}_P\} = [x_P \;\; y_P \;\; z_P]^{\mathrm{T}}, \tag{7}$$

$$\{\mathbf{R}_P\} = \{\mathbf{R}_O\} + [\mathbf{A}]\{\mathbf{r}_p\}. \tag{8}$$

Consequently, writing

$$\{\mathbf{F}_{el}\} = [F_{elX} \;\; F_{elY} \;\; F_{elZ}]^{\mathrm{T}} \tag{9}$$

with the aid of its projections onto the axes O_0X, O_oY, and O_0Z, the projections of the elastic force onto the mobile axes are

$$\{\mathbf{f}_{el}\} = [\mathbf{A}]^{\mathrm{T}}\{\mathbf{F}_{el}\}. \tag{10}$$

The moment of the elastic force relative to the center O of the ellipsoid is

$$\mathbf{m}_{el} = \mathbf{OA} \times \mathbf{f}_{el}, \tag{11}$$

$$\{\mathbf{m}_{el}\} = \begin{bmatrix} 0 & -z_A & y_A \\ z_A & 0 & -x_A \\ -y_A & x_A & 0 \end{bmatrix}\begin{bmatrix} f_{elx} \\ f_{ely} \\ f_{elz} \end{bmatrix} = \begin{bmatrix} 0 & -c & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} f_{elx} \\ f_{ely} \\ f_{elz} \end{bmatrix} = \begin{bmatrix} -cf_{ely} \\ cf_{elx} \\ 0 \end{bmatrix}. \tag{12}$$

The equations of motion of the ellipsoid are

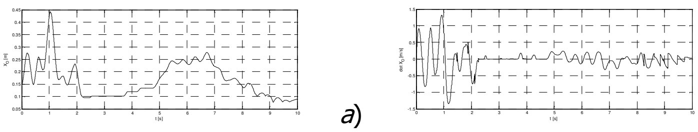
$$m\ddot{X} = F_{elX},\ m\ddot{Y} = F_{elY},\ m\ddot{Z} = F_{elZ} - mg,$$
$$J_x\dot{\omega}_x - \omega_y\omega_z(J_y - J_z) = m_{elx} = -cf_{ely},$$
$$J_y\dot{\omega}_y - \omega_z\omega_x(J_z - J_x) = m_{ely} = cf_{elx},\ J_z\dot{\omega}_z - \omega_x\omega_y(J_x - J_y) = m_{elz} = 0, \tag{13}$$

where

$$\omega_x = \dot{\psi}\cos\varphi\cos\theta + \dot{\theta}\sin\varphi,\ \omega_y = -\dot{\psi}\sin\varphi\cos\theta + \dot{\theta}\cos\varphi,$$
$$\omega_z = \dot{\psi}\sin\theta + \dot{\varphi}. \tag{}$$

2. Simulation

Two cases are considered for the simulations. In the first case, the working parameters have the following values: the coefficient of restitution with the plane O_0XY, $k_1 = 0.5$, the coefficient of restitution with the plane O_0XZ, $k_2 = 0.6$, the coefficient of restitution with the plane O_0YZ, $k_3 = 0.7$, the initial position of the point B, $(X_B)_0 = 0.15\,\mathrm{m}$, $(Y_B)_0 = 0.15\,\mathrm{m}$, $(Z_B)_0 = 0.6\,\mathrm{m}$, the pulsations $\omega_{XB} = 1\,\mathrm{s}^{-1}$, $\omega_{YB} = 1\,\mathrm{s}^{-1}$, $\omega_{ZB} = 3\,\mathrm{s}^{-1}$, the amplitudes of the excitation $A_{XB} = 0.1\,\mathrm{m}$, $A_{YB} = 0.1\,\mathrm{m}$, $A_{ZB} = 0.05\,\mathrm{m}$, the non-deformed length of the spring $l_0 = 0.05\,\mathrm{m}$, the stiffness of the spring $k = 1.2 \times 10^4\ \mathrm{N/m}$, the initial values for the simulation $(X_O)_0 = 0.15\,\mathrm{m}$, $(Y_O)_0 = 0.15\,\mathrm{m}$, $(Z_O)_0 = 0.25\,\mathrm{m}$, $\psi_0 = 0\,\mathrm{rad}$, $\theta = 0\,\mathrm{rad}$, $\varphi = 0\,\mathrm{rad}$, $(\dot{X}_O)_0 = 0\,\mathrm{m/s}$, $(\dot{Y}_O)_0 = 0\,\mathrm{m/s}$, $(\dot{Z}_O)_0 = 0\,\mathrm{m/s}$, $\dot{\psi}_0 = 0\,\mathrm{rad/s}$, $\dot{\theta}_0 = 0\,\mathrm{rad/s}$, $\dot{\varphi}_0 = 0\,\mathrm{rad/s}$. For the second case the exception is $k_3 = 0.6$. In both cases the time of simulation is $t = 10\,\mathrm{s}$, while the step of iteration is $\Delta t = 0.0025\,\mathrm{s}$. The results of the simulation are given in the next figures.



a) *b*)

Figure 2. Time history: *a*) $X_O = X_O(t)$; *b*) $\dot{X}_O = \dot{X}_O(t)$ in the first case.

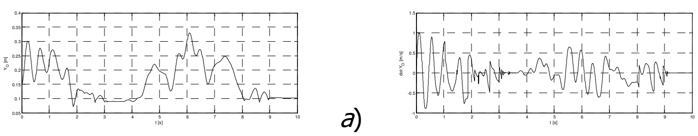


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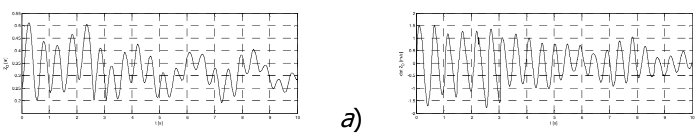
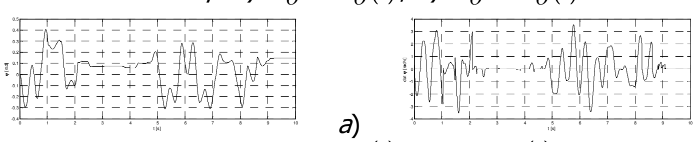
Figure 3. Time history: *a*) $Y_O = Y_O(t)$; *b*) $\dot{Y}_O = \dot{Y}_O(t)$ in the first case.



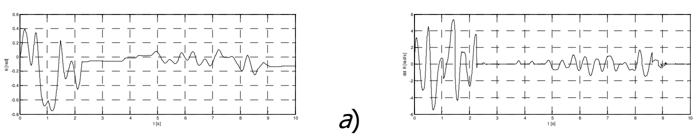
a) *b*)

Figure 4. Time history: *a*) $Z_O = Z_O(t)$; *b*) $\dot{Z}_O = \dot{Z}_O(t)$ in the first case.



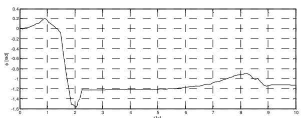
a) *b*)

Figure 5. Time history: *a*) $\psi = \psi(t)$; *b*) $\dot{\psi} = \dot{\psi}(t)$ in the first case.



a) *b*)

Figure 6. Time history: *a*) $\theta = \theta(t)$; $\dot{\theta} = \dot{\theta}(t)$ in the first case.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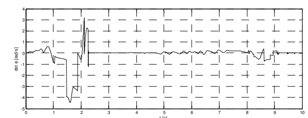
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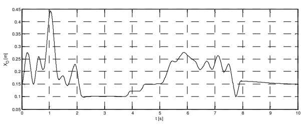
Figure 7. Time history: *a*) $\varphi = \varphi(t)$; *b*) $\dot{\varphi} = \dot{\varphi}(t)$ in the first case.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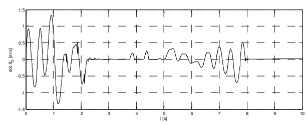
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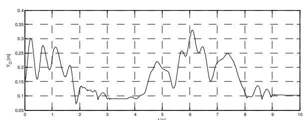
Figure 8. Time history: *a*) $X_O = X_O(t)$; *b*) $\dot{X}_O = \dot{X}_O(t)$ in the second case.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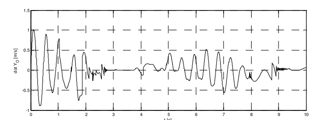
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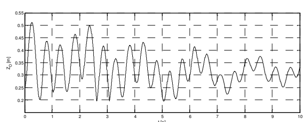
Figure 9. Time history: *a*) $Y_O = Y_O(t)$; *b*) $\dot{Y}_O = \dot{Y}_O(t)$ in the second case.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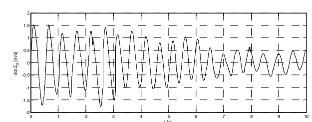
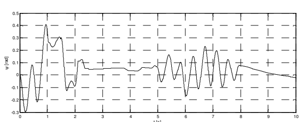

b)

Figure 10. Time history: *a*) $Z_O = Z_O(t)$; *b*) $\dot{Z}_O = \dot{Z}_O(t)$ in the second case.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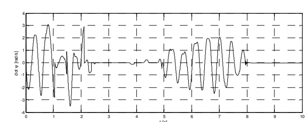
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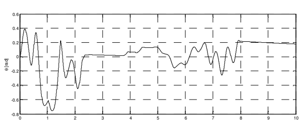
Figure 11. Time history: *a*) $\psi = \psi(t)$; *b*) $\dot{\psi} = \dot{\psi}(t)$ in the second case.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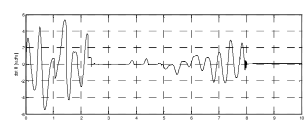
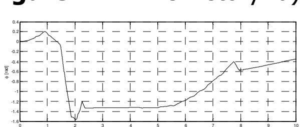

b)

Figure 12. Time history: *a*) $\theta = \theta(t)$; $\dot{\theta} = \dot{\theta}(t)$ in the second case.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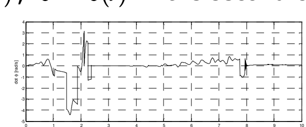
b)

Figure 13. Time history: *a*) $\varphi = \varphi(t)$; *b*) $\dot{\varphi} = \dot{\varphi}(t)$ in the second case.

3. Conclusion

The first case offers only simple and double collisions. An example of a collision with the planes O_0YZ and O_0XZ is at $t = 3.1025\,\text{s}$. A series of successive collisions with the plane O_0YZ appears for $t \in [2.2725, 2.4375]\,\text{s}$. The second case offers triple collision at $t = 3.0825\,\text{s}$.

The shape of the diagrams in Figs. 2–13 suggests the possibility of the appearance of chaotic dynamics in the motion of the ellipsoid. A study concerning this aspect will be the goal of a future paper.

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