Dynamic Modelling of Biped Robot

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Abstract— Goal of dynamics is to obtain equations of motion of biped. Dynamic model of 11 Degree of freedom biped is formulated. Euler - Lagrange formulation combined with homogeneous transformation matrices are used to derive Equations of motion. Jacobian matrixes are the basic elements in building dynamic model. Jacobian matrixes and Euler- Lagrange equations used to determines its dynamic characteristics such as velocity, acceleration and torque. This is necessary for design as well as for estimation of energy consumption of biped. This paper illustrates the mathematical model of the dynamics equations for the legs into the Sagittal and Frontal planes by applying the principle of Lagrangian dynamics.

Keywords— Degree of freedom (DOF), Denavit–Hartenberg (D-H) parameters, Jacobian, Lagrangian.

1. INTRODUCTION

In the last decade rapid growth in use of Humanoid Robotics results in autonomous research field. Humanoid robots are used in all situations of human's everyday life, cooperating with us. They will work in services, in homes and hospitals, and they are even expected to get involved in sports. Hence, they will have to be capable of doing a diversity of tasks. Humanoid robots resembles human-like in their shape and behavior. They have included in number of applications like replacement of humans in hazardous works such as rescue operations, military operations, disaster scenarios, or restoration movement in people with disabilities.

The first dynamically balanced biped was developed by Katoin 1983. It was named as quasi-dynamic due to static walking. This achievement shifted research to from static to dynamic walking [1].In 1984, Miura and Shimoyama [2] Modeled BIPER-3 depicting true active balance. It has only three actuators; one to change the angle separating the legs in the direction of motion, and the remaining two which lifted the legs out to the side in the lateral plane. Placement is done using an inverted pendulum .Later this was modified to the seven degree of-freedom BIPER-4 robot. Raibert developed a planar hopping robot [3].It has a pneumatically driven leg and has three degrees of freedom (pitch motion, and vertical and horizontal translation). A state machine was used to track the current progress.

A dynamic running robot was developed by Hodgins, Koechling and Raibert [4].This robot was constrained to two dimensional motions. Control system has the three important parameters like body height, foot placement and body attitude; these parameters are controlled through the use of a state machine. The robot was controlled depending upon its current state. Research around this time was focused on developing analytical techniques for designing and controlling robot motion. This leads to complex equations governing the motion of the robot which had no solution and been needed to be approximated or linearized. The zero moment point (ZMP) principle with a control system is used by Takanishiet al [5] to achieve dynamic stability of seven link robot. McGeer [6] showed that a correctly designed biped walker with no actuation and no control could walk down gentle slopes. He considered the pair of pendulum that walk naturally as wheel rolls.

Akihito Sano Junji Furusho [7] used angular momentum of the whole system for feedback-controlled walking. But model fails on sloped surfaces. Zheng et al [8] developed methodology for biped robot control .Estimation of the inclination of supporting foot allows robot to transfer from level walking to climbing the slope. But this methodology fails for walking from a level to a negative slope. Kajitaet al [9] restricted the movement of the center of mass (COM) to the horizontal plane to control bipedal dynamic walking. But it was unable to move on uneven surface. Yamaguchi et al. [10] added the feature of a yaw-axis movement to robot WL-12RV. This eliminates the unwanted behavior of the robot to turn at higher velocities that result in 50 percent faster movement than previous robot.

Jong H. Park and Kyoung D. Kim [11] provided solution to move on uneven surface by using gravity compensated inverted pendulum mode (GCIPM).Garcia et al [12] used a double pendulum to show the same. This work concluded that mechanical design is equally, if not more important, than the control method used. This shows that more effort on a correct mechanical system design will simplify the complexity of the control system required. Shadow Robot Group performed research in the United Kingdom and

Developed the Shadow Walker prototype having twelve degrees of freedom. Concept of anthropomorphic design and use of a wooden frame, they have constructed a biped robot using 'air' muscles. Research project done by the University of Waseda in Japan leads to WABIAN Humanoid. They worked on human motion dynamics, human-like mechanism design. Sardain et al [13] generated walking with large velocities.

Zonfrilli et al [14] suggested different biped mechanism to for passive, static, dynamic or purely dynamic walking. Sakagami et al [15] made one of most advanced features biped robot, ASIMO. It process instructions from various types of raw sensory data, able to detect obstacle and identify peoples, Posses map management system for navigation. Its processing is slower due to manipulation of huge database. The compensation for possibility of falling the robot over uneven surfaces is done by Hirukawa et al [16]. Sugahara et al included applications like to take care of elderly people. Zoss et al shows that exoskeletons used for transporting heavy objects. Ekkelen kamp et al design robot for treadmill training so as to reduce physical load on the recovering therapist patient and to offer assistance in leg movements in the forward direction and in keeping lateral balance.

Daan et al implemented pure dynamic approach with open-loop strategy .A predefined time trajectory for the swing leg makes the swing leg move backwards just prior to foot impact. So it moves without requiring local control. Fariz et al [17] had made kinematic formulations by considering position and orientation that results in walking pattern for flat surfaces as well as inclined surfaces.

The research on humanoid biped robot includes various areas such as mechanical design, mathematical modeling, and simulation of biped locomotion. There are many problems that involve kinematics, dynamics, balance and Stability. It makes the study of bipedal robot a complex subject. However, with technological development based on theoretical and experimental research, we have managed to do it. Since the robot have more joints, so the robot has more degrees of freedom. Analysis is related to more variables and the derivation of correlation formulas are also more complex. When we built a humanoid biped robot, we need to make the design of each component by obtaining the mathematical model of each part. The mechanical structure and drive of the biped robot affect their movement speed, power consumption and load directly.

In this paper, we proposed a dynamic model by viewing the kinematic chain of a leg of a biped in forward order. This paper is organized as follows: the forward kinematics for the proposed humanoid robot is obtained using the Denavit-Hartenberg convention.in Section 2. The jacobian matrix which is prerequisite for dynamics is described in Section 3. The discussion about the dynamic model in the Sagittal and Frontal planes using Lagrange equations is in Section 4. Finally, Section 5 presents some important conclusions for biped robot.

2. FORWARD KINEMATICS

Forward kinematics is the task in which the position and orientation of the end-effector is to be determined by giving the configurations for the joints of the robot. The design of biped is based on human body in terms of ratios, body proportions, and range of motion. This paper propose to have sufficient DOF to imitate human motion. The model used consists of 5-links connected through revolute joints, 2-links for each leg and 1-link for torso. The identical legs have hip joint between torso and thigh, knee joints between the thigh and shank, ankle joint between shank and foot, and a rigid body forms the torso. The joint structure of the biped has eleven degrees of freedom, 5 DOF for each leg and 1 DOF for waist or torso. DOF for waist is shared between legs. The Hip joint has 2-DOF, which allows it motion in the sagittal and the lateral plane.

Servos mounted on robot serves as actuators. One servo is mounted on torso, two servos are attached to the hip, one servo is attached to the knee and two servos are attached to the ankle. The mechanical design of the bipedal robot is modular, making it easy to change and replace parts. Forward kinematics is the first step to derive dynamic model of biped. Once the model has been prepared, it is necessary is to attach the frames to each joint. The link frame assignment is the basic requirement of Denavit–Hartenberg(D-H)parameter.

Fig 1: Basic model of biped

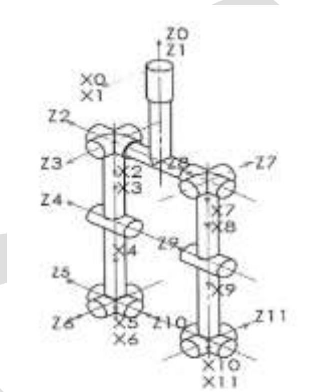

Fig 2: D-H Parameter frame assignment

A 4×4 transformation matrix relating i frame to i-1 frame is given by,

$$^{i-1}H_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i-1} & \sin\theta_i \sin\alpha_{i-1} & a_{i-1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i-1} & -\cos\theta \sin\alpha_{i-1} & a_{i-1}\sin\theta_i \\ 0 & \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where,

θ_i = Rotation angle is angle between X_{i-1} and X_i measured about Z_i.

α_{i-1} = Twist angle is angle between lines along joints i-1 and i measured about common perpendicular X_{i-1}.

a_{i-1} = link length is the distance between the lines along joints i-1 and i along common perpendicular.

d_i = link offset is distance along Z_i from line parallel to X_{i-1} to the line parallel to X_i and are called as Denavit-hartenberg (D-H) parameters.

These are known as D -H parameters and are used to calculate transformation matrix of one link with respect to previous link. Homogeneous transformation from one link to another link is obtained by multiplying continuous chain of matrixes form one to another link.

3. JACOBIAN MATRIX

In this we are interested to derive the velocity relationships that relate the linear and angular velocities of the end-effector to the joint velocities. We will find the angular velocity of the end-effector frame which gives the rate of rotation of the frame and the linear velocity of the origin. Then we relate these velocities to the joint velocities. Jacobian matrix forms the basic elements in building a dynamic model of biped walking. On the basis of motion i.e rectilinear or rotary, the jacobian matrixes are divided as linear or revolutary. In this design, all joints are revolute, so general form of matrix can be written as,

$$J_i = \begin{bmatrix} J\nu_i \\ J\omega_i \end{bmatrix} = \begin{bmatrix} \frac{Z_{i-1} \times (On - O_{i-1})}{Z_{i-1}} \end{bmatrix} \quad \text{........ (1)}$$

Masses are considered as two concentrated material points such as thigh, shin or leg. We can define the dynamic system as figure 3.

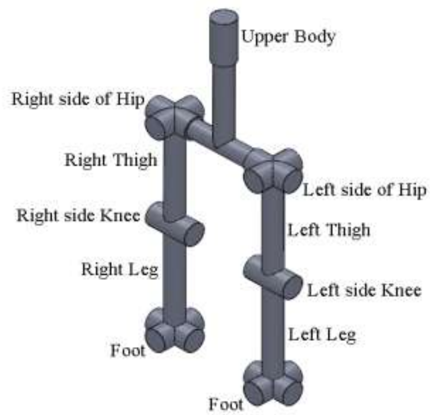

Fig 3: Masses and joints for thigh and shin or legs.

Each mass needs a jocobian matrix J_4 and J_6 means jacobian of thigh and shin. By separating this jacobian matrix into rectilinear and rotary elements, jacobian matrix can be written as shown below. Linear and rotary components of jacobian of thigh are given by,

$$J\nu_4 = [Z_0 \times (O_4c - O_0) \quad Z_1 \times (O_4c - O_1) \quad Z_2 \times (O_4c - O_2) \quad Z_3 \times (O_4c - O_3) \quad 0 \quad 0] \quad \text{........ (2)}$$

$J\nu_4$ = [[- l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)),- l_4 s(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), 0, 0],[l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)), l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2)),- l_4 s(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1)), 0, 0],[0, 0, 0, - l_4 c(θ_3) (c(θ_1) s(θ_2) + c(θ_2) s(θ_1))² - l_4 c(θ_3) (c(θ_1) c(θ_2) - s(θ_1) s(θ_2))², 0, 0]]

$$J\omega_4 = [Z_0 \quad Z_1 \quad Z_2 \quad Z_3 \quad 0 \quad 0\,] \quad \text{........ (3)}$$

$J\omega_4$ = [[0, 0, 0, - c(θ_1) s(θ_2) - c(θ_2) s(θ_1), 0, 0],[0, 0, 0, c(θ_1) c(θ_2) - s(θ_1) s(θ_2), 0, 0][1, 1, 1,0, 0, 0]]

Similarly, Linear and rotary components of jacobian of shin can be calculated as,

$$J\nu_6 = [Z \times (O_6c - O_0) \quad Z_1 \times (O_6c - O_1) \quad Z_2 \times (O_{64}c - O_2) \quad Z_3 \times (O_6c - O_3) \quad Z_4 \times (O_6c - O_4) \quad 0]$$

$$J\omega_{\,6} = [Z_0 \quad Z_1 \quad Z_2 \quad Z_3 \quad Z_4 \quad 0\,]$$

Where,

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} {}^0H_1(1,3) \\ {}^0H_1(2,3) \\ {}^0H_1(3,3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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$$Z_2 = \begin{bmatrix} {}^0H_2(1,3) \\ {}^0H_2(2,3) \\ {}^0H_2(3,3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} {}^0H_3(1,3) \\ {}^0H_3(2,3) \\ {}^0H_3(3,3) \end{bmatrix} = \begin{bmatrix} -c(\theta_1)s(\theta_2) - c(\theta_2)s(\theta_1) \\ c(\theta_1)c(\theta_2) - s(\theta_1)\, s(\theta_2) \\ 0 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} {}^0H_4(1,3) \\ {}^0H_4(2,3) \\ {}^0H_4(3,3) \end{bmatrix} = \begin{bmatrix} -c(\theta_1)\, s(\theta_2) - c(\theta_2)\, s(\theta_1) \\ c(\theta_1)c(\theta_2) - s(\theta_1)s(\theta_2) \\ 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} {}^0H_1(1,4) \\ {}^0H_1(2,4) \\ {}^0H_1(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} {}^0H_2(1,4) \\ {}^0H_2(2,4) \\ {}^0H_2(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} {}^0H_3(1,4) \\ {}^0H_3(2,4) \\ {}^0H_3(3,4) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_4 = O_4c = \begin{bmatrix} {}^0H_4(1,4) \\ {}^0H_4(2,4) \\ {}^0H_4(3,4) \end{bmatrix} = \begin{bmatrix} l_4\, c(\theta_3)(c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) \\ l_4\, c(\theta_3)\, (c(\theta_1)s(\theta_2) + c(\theta_2)s(\theta_1)) \\ -l_4\, s(\theta_3) \end{bmatrix}$$

$$O_6c = \begin{bmatrix} {}^0H_6(1,4) \\ {}^0H_6(2,4) \\ {}^0H_6(3,4) \end{bmatrix}$$

= [{l5 $(c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, c(\theta_2)-s(\theta_1)\, s(\theta_2))\, s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, c(\theta_2)s(\theta_1)\, s(\theta_2)))+ l_4\, c(\theta_3)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2))$},{l5 $(c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, s(\theta_2)+c(\theta_2)\, s(\theta_1))s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))) + l_4\, c(\theta_3)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))$},{- l5 $(c(\theta_3)\, s(\theta_4) + c(\theta_4)\, s(\theta_3)) - l_4\, s(\theta_3)$}]

4. LAGRANGE FORMULATION

The aim to solve dynamics is to obtain equation of motion of system. Because of multiple degree of freedom system, it is difficult to obtain equation of motion. In this paper, principles of Lagrangian dynamics is used for determining the gait locomotion equations for obtaining the torque in each joint of the biped. By representing variables of system as generalized coordinate, we can write equation of motion for an n-DOF system using Euler-Lagrange Equation as,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \qquad \text{…….. (4)}$$

$$L = K - P \qquad \text{…….. (5)}$$

Where L is Lagrangian, K is kinetic energy and P is potential energy.

The kinetic energy of a rigid body is sum of two terms.

$K=\frac{1}{2}mv^{T}v+\frac{1}{2}\omega^{T}I\omega$ …….. (6)

The inertia tensor is required to be transferred into global coordinate, so equation 6 should be multiplied by rotational transfer matrix R.

$K=\frac{1}{2}mv^{T}v+\frac{1}{2}\omega^{T}RIR^{T}\omega$ …….. (7)

Total kinetic energy is sum of each links.

$K=\sum_{i=1}^{n}\left\{\frac{1}{2}m_i v_i{}^{T}v_i+\frac{1}{2}\omega_i{}^{T}R_i I_i R_i{}^{T}\omega_i\right\}$ …….. (8)

By using the Jacobian matrix, the kinetic energy can be written as the function of the joint variables like Equation 9

$K=\frac{1}{2}\ \dot{q}^{T}[\sum_{i=1}^{n}\{m_i\ J\nu_i\ (q)^{T}J\nu_i(q)+J\omega_i(q)^{T}R_i(q)I_iR_i(q)^{T}J\omega_i(q)\}]\dot{q}$ …….. (9)

$K=\frac{1}{2}\ \dot{q}^{T}\{m_4\ J\nu_4(q)^{T}J\nu_4(q)+m_6\ J\nu_6(q)^{T}J\nu_6(q)+\ J\omega_4(q)^{T}R_4(q)I_4R_4(q)^{T}J\omega_4(q)+J\omega_6(q)^{T}R_6(q)I_6R_6(q)^{T}J\omega_6(q)\}\ \dot{q}$ ….. (10)

Inertia matrix $D(q)$ can be given by equation 11

$D(q)=m_4\ J\nu_4(q)^{T}J\nu_4(q)+m_6\ J\nu_6(q)^{T}J\nu_6(q)+\ J\omega_4(q)^{T}R_4(q)I_4R_4(q)^{T}J\omega_4(q)+J\omega_6(q)^{T}R_6(q)I_6R_6(q)^{T}J\omega_6(q)\}$ ….. (11)

Kinetic energy can be written as,

$K=\frac{1}{2}\ \dot{q}^{T}\ D(q)\ \dot{q}=\frac{1}{2}\sum_{j=1}^{n}d_{ij}(q)\,)\dot{q}_i\ \dot{q}_j$ …….. (12)

Where D(q) is 6×6 symmetric matrix. Equation 13 shows its elements.

$$D(q)=\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ * & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ * & * & d_{33} & d_{34} & d_{35} & d_{36} \\ * & * & * & d_{44} & d_{45} & d_{46} \\ * & * & * & * & d_{55} & d_{56} \\ * & * & * & * & * & d_{66} \end{bmatrix}$$ …….. (13)

Potential Energy of leg is,

$P=m\ g\ h=\sum_{i=1}^{n}m_i\ g\ h_{ci}$ …….. (14)

Lagrangian L is the function of the joint variables given by Equation 15.

$L=K-P=\frac{1}{2}\ \dot{q}^{T}\ D(q)\ \dot{q}-P(q)=\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}d_{ij}(q)\,)\dot{q}_i\ \dot{q}_j-\sum_{i=1}^{n}m_i\ g\ h_{ci}(q)$ …….. (15)

The partial derivatives of the Lagrangian with respect to the velocity is,

$$\frac{\partial L}{\partial \dot{q}k}=\frac{\partial}{\partial \dot{q}k}\left(\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}d\,ij\ (q)\ )\dot{q}i\ \dot{q}j\right)-\frac{\partial}{\partial \dot{q}k}\ P(q)=$$

$\sum_{j=1}^{n}dkj(q)\dot{q}j$ …….. (16)

Differential of equation 16 is,

$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}k}=\sum_{j=1}^{n}dkj\ \ddot{q}j+\sum_{j=1}^{n}\frac{d}{dt}\ dkj\ \dot{q}j=\sum_{j=1}^{n}dkj\ \ddot{q}j+\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial dkj}{\partial qi}\ \dot{q}i\ \dot{q}j$ …….. (17)

The partial derivatives of the Lagrangian with respect to the position is

$$\frac{\partial L}{\partial qk} = \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{\partial ij}{\partial qk} \dot{q}i\, \dot{q}j - \frac{\partial P}{\partial k} \quad …….. (18)$$

Euler-Lagrangian equation can be obtained by substraction of equation 18 from equation 17

$$\sum_{j=1}^{n} dkj\, \ddot{q}j + \sum_{j=1}^{n} \sum_{i=1}^{n} \left\{\frac{\partial dkj}{\partial qi} - \frac{1}{2}\frac{\partial dij}{\partial qk}\right\} \dot{q}i\, \dot{q}j + \frac{\partial P}{\partial qk} = \tau_k \quad …….. (19)$$

If we define the Christoffel symbols C_{ijk} and gravity force $g_k(q)$ as in Equation and

$$C_{ijk} = \frac{1}{2}\left(\frac{\partial dkj}{\partial qi} + \frac{\partial dki}{\partial qj} - \frac{\partial dij}{\partial qk}\right) \quad …….. (20)$$

$$g_k(q) = \frac{\partial P(q)}{\partial qk} \quad …….. (21)$$

Equations of motion are given by equation 22,

$$\sum_{j=1}^{6} dkj(q)\, \ddot{q}j + \sum_{j=1}^{6} \sum_{i=1}^{6} Cijk(q)\, \dot{q}i\, \dot{q}j + gk(q) = \tau_k \quad …….. (22)$$

The second term of equation 22, $\sum_{j=1}^{6} \sum_{i=1}^{6} Cijk(q)\, \dot{q}i\, \dot{q}j$ has two meanings. When i □□j, term indicates centrifugal force. When i □□j, term indicates Coriolis Effect. Since the product of inertia is much smaller than moment of inertia, Coriolis effect can be disregarded. So the equations of motion can written as equation 23.

$$\tau_1 = d_{11}\ddot{\theta}_1 + d_{12}\ddot{\theta}_2 + d_{13}\ddot{\theta}_3 + d_{14}\ddot{\theta}_4 + d_{15}\ddot{\theta}_5 + d_{16}\ddot{\theta}_6 + C_{111}\dot{\theta}_1^2 + C_{221}\dot{\theta}_2^2 + C_{331}\dot{\theta}_3^2 + C_{441}\dot{\theta}_4^2 + C_{551}\theta_5^2 + C_{661}\theta_6^2 + g_1.$$

$$\tau_2 = d_{21}\ddot{\theta}_1 + d_{22}\ddot{\theta}_2 + d_{23}\ddot{\theta}_3 + d_{24}\ddot{\theta}_4 + d_{25}\ddot{\theta}_5 + d_{26}\ddot{\theta}_6 + C_{112}\dot{\theta}_1^2 + C_{222}\dot{\theta}_2^2 + C_{332}\dot{\theta}_3^2 + C_{442}\dot{\theta}_4^2 + C_{552}\theta_5^2 + C_{662}\theta_6^2 + g_2.$$

$$\tau_3 = d_{31}\ddot{\theta}_1 + d_{32}\ddot{\theta}_2 + d_{33}\ddot{\theta}_3 + d_{34}\ddot{\theta}_4 + d_{35}\ddot{\theta}_5 + d_{36}\ddot{\theta}_6 + C_{113}\dot{\theta}_1^2 + C_{223}\dot{\theta}_2^2 + C_{333}\dot{\theta}_3^2 + C_{443}\dot{\theta}_4^2 + C_{553}\theta_5^2 + C_{663}\theta_6^2 + g_3.$$

$$\tau_4 = d_{41}\ddot{\theta}_1 + d_{42}\ddot{\theta}_2 + d_{43}\ddot{\theta}_3 + d_{44}\ddot{\theta}_4 + d_{45}\ddot{\theta}_5 + d_{46}\ddot{\theta}_6 + C_{114}\dot{\theta}_1^2 + C_{224}\dot{\theta}_2^2 + C_{334}\dot{\theta}_3^2 + C_{444}\dot{\theta}_4^2 + C_{554}\theta_5^2 + C_{664}\theta_6^2 + g_4.$$

$$\tau_5 = d_{51}\ddot{\theta}_1 + d_{52}\ddot{\theta}_2 + d_{53}\ddot{\theta}_3 + d_{54}\ddot{\theta}_4 + d_{55}\ddot{\theta}_5 + d_{56}\ddot{\theta}_6 + C_{115}\dot{\theta}_1^2 + C_{225}\dot{\theta}_2^2 + C_{335}\dot{\theta}_3^2 + C_{445}\dot{\theta}_4^2 + C_{555}\theta_5^2 + C_{665}\theta_6^2 + g_5.$$

$$\tau_6 = d_{61}\ddot{\theta}_1 + d_{62}\ddot{\theta}_2 + d_{63}\ddot{\theta}_3 + d_{64}\ddot{\theta}_4 + d_{65}\ddot{\theta}_5 + d_{66}\ddot{\theta}_6 + C_{116}\dot{\theta}_1^2 + C_{226}\dot{\theta}_2^2 + C_{336}\dot{\theta}_3^2 + C_{446}\dot{\theta}_4^2 + C_{556}\theta_5^2 + C_{666}\theta_6^2 + g_6 ……… (23)$$

Element of inertia matrix is,

$$d_{11} = (I_{xx4}\, (c(\theta_3)\, (1 - l_4^2\, l_5^2\, ((l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))) + l_4\, c(\theta_3)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)))^2/2 + (l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2))) + l_4\, c(\theta_3)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\, (c(\theta_3)\, s(\theta_4) + c(\theta_4)\, s(\theta_3)) + l_4\, s(\theta_3))^2/2)^2)^{½} + l_4\, l_5\, s(\theta_3)\, ((l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))) + l_4\, c(\theta_3)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)))^2/2 + (l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2))) + l_4\, c(\theta_3)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\, (c(\theta_3)\, s(\theta_4) + c(\theta_4)\, s(\theta_3)) + l_4\, s(\theta_3))^2/2)) - Iyx4\, (s(\theta_3)\, (1 - l_4^2\, l_5^2\, ((l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))) + l_4\, c(\theta_3)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)))^2/2 + (l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2))) + l_4\, c(\theta_3)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\, (c(\theta_3)\, s(\theta_4) + c(\theta_4)\, s(\theta_3)) + l_4\, s(\theta_3))^2/2)\, ^2)^{½} - l_4\, l_5\, c(\theta_3)\, ((l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1))) + l_4\, c(\theta_3)\, (c(\theta_1)\, s(\theta_2) + c(\theta_2)\, s(\theta_1)))^2/2 + (l_5\, (c(\theta_3)\, c(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)) - s(\theta_3)\, s(\theta_4)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2))) + l_4\, c(\theta_3)\, (c(\theta_1)\, c(\theta_2) - s(\theta_1)\, s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5\, (c(\theta_3)\, s(\theta_4) + c(\theta_4)\, s(\theta_3)) + l_4\, s(\theta_3))^2/2))).$$

Similarly all other $d_{12}, d_{13}, d_{14}, d_{15}, d_{16}, d_{21}, d_{22}, d_{23}, d_{24}, d_{25}, d_{26}, d_{31}, d_{32}, d_{33}, d_{34}, d_{35}, d_{36}, d_{41}, d_{42}, d_{43}, d_{44}, d_{45}, d_{46}, d_{51}, d_{52}, d_{53}, d_{54}, d_{55}, d_{56}, d_{61}, d_{62}, d_{63}, d_{64}, d_{65}, d_{66}$ can be found out.

Each element of the Christoffel symbols is,

$$C_{113} = \frac{1}{2}\left\{\frac{\partial d31}{\partial q1} + \frac{\partial d31}{\partial q1} - \frac{\partial d11}{\partial q3}\right\} = \frac{\partial d13}{\partial q1} - \frac{1}{2}\frac{\partial d11}{\partial q3}$$

$$= ((I_{xx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2) + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) + c(\theta_4) s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) - (l_5 (c(\theta_3) c(\theta_4) - s(\theta_3) s(\theta_4)) + l_4 c(\theta_3)) (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3)) + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) + c(\theta_4) s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))) - (l_4^2 l_5^2 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) + c(\theta_4) s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) - (l_5 (c(\theta_3) c(\theta_4) - s(\theta_3) s(\theta_4)) + l_4 c(\theta_3)) (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3)) + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) (l_5 (c(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) + c(\theta_4) s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))/(1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))$$

Equation 14 is about potential energy which is function of joint angles and is given by,

$$P = m_4\, g\, (h_0 - l_{4}c\, c(\theta_2)\, c(\theta_3)) + m_6\, g\, (h_0 - c(\theta_2)(l_4\, c(\theta_3) + l_{6c}\, c(\theta_3 + \theta_4)$$

$$P = (981\, m_4 (h_0 - l_{4}c\, c(\theta_2)\, c(\theta_3)))/100 + (981\, m_6 (h_0 - c(\theta_2) (l_{6c}\, c(\theta_3 + \theta_4) + l_4\, c(\theta_3))))/100$$

Element of gravity force is given by,

$$g_2 = \frac{\partial P}{\partial q2} = (981\, m_6\, s(\theta_2) (l_{6c}\, c(\theta_3 + \theta_4) + l_4\, c(\theta_3)))/100 + (981\, l_{4c}\, m_4\, c(\theta_3)\, s(\theta_2))/100$$

Similarly each element of gravity force g_1, g_3, g_4, g_5, g_6 can be calculated.

Torque required for yaw motion of torso is given by,

$$\tau_1 = \dot{\theta}_1 ((I_{xx4} (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - I_{yx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))).$$

Torque required for rolling motion at hip joint is given by,

$$\tau_2 = \dot{\theta}_1 ((I_{xx4} (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - I_{yx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^2 - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))$$

$+ l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)))$.

Torque required for pitching motion at hip joint is given by,

$\tau_3 = \ddot{\theta}_1 ((I_{xx4} (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))+ l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2) ^2)^{1/2} + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - I_{yx4} (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}) \theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - s(\theta_5) (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 -l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))))$.

Torque required for pitching motion at knee joint is given by,

$\tau_4= \ddot{\theta}_4 ((c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) (s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))) + c(\theta_2) s(\theta_1)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - (c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}+ l_4 l_5 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)))$.

Torque required for pitching motion at ankle joint is given by,

$$\tau_5 = \ddot{\theta}_4 ((c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}) (c(\theta_5) (c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - s(\theta_5) (s(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2))) - (c(\theta_5) (c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)).$$

Torque required for rolling motion at ankle joint is given by,

$$\tau_6 = \dot{\theta}_1 ((1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}) + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2}) + ((I_{yz6}\ \mathrm{pi}\ c((l_4 c(\theta_4) c(\theta_5) s(\theta_6) - l_4 s(\theta_4) s(\theta_5) s(\theta_6) + l_5 c(\theta_4)^2 c(\theta_5) s(\theta_6) + l_5 c(\theta_5) s(\theta_4)^2 s(\theta_6))/(c(\theta_4)^2 c(\theta_5)^2 c(\theta_6)^2 + c(\theta_4)^2 c(\theta_5)^2 s(\theta_6)^2 + c(\theta_4)^2 c(\theta_6)^2 s(\theta_5)^2 + c(\theta_5)^2 c(\theta_6)^2 s(\theta_4)^2 + c(\theta_4)^2 s(\theta_5)^2 s(\theta_6)^2 + c(\theta_5)^2 s(\theta_4)^2 s(\theta_6)^2 + c(\theta_6)^2 s(\theta_4)^2 s(\theta_5)^2 + s(\theta_4)^2 s(\theta_5)^2 s(\theta_6)^2)) c((\mathrm{pi}\ s((l_4 c(\theta_4) c(\theta_5) s(\theta_6) - l_4 s(\theta_4) s(\theta_5) s(\theta_6) + l_5 c(\theta_4)^2 c(\theta_5) s(\theta_6) + l_5 c(\theta_5) s(\theta_4)^2 s(\theta_6))/(c(\theta_4)^2 c(\theta_5)^2 c(\theta_6)^2 + c(\theta_4)^2 c(\theta_5)^2 s(\theta_6)^2 + c(\theta_4)^2 c(\theta_6)^2 s(\theta_5)^2 + c(\theta_5)^2 c(\theta_6)^2 s(\theta_4)^2 + c(\theta_4)^2 s(\theta_5)^2 s(\theta_6)^2 + c(\theta_5)^2 s(\theta_4)^2 s(\theta_6)^2 + c(\theta_6)^2 s(\theta_4)^2 s(\theta_5)^2 + s(\theta_4)^2 s(\theta_5)^2 s(\theta_6)^2)))/2) (c(\theta_5) (c(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} + l_4 l_5 s(\theta_3) ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)) - s(\theta_5) (s(\theta_3) (1 - l_4^2 l_5^2 ((l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)) - s(\theta_3) s(\theta_4) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1))) + l_4 c(\theta_3) (c(\theta_1) s(\theta_2) + c(\theta_2) s(\theta_1)))^2/2 + (l_5 (c(\theta_3) c(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)) - s(\theta_3) s(\theta_4) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2))) + l_4 c(\theta_3) (c(\theta_1) c(\theta_2) - s(\theta_1) s(\theta_2)))^2/2 - l_4^2/2 - l_5^2/2 + (l_5 (c(\theta_3) s(\theta_4) + c(\theta_4) s(\theta_3)) + l_4 s(\theta_3))^2/2)^2)^{1/2} - l_4 l_5 c(\theta_3)).$$

5. CONCLUSION

Biped which had been previously modeled have less number of DOF, which restrict the motion of biped in working environment. Also the mathematical model which had been used, take into account either kinematic or dynamic approach. Both approaches were not considered. In this work, design of 11 DOF bipeds is done considering kinematic and dynamic aspects.

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