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ON RIGHT TERNARY Γ-IDEALS OF TERNARY Γ-SEMIRING

M. SAJANI LAVANYA[1], D. MADHUSUDHANA RAO[2] & VB SUBRAHMANYESWARA RAO SEETMRAJU [3]

[1]Lecturer, Department of Mathematics, A. C. College, Guntur, A.P. India

[2]Selection Grade Lecturer, Department of Mathematics, V. S. R & N.V.R. College, Tenali, A.P. India

[3] Associate Professor, Department of Mathematics, V K R, V N B & A G K College of Engineering, Gudivada, A.P. India

ABSTRACT

In this paper we introduce the concepts of a right weakly regular Ternary Γ-semiring and a fully prime right Ternary Γ-semiring.

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KEYWORDS: Ternary Γ-semiring, Fully Prime Right Ternary Γ-semiring, Right Weakly Regular Ternary Γ-semiring, Semiprime Right Ternary Γ-ideal, Strongly Irreducible Right Ternary Γ-ideal, Maximal Right Ternary Γ-ideal, Right Pure Ternary Γ-ideal

INTRODUCTION

The notion of ternary Γ-Semiring has been introduced by D. Madhusudhana Rao and M. Sajani Lavanya [5] in the year 2015. The notion of Strongly prime ring has been introduced by Handelman and Lawrence [3]. The notion of Ternary Semiring was introduced by T. K. Dutta and S. Kar [1] in the year 2003 as a natural generalization of ternary ring which was introduced by W. G. Lister [4] in 1971. The earlier works of D. Madhusudhana Rao and M. Sajani Lavanya on Ternary Γ-Semiring may be found in [5, 6, 7, 8]. In 2007, T. K. Dutta and M. L. Das [2] introduced and studied right strongly prime Semiring.

2. PRELIMINARIES

- **Definition 2.1[5]:** Let T and Γ be two additive commutative semigroups. T is said to be a ***Ternary Γ-Semiring*** if there exist a mapping from T ×Γ× T ×Γ× T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \to [x_1 \alpha x_2 \beta x_3]$ satisfying the conditions:

  i) $[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$

  ii) $[(a + b)\alpha c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$

  iii) $[a\alpha (b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$

  iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Obviously, every ternary semiring T is a ternary Γ-semiring. Let T be a ternary semiring and Γ be a commutative ternary semigroup. Define a mapping T ×Γ× T ×Γ× T ⟶ T by $a\alpha b\beta c = abc$ for all $a, b, c \in$ T and $\alpha, \beta \in \Gamma$. Then T is a ternary Γ-semiring.

- **Definition 2.2[5]:** An element 0of a ternary Γ-semiring T is said to be an ***absorbing zero*** of T provided $0 + x = x = x + 0$and $0\alpha a\beta b = a\alpha 0\beta b = a\alpha b\beta 0 = 0\ \forall\ a, b, x \in$ T and $\alpha, \beta \in \Gamma$.

- **Definition 2.3[5]** : A ternary Γ-semiring T is said to be ***commutative ternary* Γ*-semiring*** provided $a\Gamma b\Gamma c = b\Gamma c\Gamma a = c\Gamma a\Gamma b = b\Gamma a\Gamma c = c\Gamma b\Gamma a = a\Gamma c\Gamma b$for all $a, b, c \in$ T.

- **Definition 2.4[5]:** A non-empty subset S of a ternary Γ-semiring T is a ***ternary sub Γ-semiring*** if and only if S + S ⊆ S and SΓSΓS ⊆ S.

- **Definition 2.5[5]**: A nonempty subset A of a ternary Γ-semiring T is a ***left ternary Γ-ideal*** of T if and only if A is additive sub semigroup of T and TΓTΓA ⊆ A.

- **Definition 2.6[5]**: A nonempty subset of A of a ternary Γ-semiring T is a ***lateral ternary Γ-ideal*** of T if and only if A is additive sub semigroup of T and TΓAΓT ⊆ A.

- **Definition 2.7[5]**: A nonempty subset A of a ternary Γ-semiring T is a ***right ternary Γ-ideal*** of T if and only if A is additive sub semigroup of T and AΓTΓT ⊆ A.

- **Definition 2.8[5]**: A nonempty subset A of a ternary Γ-semiring T is a ***ternary Γ-ideal*** of T if and only if it is left ternary Γ-ideal, lateral ternary Γ-ideal and right ternary Γ-ideal of T.

3. PRIME RIGHT TERNARY Γ-IDEAL

- **Definition 3.1:** A right ternary Γ-ideal P of ternary Γ-semiring T is said to be a ***prime right ternary Γ-ideal*** provided AΓBΓC ⊆ P implies A ⊆ P or B ⊆ P or C ⊆ P, for any right ternary Γ-ideals A, B and C of T.

- **Theorem 3.2: A right ternary Γ-ideal P of ternary Γ-semiring T is a prime right ternary Γ-ideal of T if and only if $a\Gamma T\Gamma b\Gamma T\Gamma c \subseteq$ P implies a ∈ P or b ∈ P or $c \in$ P, for any $a, b, c \in$ T.**

***Proof*:** Suppose that P is a prime right ternary Γ-ideal of T. Let $a\Gamma T\Gamma b\Gamma T\Gamma c \subseteq$ P, for $a, b, c \in$ T. Then $a\Gamma T\Gamma T\Gamma b\Gamma T\Gamma T\Gamma c\Gamma T\Gamma T \subseteq P \Rightarrow (a\Gamma T\Gamma T)\Gamma(b\Gamma T\Gamma T)\Gamma(c\Gamma T\Gamma T) \subseteq$ P. By $a\Gamma T\Gamma T$, $b\Gamma T\Gamma T$ and $c\Gamma T\Gamma T$ are right ternary Γ-ideals of T and P is a prime right ternary Γ-ideal, $a\Gamma T\Gamma T \subseteq$ P or $b\Gamma T\Gamma T \subseteq$ P or $c\Gamma T\Gamma T \subseteq$ P. Therefore, $a \in$ P or $b \in$ P or $c \in$ P.

Conversely, assume the given statement holds. Let A, B and C be any three right ternary Γ-ideal of T such that AΓBΓC ⊆ P. If A ⊆ P, then the result holds. Suppose that A ⊈ P. Hence, there exists an element $a \in$ A such that a ∉ P. For any $b \in$ B, and $c \in$ C, $a\Gamma T\Gamma b\Gamma T\Gamma c = (a\Gamma T)\Gamma b\Gamma T\Gamma c \subseteq$ AΓBΓC ⊆ P. Therefore, by the assumption $b \in$ P or $c \in$ P implies B ⊆ P or C ⊆ P. Therefore, P is a prime right ternary Γ-ideal of T.

- **Definition 3.3:** A right ternary Γ-ideal P of ternary **Γ**-semiring T is said to be a ***semiprime right ternary Γ-ideal*** Provided AΓAΓA ⊆ P implies A ⊆ P, for any right ternary Γ-ideal A of T.

  Obviously, every prime right ternary Γ-ideal in T is a semiprime right ternary Γ-ideal.

- **Theorem 3.4: A right ternary Γ-ideal P of a ternary Γ-semiring T is a semiprime right ternary Γ-ideal of T if and only if $a\Gamma T\Gamma a\Gamma T\Gamma a \subseteq$ P implies a ∈ P, for any $a \in$ T.**

Proof: Suppose that P is a semiprime right ternary Γ-ideal of T. Let $a\Gamma T\Gamma a\Gamma T\Gamma a \subseteq P$, for $a \in T$. Then, $a\Gamma T\Gamma T\Gamma a\Gamma T\Gamma T\Gamma a\Gamma T\Gamma T \subseteq P \Rightarrow (a\Gamma T\Gamma T)\Gamma(a\Gamma T\Gamma T)\Gamma(a\Gamma T\Gamma T) \subseteq P$. By $a\Gamma T\Gamma T$ is a right ternary Γ-ideal of ternary Γ-semiring T and P is a semiprime right ternary Γ-ideal, $a\Gamma T\Gamma T \subseteq P$. Then $a \in P$.

Conversely, assume given statement holds. Let A be any right ternary Γ-ideal of ternary Γ-semiring T such that $A\Gamma A\Gamma A \subseteq P$. For any $a \in A$, $a\Gamma T\Gamma a\Gamma T\Gamma a = (a\Gamma T)\Gamma(a\Gamma T)\Gamma a \subseteq A\Gamma A\Gamma A \subseteq P$. Therefore, by assumption $a \in P$ implies $A \subseteq P$. Hence P is a semiprime right ternary Γ-ideal of T.

Theorem 3.5: If P is a prime right ternary Γ-ideal of ternary Γ-semiring T, then $(P : a) = \{x \in T : a\Gamma t\Gamma x \subseteq P\}$ is also a prime right ternary Γ-ideal of T for any, $a \in T\backslash P$, $t \in T$.

Proof. Let P be a prime right ternary Γ-ideal of ternary Γ-semiring T and $(P : a) = \{ x \in T : a\Gamma t\Gamma x \subseteq P \}$. Let $x, y \in (P : a)$. Therefore, $a\Gamma t\Gamma x \subseteq P$, $a\Gamma t\Gamma y \subseteq P$. $a\Gamma t\Gamma(x + y) = a\Gamma t\Gamma x + a\Gamma t\Gamma y \subseteq P$ implies $x + y \in (P : a)$. Let $x \in (P : a)$, $t, s \in T$ and $\alpha, \beta \in \Gamma$. Then, $a\Gamma(x\alpha t\beta s) \subseteq a\Gamma(x\Gamma t\Gamma s) = (a\Gamma x\Gamma t)\Gamma s = (a\Gamma t\Gamma x)\Gamma s \subseteq P$ gives $x\alpha t \in (P : a)$. This shows $(P : a)$ is a right ternary Γ-ideal. To show (P : a) is a prime right ternary Γ-ideal, let A, B and C be any three right ternary Γ-ideals of T such that $A\Gamma B\Gamma C \subseteq (P : a)$. Then, $a\Gamma b\Gamma(A\Gamma B\Gamma C) \subseteq P$. $a\Gamma b\Gamma A$, $a\Gamma b\Gamma B$ and $a\Gamma b\Gamma C$ are right ternary Γ-ideals of T. $(a\Gamma b\Gamma A)\Gamma(a\Gamma b\Gamma B)\Gamma(a\Gamma b\Gamma C) \subseteq a\Gamma b\Gamma A\ \Gamma B\Gamma C = a\Gamma b\Gamma(A\Gamma B\Gamma C) \subseteq P$. As P is a prime right ternary Γ-ideal of T, $a\Gamma b\Gamma A \subseteq P$ or $a\Gamma b\Gamma B$ or $a\Gamma b\Gamma C \subseteq P$. Therefore, $A \subseteq (P : a)$ or $B \subseteq (P : a)$ or $C \subseteq (P : a)$, which shows that $(P : a)$ is a prime right ternary Γ-ideal of T.

Lemma 3.6: Every prime right ternary Γ-ideal A of a ternary Γ-semiring T is Semiprime right ternary Γ-ideal of T.

Proof: Suppose that A is a prime right ternary Γ-ideal of a ternary Γ-semiring T. Let X be a right ternary Γ-ideal of T such that $X\Gamma X\Gamma X \subseteq A$. Since A is prime, $X \subseteq A$. Hence A is Semiprime right ternary **Γ**-ideal of T.

The following example will show that there exist semi-prime ternary **Γ**-ideal that are not prime ternary **Γ**-ideal.

Example 3.7: A set Z^+ of non-negative integers and $Z^+ = \Gamma$ is ternary Γ-semiring.

Let < 6 > denote the ternary Γ-ideal generated by $6 \in Z^+$ and P = < 12 > is a ternary Γ-ideal generated by $12 \in Z^+$ For $1 \in Z^+$ it follows that $< 6 > = \{6\alpha n : n \in Z^+, \alpha \in \Gamma\}$.

Since $2 \notin < 6 >$, $3 \notin < 6 >$ and $2 .4.2.5.3 = 240 \in < 6 >$. It is clear that < 6 > is not prime.

The only prime ternary Γ-ideals in Z^+ that contains < 6 > are < 2 >, < 3 > and $\{0\} \cup \{x \in Z^+ : x > 1\}$. Since $< 2 > \cap < 3 > \cap \{x \in Z^+ : x > 1\} \subseteq < 6 >$. It follows that $< 6 > = \sqrt{< 6 >}$. Therefore < 6 > is semi-prime.

Definition 3.8: A right ternary Γ-ideal P of T is said to be an ***irreducible right ternary Γ-ideal*** provided $A \cap B \cap C = P$ implies A = P or B = P or C = P, for any right ternary Γ-ideals A, B and C of T.

Definition 3.9: A right ternary Γ-ideal P of T is said to be a ***strongly irreducible right ternary Γ-ideal*** if $A \cap B \cap C \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$, for any right ternary Γ-ideals A, B and C of T.

The necessary condition for a right ternary Γ-ideal to be prime is given in the following theorem.

Theorem 3.10: Every semiprime and strongly irreducible right ternary Γ-ideal is a prime right ternary Γ-

ideal of ternary Γ-semiring T.

Proof: Let P be a strongly irreducible and a semiprime right ternary Γ-ideal of ternary Γ-semiring T. For any right ternary Γ-ideals A, B and C of T, (AΓBΓC) ⊆ P. A∩B⋂C is a right ternary Γ-ideal of T. Hence (A∩B⋂C)Γ(A∩B⋂C) ⊆ A∩B⋂C ⊆ P. By P is a semiprime right ternary Γ-ideal, A∩B⋂C ⊆ P. Therefore, A ⊆ P or B ⊆ P or C ⊆ P, since P is a strongly irreducible right ternary Γ-ideal. Thus P is a prime right ternary Γ-ideal of T.

Definition 3.11 A proper ternary Γ-ideal M of ternary Γ-semiring T is said to be a ***maximal ternary Γ-ideal*** provided there does not exist any other proper ternary Γ-ideal of T containing M properly.

Theorem 3.12: Any maximal right ternary Γ-ideal of ternary Γ-semiring T is a prime right ternary Γ-ideal.

Proof: Let M be any maximal ternary Γ-ideal of T. To show that M is a prime let *a*ΓTΓ*b*ΓC ⊆ M. Suppose that *a* ∉ M. *a*ΓTΓT is a right ternary Γ-ideal of T which contains an element *a*. By M is a maximal right ternary Γ-ideal, M + *a*ΓTΓT = T. As 1 ∈ S, $1 = m + \sum_i a\alpha_i x_i \beta_i y_i$. Then, $1\alpha b\beta c$ = $m\alpha b\beta c$ + ($\sum_i a\alpha_i x_i \beta_i y_i$)$\alpha b\beta c$ ⊆ M + *a*ΓTΓ*b*Γ*c* ⊆ M. Therefore, *b*, *c* ∈ M. This shows that M is a prime ternary Γ-ideal.

Theorem 3.13: If R is a right ternary Γ-ideal of ternary Γ-semiring T and *a* is a nonzero element of T such that *a* ∉ R, then there exists an irreducible right ternary Γ-ideal P of T such that R ⊆ P and *a* ∉ P.

Proof: Let B be the family of all right ternary Γ-ideals of S containing I and not containing an element *a*. Then B is nonempty as R ∈ B. This family of all right ternary Γ-ideals of T forms a partially ordered set under the inclusion of sets. Hence, by Zorn's lemma there exists a maximal right ternary Γ-ideals P in B. Therefore, R ⊆ P and *a* ∉ P. Now, to show that P is an irreducible right ternary Γ-ideal of T let A, B and C be any three right ternary Γ-ideals of T such that A∩B⋂C = P. Suppose that A, B and C are contained in P properly. Since P is a maximal right ternary Γ-ideal in B, we get *a* ∈ A, *a* ∈ B and *a* ∈ C. Therefore, *a* ∈ A∩B⋂C = P which is an absurd. Thus, either A = P or B = P or C = P. Therefore, P is an irreducible right ternary Γ-ideal of T.

Theorem 3.14: Any proper right Γ-ideal of T is the intersection of irreducible right Γ-ideal of T which contain it.

Proof. Let R be any proper ternary Γ-ideal of T and $\{X_I / I \in \Delta\}$be a family of irreducible right ternary Γ-ideals of T which contain R, where Δ denotes the indexed set. Then clearly R $\subseteq \cap_I X_I$. To show that $\cap_I X_I \subseteq$ R. Suppose that $\cap_I X_I \subset$ R. Therefore, there is an element $a \in \cap_I X_I$ such that *a* ∉ R. Then by theorem 3.13, there exists an irreducible ternary Γ-ideal P such that R ⊆ P and *a* ∉ P. This establishes the existence of irreducible right ternary Γ-ideal P such that *a* ∉ P and R ⊆ P. Therefore, $a \notin \cap_I X_I$ for every *a* ∉ R. Hence, by the contrapositive method $\cap_I X_I \subseteq$ R. Therefore $\cap_I X_I$ = R.

4. RIGHT WEAKLY REGULAR TERNARY Γ-SEMIRING

Definition 4.1: A ternary Γ-semiring T is said to be ***right weakly regular*** if *a* ∈ (*a*ΓT**Γ**T)**Γ**(*a*ΓT**Γ**T) Γ(*a*ΓT**Γ**T), for any *a* ∈ T.

In the following theorems we characterize for a right weakly regular ternary Γ-semiring.

Theorem 4.2: In the ternary Γ-semiring T, the following statements are equivalent.

(1) T is right weakly regular.

(2) RΓRΓR = R, for each right ternary Γ-ideal R of T.

(3) R∩I = RΓIΓI, for any right ternary Γ-ideal R and ternary Γ-ideal I of T.

Proof: (1) ⇒ (2) Suppose that T is right weakly regular.

For any right ternary Γ-ideal R of T, RΓRΓR ⊆ RΓTΓT ⊆ R.

Conversely, let a ∈ R. As T is right weakly regular, a ∈ (aΓT**Γ**T)**Γ**(aΓT**Γ**T)Γ(aΓT**Γ**T).

Then a ∈ (aΓT**Γ**T)**Γ**(aΓT**Γ**T)Γ(aΓT**Γ**T) ⊆ (RΓTΓT)Γ(RΓTΓT)Γ(RΓTΓT) ⊆ RΓRΓR.

Thus, RΓRΓR = R, for each right ternary Γ-ideal R of T.

(2) ⇒ (1) Suppose that RΓRΓR = R, for each right ternary Γ-ideal R of T. For any a ∈ T, a ∈ aΓT**Γ**T and aΓT**Γ**T is a right ternary Γ-ideal of T.

Therefore, a ∈ (aΓT**Γ**T)**Γ**(aΓT**Γ**T)Γ(aΓT**Γ**T), which shows that T is right weakly regular.

(2) ⇒ (3) Let R be a right ternary Γ-ideal and I be a ternary Γ-ideal of T. Then R∩I⋂I is a right ternary Γ-ideal of T. By assumption (R∩I)Γ(R∩I)⋂(R∩I) ⊆ RΓIΓI.

Clearly, RΓIΓI ⊆ R and RΓIΓI ⊆ I. Therefore, RΓIΓI ⊆ R∩I. Thus we get R∩I = RΓIΓI.

(3) ⇒ (2) Let R be a right ternary Γ-ideal of T and (R) be a ternary Γ-ideal generated by R. Then we write (R) = TΓTΓRΓTΓT. By assumption R∩(R)⋂(R) = RΓ(R) Γ(R).

Then, R = RΓ(TΓTΓRΓTΓT)Γ(TΓTΓRΓTΓT) = (RΓTΓT)Γ(RΓTΓTΓTΓT)Γ(RΓTΓT)

= (RΓTΓT)Γ(RΓTΓT)Γ(RΓTΓT) ⊆ RΓRΓR = R. Therefore, RΓRΓR = R.

Theorem 4.3: A ternary Γ-semiring T is right weakly regular if and only if every right ternary Γ-ideal of T is semiprime.

Proof: Suppose that T is right weakly regular. Let R be a right ternary Γ-ideal of T such that AΓAΓA ⊆ R, for any right ternary Γ-ideal A of T. A = AΓAΓA as T is right weakly regular. Therefore, A ⊆ R. Hence R is a semiprime right ternary Γ-ideal of T.

Conversely, suppose that every right ternary Γ-ideal of T is semiprime. Let R be right ternary Γ-ideal of T. RΓRΓR is also a right ternary Γ-ideal of T. By assumption RΓRΓR is a semiprime right ternary Γ-ideal of T. RΓRΓR ⊆ RΓRΓR implies R ⊆ RΓRΓR. Therefore, RΓRΓR = R. Hence, T is right weakly regular.

Theorem 4.4: If ternary Γ-semiring T is right weakly regular, then a ternary Γ-ideal P of T is prime if and only if P is irreducible.

Proof: Let T be a right weakly regular ternary Γ-semiring and P be a ternary Γ-ideal of T. If P is a prime ternary Γ-ideal of T, then clearly P is an irreducible ternary Γ-ideal. Suppose that P is an irreducible ternary Γ-ideal of T. To show P is a prime ternary Γ-ideal, let A, B and C be any three ternary Γ-ideals of T such that AΓBΓC ⊆ P. Then, by Theorem

4.2, we have A∩B⋂C ⊆ P. Therefore, (A∩B⋂C)+P = P. But L_T lattice of all ternary Γ-ideals of T being distributive (A + P)⋂(B + P)⋂(A + C) = P. As P is an irreducible ternary Γ-ideal, A+P = P or B+P = P or A + C = P. Then A ⊆ P or B ⊆ P or C ⊆ P. Therefore, P is a prime ternary Γ-ideal of T.

Now we define a fully prime right ternary Γ-semiring and a fully semiprime right ternary Γ-semiring.

Definition 4.5: A ternary Γ-semiring T is said to be a ***fully prime right ternary Γ-semiring*** provided all right ternary Γ-ideals of T are prime right ternary Γ-ideals.

Definition 4.6: A ternary Γ-semiring T is said to be a ***fully semiprime right ternary Γ-semiring*** if all right ternary Γ-ideals of T are semiprime right ternary Γ-ideals.

The relation between a fully prime right ternary Γ-semiring and a right weakly regular ternary Γ-semiring is furnished in the following theorems.

Theorem 4.7: If a ternary Γ-semiring T is a fully prime right ternary Γ-semiring, then T is right weakly regular and the set of ternary Γ-ideals of T is totally ordered.

Proof: Let T be a fully prime right ternary Γ-semiring. Therefore, every right ternary Γ-ideal of T is a prime right ternary Γ-ideal. But every prime right ternary Γ-ideal is a semiprime right ternary Γ-ideal. Hence, by theorem 4.3, T is right weakly regular. Let A, B and C be any three ternary Γ-ideals of T. Then A∩B⋂C is a right ternary Γ-ideal of T. By hypothesis A∩B⋂C is a prime right ternary Γ-ideal of T. AΓBΓC ⊆ A∩B⋂C implies A ⊆ A∩B⋂C or B ⊆ A∩B⋂C or C ⊆ A∩B⋂C. Therefore, A∩B⋂C = A or A∩B⋂C = B or A∩B⋂C = C. Thus we get either A ⊆ B, C or B ⊆ A, C or C ⊆ A, B. Hence, the set of ternary Γ-ideals of T is totally ordered.

Theorem 4.8: If a ternary Γ-semiring T is right weakly regular and the set of ternary Γ-ideals of T is totally ordered, then T is a fully prime right ternary Γ-semiring.

Proof: Let T be a right weakly regular ternary Γ-semiring and the set of ternary Γ-ideals of T is totally ordered. To show that T is a fully prime right ternary Γ-semiring, let P be any right ternary Γ-ideal of T. To prove P is a prime right ternary Γ-ideal of T, let A, B and C be any three ternary Γ-ideals of T such that AΓBΓC ⊆ P. By assumption, either A ⊆ B, C or B ⊆ A, C or C ⊆ A, B and AΓAΓA = A, BΓBΓB = B and CΓCΓC = C. We consider A ⊆ B, C. Then, A = AΓAΓA ⊆ AΓBΓC ⊆ P. Therefore, P is a prime right ternary Γ-ideal of T. Hence, T is a fully prime right ternary Γ-semiring.

Definition 4.9: An element a of a ternary Γ-semiring. T is said to be ***regular*** if there exist $x, y \in$ T and $\alpha, \beta, \gamma, \delta \in$ Γ such that $a\alpha x\beta a\gamma y\delta a = a$.

Definition 4,10: A ternary Γ-semigroup T is said to be ***regular ternary Γ-semiring*** provided every element is regular.

Note 4.11: A ternary Γ-semiring is said to be regular if $a \in a$ΓTΓaΓTΓa, for any $a \in$ T.

Note 4.12: In general, the family of regular ternary Γ-semirings forms a proper subclass of the family of right weakly regular ternary Γ-semirings. But if T is a commutative ternary Γ-semiring, then T is regular ternary Γ-semiring if and only if T is right weakly regular ternary Γ-semiring.

Theorem 4.13: If T is a commutative ternary Γ-semiring, then T is regular if and only if T is right weakly regular.

Proof: Let T be a commutative ternary Γ-semiring. Suppose that T is a right weakly regular ternary Γ-semiring. Therefore, for any $a \in$ T, $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T)$.

$a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T) \subseteq a\Gamma T\Gamma a\Gamma T\Gamma a$. Therefore, T is a regular ternary Γ-semiring. Conversely, suppose T is a regular ternary Γ-semiring. Let $a \in$ T. Hence, $a \in a\Gamma T\Gamma a\Gamma T\Gamma a$. Then, $a \in a\Gamma T\Gamma a\Gamma T\Gamma a \subseteq (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T)$. This shows that T is a right weakly regular ternary Γ-semiring.

Theorem 4.14: Each ternary Γ-ideal of a right weakly regular ternary Γ-semiring T is a right weakly regular (as a ternary Γ-semiring).

Proof: Let R be any ternary Γ-ideal of a right weakly regular ternary Γ-semiring T. Hence R itself is a ternary sub-Γ-semiring of T. For any element $a \in$ R, $a\Gamma R\Gamma R$ is a right ternary Γ-ideal of T. T is a right weakly regular ternary Γ-semiring implies $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T)$ and $(a\Gamma R\mathbf{\Gamma}R)\mathbf{\Gamma}(a\Gamma R\mathbf{\Gamma}R)\Gamma(a\Gamma R\mathbf{\Gamma}R) = a\Gamma R\mathbf{\Gamma}R$. Hence we have, $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\Gamma(a\Gamma T\mathbf{\Gamma}T) = a\Gamma(T\mathbf{\Gamma}T\mathbf{\Gamma}a\Gamma T\mathbf{\Gamma}T)\Gamma(a\Gamma T\mathbf{\Gamma}T) \subseteq a\Gamma(T\Gamma R\Gamma T\Gamma R) \subseteq a\Gamma R\mathbf{\Gamma}R = (a\Gamma R\mathbf{\Gamma}R)\mathbf{\Gamma}(a\Gamma R\mathbf{\Gamma}R)\Gamma(a\Gamma R\mathbf{\Gamma}R)$. Therefore, $a \in (a\Gamma R\mathbf{\Gamma}R)\mathbf{\Gamma}(a\Gamma R\mathbf{\Gamma}R)\Gamma(a\Gamma R\mathbf{\Gamma}R)$ implies R is itself a right weakly regular ternary Γ-semiring.

Bi-ternary Γ-ideals of a ternary Γ-semiring are defined by Sajani Lavanya, Madhusudhana Rao and Syam Julius Rajendra in [7] as follows:

Definition 4.15: A ternary Γ-subsemiring B of a ternary Γ-semiring T is called a ***bi-ternary* Γ*-ideal*** of T if $B\Gamma T\Gamma B\Gamma T B \subseteq B$.

Theorem 4.16: T is right weakly regular if and only if B∩I∩I ⊆ BΓIΓI, for any bi-ideal B and an ideal I of T.

Proof: Suppose that T is a right weakly regular ternary Γ-semiring. Let B be a bi-ternary Γ-ideal and I be a ternary Γ-ideal of T. Let $a \in B\cap I\cap I$. Therefore, $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T)$, since T is a right weakly regular. Then $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T) \subseteq (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\Gamma(a\Gamma T\mathbf{\Gamma}T)\Gamma T\Gamma T \subseteq (B\Gamma T\Gamma B\Gamma T\Gamma B)\Gamma(T\Gamma I\Gamma T\Gamma T\Gamma T\Gamma I) \subseteq B\Gamma I\Gamma I$. Therefore, $B\cap I\cap I \subseteq B\Gamma I\mathbf{\Gamma}I$.

Conversely, suppose that $B\cap I\cap I \subseteq B\Gamma I\Gamma I$, for any bi- ternary Γ-ideal B and a ternary Γ-ideal I of T. Let R be a right ternary Γ-ideal of T. Then R itself a bi- ternary Γ-ideal of T. By assumption $R = R\Gamma(T\Gamma T\Gamma R\Gamma T\Gamma T)\Gamma(T\Gamma T\Gamma R\Gamma T\Gamma T) = (R\Gamma T\Gamma T)\Gamma(R\Gamma T\Gamma T\Gamma T\Gamma T)\Gamma(R\Gamma T\Gamma T)$

$= (R\Gamma T\Gamma T)\Gamma(R\Gamma T\Gamma T)\Gamma(R\Gamma T\Gamma T) \subseteq R\Gamma R\Gamma R = R$. Therefore, $R = R\Gamma R\Gamma R$. Then by Theorem 4.2, T is a right weakly regular ternary Γ-semiring.

Theorem 4.17: A ternary Γ-semiring T is right weakly regular if and only if B∩I∩R ⊆ BΓIΓR, for any bi-ternary Γ-ideal B, a ternary Γ-ideal I and a right ternary Γ-ideal R of T.

Proof: Suppose that T is a right weakly regular ternary Γ-semiring. Let B be a bi-ternary Γ-ideal, I be a ternary Γ-ideal and R be a right ternary Γ-ideal of T. Let $a \in B \cap I \cap R$. Therefore, $a \in (a\Gamma T\mathbf{\Gamma}T)\mathbf{\Gamma}(a\Gamma T\mathbf{\Gamma}T)\,\Gamma(a\Gamma T\mathbf{\Gamma}T)$, since T is a right weakly regular.

Then $a \in (a\Gamma T\boldsymbol{\Gamma}T)\boldsymbol{\Gamma}(a\Gamma T\boldsymbol{\Gamma}T)\Gamma(a\Gamma T\boldsymbol{\Gamma}T) \subseteq (a\Gamma T\boldsymbol{\Gamma}T)\boldsymbol{\Gamma}(a\Gamma T\boldsymbol{\Gamma}T)\Gamma(a\Gamma T\boldsymbol{\Gamma}T)\Gamma T\boldsymbol{\Gamma}T$

$\subseteq B\Gamma(T\Gamma T\Gamma I\Gamma T\Gamma T)\Gamma(R\Gamma T\Gamma T) \subseteq B\Gamma I\Gamma R$. Therefore, $B \cap I \cap R \subseteq B\Gamma I\Gamma R$.

Conversely, suppose $B \cap I \cap R \subseteq B\Gamma I\Gamma R$, for any bi-ternary Γ-ideal B and a ternary Γ-ideal I and a right ternary Γ-ideal R of T. For a right ternary Γ-ideal R of T, R itself being a bi- ternary Γ-ideal and T itself is being a ternary Γ-ideal of T. By assumption $R\cap T \cap R \subseteq R\Gamma T\Gamma R = (R\Gamma T)\Gamma R \subseteq R\Gamma R\Gamma R$. Therefore, $R \subseteq R\Gamma R\Gamma R$. Therefore, $R = R\Gamma R\Gamma R$. Then, by Theorem 4.2, T is a right weakly regular ternary Γ-semiring.

5. RIGHT PURE TERNARY Γ-IDEALS

In this section we define a right pure ternary Γ-ideal of a ternary Γ-semiring T and furnish some of its characterizations.

Definition 5.1: Let T be a ternary Γ-semiring. A ternary Γ- ideal A of T is said to be ***right pure ternary* Γ*-ideal*** if for each $x \in A$ there exist y_i, $z_i \in A$, α_i, $\beta_i \in \Gamma$ where $i \in \Delta$ such that $x = \sum_{i=1}^{n} x\alpha_i y_i \beta_i z_i$. Similarly, we define one-sided right pure ternary Γ-ideals.

Note 5.2: A ternary Γ- ideal A of ternary Γ-semiring T is said to be a right pure ternary Γ- ideal if for any $x \in A$, $x \in x\Gamma A\Gamma A$.

Theorem 5.3: A ternary Γ- ideal I of T is right pure if and only if R∩I = RΓIΓI, for any right ternary Γ-ideal R of T.

Proof: Let I be a right pure ternary Γ- ideal and R be a right ternary Γ- ideal of T. Then clearly $R\Gamma I\Gamma I \subseteq R \cap I$. Now let $a \in R \cap I$, gives $a \in R$ and $a \in I$. As I is a right pure ternary Γ- ideal, $a \in a\Gamma I\Gamma I \subseteq R\Gamma I\Gamma I$. This gives $R \cap I \subseteq R\Gamma I\Gamma I$. By combining both inclusions we get $R\cap I = R\Gamma I\Gamma I$.

Conversely, suppose $R\cap I = R\Gamma I\Gamma I$, for a right ternary Γ- ideal R and a ternary Γ- ideal I of T. Let I be a ternary Γ-ideal of T and $a \in I$. $(a)_r$ denotes the right ternary Γ- ideal generated by a and given by $(a)_r = N_0 a + a\Gamma I\Gamma I$, where N_0 is a set of non-negative integers. Then, $a \in (a)_r\Gamma I\Gamma I = (N_0 a + a\Gamma I\Gamma I)\Gamma I\Gamma I \subseteq a\Gamma I\Gamma I$. Therefore, I is a right pure ternary Γ- ideal of T.

Theorem 5.4: The intersection of right pure ternary Γ- ideals of ternary Γ-semiring T is a right pure ternary Γ- ideal of T.

Proof: Let A and B be right pure ternary Γ- ideals of T. Then for any right ternary Γ- ideal R of T we have, $R \cap A = R\Gamma A\Gamma A$ and $R \cap B = R\Gamma B\Gamma B$ by theorem 5.2, We consider $R\cap(A\cap B) = (R\cap A)\cap B = (R\Gamma A\Gamma A)\cap B = (R\Gamma A\Gamma A)\Gamma B\Gamma B = R\Gamma(A\Gamma A)\Gamma(B\Gamma B) = R\Gamma(A\Gamma B) = R\Gamma(A\cap B)$. Therefore, A∩B is a right pure ternary Γ- ideal of T.

We characterize right weakly regular ternary Γ-semiring in terms of right pure ternary Γ- ideals in the following theorem.

Theorem 5.5: A ternary Γ-semiring T is right weakly regular if and only if any ternary Γ- ideal of T is right pure.

Proof: Suppose that T is a right weakly regular ternary Γ-semiring. Let I be a ternary Γ- ideal and R be a right ternary Γ- ideal of T. Then by Theorem 4.2, R∩I = RΓIΓI. Therefore, a ternary Γ- ideal I of T is right pure by theorem 5.3.

Conversely, suppose that any ternary Γ- ideal of T is right pure. Then, from theorem 5.3 and Theorem 4.2 we get T is a right weakly regular ternary Γ-semiring.

CONCLUSIONS

In this paper, efforts are made to introduce and characterize a right weakly regular ternary Γ-semiring and a fully prime right ternary Γ-semiring.

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