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COMMON FIXED POINT THEOREM OF OCCASIONALLY WEAKLY COMPATIBLE MAPPING IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In this paper, we prove common fixed point theorem in intuitionistic fuzzy metric space with the help of. occasionally weakly compatible mapping.

KEYWORDS: Occasionally Weakly Compatible Mapping, Intuitionistic Fuzzy Metric Space, Weakly Compatible Mapping, Common Fixed Point

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INTRODUCTION

Fuzzy set was defined by Zadeh [1]. The.theory of fixed point is one of the basic tools to handleVarious physical formulations. Later on the concept of fuzzy set was modified by George and Veeramani.[2].Fixed point theory is one of the most dynamic research are in nonlinear analysis. It has a wide range of applicationin the fields such as economics, computer science and many other.

Alaca et. al.[3] in 2006, redefined the notion of Intuitionistic fuzzy metric space as a generalization of KM –fuzzy metric space. Alaca et. al. [3], further proved well known fixed point theorem of Banach [4], in intuitionistic fuzzy metric space with the help of Grabic[5]. Turkogh et.al.[6] introduced the concept of compatible maps amd compatible maps of type (α).and (β).in intuitionistic fuzzy metric space and gave some relation between compatible maps of type.(α).and (β).

Saadati et al [7], Singalotti et. al. [8], Sharma and Deshpande [9] and many other studied the concept of Intuitionistic fuzzy metric space and its applications..Aamri and Moutwakil [10] introduced the property E.A.

Sharma, Kutukcu and Pathak [11] introduced the property (S-B) in intuitionistic fuzzy metric space.

In 2008 Al- Thagafi and N. Shahzad [12] introduced the notion of occasionally.weakly compatible mappings..

PRELIMINARIES

Definition 2.1 [13]

A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$.is continuous t – norm if $*$ satisfies the following conditions :

- is continuous,
- is commutative and associative,

- .a * 1 = a, for all a ∈ [0,1],
- a * b ≤ c * d,whenever a ≤ c and b≤d, for all a,b,c,d ∈ [0,1].

Definition 2.2 [13]

A binary operation ♢ : [0,1] × [0,1] →[0,1].is continuous t- conorm if ♢ satisfies the following conditions :

- ♢ is commutative and associative,
- ♢ is continuous,
- a ♢0 =a,for all a ∈ [0,1],
- a ♢ b≤ c ♢ d, whenever a ≤ c and b≤d, for all a,b,c,d ∈ [0,1].

Remark 2.1

The concept of triangular norms (t- norms) and triangular conorm (t-conorms) are known as the Axiomatic skeletons that we use for characterizing fuzzy intersection and unions, respectively. These concepts were originally introduced by Menger [14] in his stydy of statistical metric spaces, several examples for this concepts where proposed by many authors [15], [16].

Definition 2.3 [3]

A 5- tuple (X,M,N, *, ♢) is said to be an intuitionistic fuzzy metric space.if X is an arbitrary set, * is a continuous t-norm, ♢ is a continuous t –conorm and M, N are fuzzy set on $X^2 \times [0, \infty)$ satisfying the following condition :

- M (x, y, t) + N (x, y, t) ≤ 1, for all x, y.∈ X and t > 0,
- M (x, y, t) = 0, for all x,y.∈ X,
- M(x, y, t) = 1, for all x, y.∈ X and t > 0 if and only if x =y,
- M (x, y, t) = M (y, x, t), for all x, y.∈ X and t > 0,
- M(x,y,t) * M(y,z, s) ≤ M(x,z,t+ s), for all.x,y,z ∈ X and.s,t > 0,
- For all x, y ∈ X, M(x, y,.) : [0,∞) → [0,1] is left continuous,
- $\lim_{t\to\infty}$ M(x,y,t) = 1,.for all.x,y.∈ X and t > 0,
- N(x, y, 0) = 1, for all x, y.∈ X,
- N(x, y, t) = 0, for all x, y.∈ X and t > 0 if and only if x =y,
- N(x,y,t) = N(y,x,t), for all.x,y.∈ X and t > 0,
- N(x,y,t) ♢ N(y,z,s). ≥ N(x,z,t+ s), for all.x,y,z ∈ X and.s,t > 0,
- For all x,y ∈ X, N(x,y,.) : [0,∞) → [0,1] is.right continuous,
- $\lim_{t\to\infty}$ N(x,y,t) = 0,.for all.x,y.∈ X.

(M,N).is called an intuitionistic fuzzy metric space on X. The functions M(x,y,t) and N(x,y,t) denote the degree of non- nearness between x and y with respect to t, respectively.

Remark 2.2 [3]

An intuitionistic fuzzy metric spaces with continuous t- norm * and continuous t- conorm ♢. defined.by a *a $\geq$ a and (1-a) ♢ (1-a) $\leq$ (1-a), for all. a $\in$ [0,1]. Then for all x, y $\in$ X, M(x,y, *).is non decreasing and N(x,y,t) is non-increasing.

Remark 2.3 [3]

Every fuzzy metric space (X,M,*) is an intuitionistic fuzzy metric space if X of the form (X, M, 1-M, *, ♢)such that t –norm * and t- conorm ♢ are associated,i.e. x ♢ y = 1- ((1-x) * (1-y)), for any x, y.$\in$ [0,1]. But the convers is not true.

Example 2.1 [17]

Let (X,d) be a metric space.Define a * b = min {a,b}and t- conorm a♢b = max{a,b}, for all x, y $\in$ X.and t > 0 $M_d(x,y,t) = t / t + d(x,y)$.and $N_d(x,y,t) = d(x,y) / t + d(x,y)$. Then (X, M,N,*,♢) is an intuitionistic fuzzy metric space.

Definition 2.4 [18]

A pair of self mapping (P,Q) of an intuitionistic fuzzy metric space (X,M,N,*,♢).is said to be compatible if. $\lim_{n\to\infty} M(PQx_n, QPx_n,t) = 1$ and.$\lim_{n\to\infty} N(PQx_n, QPx_n,t) = 0$, for every t>0, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$ for some z $\in$ X.

Definition 2.5 [19]

Two self maps P and Q are said to be weakly compatible if they commute at coincidence points.

Definition 2.6 [20]

Let (X,M,N,*,♢).be.an intuitionistic fuzzy metric space and P and Q be self maps on X. A point. x.$\in$ X is called a coincidence point of P and Q iff Px = Qx.

Definition 2.7

A pair of self mapping (P,Q).of an intuitionistic fuzzy metric space (X,M,N,*,♢).is said to be Occasionally weakly compatible if there. is a point in X which is coincidence point of P and Q at which P and Q commute.

Lemma 2.1 [3]

Let (X, M, N,*, ♢). be intuitionistic fuzzy metric space and for all x,y in X, t> 0 and if for a numbers $\in$ (0,1), M(x,y,st) $\leq$ M(x,y,t) and N(x,y, st) $\geq$ N(x,y,t). Then x= y.

Lemma 2.2

Let P and Q be a self maps in an intuitionistic fuzzy metric space. (X,M,N,*,♢).and let P and Q have a unique point of.coincidence, z = Px = Qx, then z is the unique common fixed point of P and Q..

Lemma 2.3 [3]

Let $\{z_n\}$ is a sequence in an intuitionistic fuzzy metric space (X, M, N,*, ◇). If there exists a.constant...s ∈ (0,1) such that

$M(z_n, z_{n+1}, st) \geq M(z_{n-1}, z_n, t)$,

$N(z_n, z_{n+1}, st) \leq N(z_{n-1}, z_n, t)$, for all n = 0,1,2,…

Then $\{z_n\}$ is a Cauchy sequence in X.

MAIN.RESULT

Let A, B, S and T be self maps of a complete intuitionistic fuzzy metric space. (X, M, N,*, ◇) satisfying the following. Conditions:

- $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,
- S and T are continuous,
- There exists k ∈.(0,1) such that for all x,y ∈ X and t>o,

$M(Ax,By,kt) \geq M(Sx,Ty,t) * ½ (M(Sx,Ax,t) + M(Ty,By,t)) * M(Sx,By,t)$

$N(Ax,By,kt) \leq N(Sx,Ty,t) ◇.½ (N(Sx,Ax,t) + N(Ty,By,t)) ◇ N(Sx,By,t)$

If the pairs (A,S).and (B,T) are occasionally weakly compatible mapping on X. Then A,B,S and T have a unique common fixed point on X.

Proof – AS $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$., then let.x_0.∈.X be arbitrary. Now construct a sequence $\{y_n\}$ such.that..$y_{2n+1} = Ax_{2n} = Tx_{2n+1}$., $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$, for n = 0,1,2,…

We first show that $\{y_n\}$ is a cauchy sequence in X.

Now by (3), we get

$M(Ax_{2n},Bx_{2n+1}, kt).= M(y_{2n+1}, y_{2n+2}, kt)$

$= M(Sx_{2n}, Tx_{2n+1}, t) * ½ (M(Sx_{2n,}, Ax_{2n}, t) + M(Tx_{2n+1}, Bx_{2n+1},t)) * M(Sx_{2n}, Bx_{2n+1},t)$

$\geq M(y_{2n}, y_{2n+1},t) *.½ (M(y_{2n},y_{2n},t) + M(y_{2n+1}, y_{2n+1}, t)) * M (y_{2n},y_{2n,}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1},t)$

Similarly, we can get

$M(_{2n+2}, y_{2n+3}, kt) \geq M (y_{2n+1}, y_{2n+2},t)$

Hence, we have

$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n,y_{n+1,}t)$........................………………………………………………………… ………(i)

$N(y_{2n+1}, y_{2n+2}, kt) = N (Ax_{2n}, Bx_{2n+1}, kt)$

$= N(Sx_{2n}, Tx_{2n+1}, t) ◇ ½ (N(Sx_{2n,}, Ax_{2n}, t) + M(Tx_{2n+1}, Bx_{2n+1},t)) ◇ N(Sx_{2n}, Bx_{2n+1},t)$

$\leq N(y_{2n}, y_{2n+1},t) \diamond .½ (M(y_{2n},y_{2n},t) + M(y_{2n+1}, y_{2n+1}, t)) \diamond M (y_{2n},y_{2n}, t)$

$N (y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1},t)$

Similarly, we have

$N(_{2n+2}, y_{2n+3}, kt). \leq .N (y_{2n+1}, y_{2n+2},t)$

Hence, we have

$N(y_{n+1}, y_{n+2}, kt).\leq. N(y_n,y_{n+1,}t)$..…… ……..(ii)

Equation (i) and (ii), shows that $\{y_n\}$ is a Cauchy sequence. Since X is complete then $\{y_n\}$ converges to some point z.∈ X and so the subsequence,

$\{Ax_{2n}\} \rightarrow z, \{Bx_{2n+1}\} \rightarrow z$..........................…………………………………………………….……(iii)

$\{Sx_{2n}\} \rightarrow z, \{Tx_{2n+1}\} \rightarrow z$..…(iv)

Since (A,S) is occasionally weakly compatible mapping,then we have.Az.= Sz., and (B,T) is owc.mapping,then

We have.Bz = Tz.

Now, by (3), we have

$M(Az, Bx_{2n+1},kt) \geq M(Sz,Tx_{2n+!},t) * ½.(M(Sz, Az,t) + M(Tx_{2n+1},Bx_{2n+1},t)) * M(Sz, Bx_{2n+1},t)$

$= M(Az, Tx_{2n+1},t) * ½.(M(Az,Az,t) + M.(Tx_{2n+1},Bx_{2n+1},t)) * M(Az, Bx_{2n+1},t)$

Taking limit $n\rightarrow\infty$,and using (iii) and (iv)., we get

$M(Az,z,kt). \geq M(Az,z,t) * ½.(1, M(z,z,t)) * M(Az, z, t)$

$M(Az,z,kt) \geq M(Az,z,t)$

Similarly, we get

$N(Az, Bx_{2n+1},kt) \leq N(Sz,Tx_{2n+!},t) \diamond ½.(N(Sz, Az,t) + N(Tx_{2n+1},Bx_{2n+1},t)) \diamond N(Sz, Bx_{2n+1},t)$

$= N(Az, Tx_{2n+1},t) \diamond ½.(N(Az,Az,t) + N (Tx_{2n+1},Bx_{2n+1},t)) \diamond N (Az, Bx_{2n+1},t)$

Taking limit $n\rightarrow\infty$,and using (iii) and (iv)., we get

$N(Az,z,kt). \leq N(Az,z,t) \diamond ½.(1, N(z,z,t)) \diamond .N (Az, z, t)$

$N(Az,z,kt) \leq N(Az,z,t)$

It follows that, from Lemma 2.1, Az= z. Since Az = Sz, therefore z =Az = Sz

Again by (3), we get

$M (x_{2n}, Bz, kt) \geq M(Sx_{2n}, Bz t) * ½ (M(Sx_{2n}, Ax_{2n},t) + M(Bz,Bz,t)) * M(Sx_{2n}, Bz,t)$

Taking limit $n\rightarrow\infty$,and using (iii) and (iv)., we get

$M(z,Bz,kt) \geq M(z,Bz,t) * ½ (M(z,z,t) + 1) * M(z,Bz,t)$

$M(z,Bz,t) \geq M(z, Bz,t)$

Similarly, we get

$N(x_{2n}, Bz, kt) \leq N(Sx_{2n}, Bz\ t) \diamond ½ (N(Sx_{2n}, Ax_{2n},t) + N(Bz,Bz,t)) \diamond N(Sx_{2n}, Bz,t)$

Taking limit $n\to\infty$, and using (iii) and (iv), we get

$N(z,Bz,kt). \leq N(z,Bz,t) \diamond ½ (M(z,z,t) + 1). \diamond N(z,Bz,t)$

$N(z,Bz,t). \leq N(z, Bz,t)$

It follows that, from Lemma2.1, z = Bz, since Bz =Tz, hence it follows that.z = Bz = Tz.

Thus, we have

z= Az = Sz =Bz =Tz, hence from this we conclude that z is a common fixed pint of A, B,S and T.

UNIQUENESS

Let z_1 be another common fixed point of A,B,S and T.

Then, z = Az= Bz =Sz =Tz..and z_1 = Az=Bz=Sz=Tz

Now, by (3),

$M(z,z_1,kt) = M(Az,Bz_1,kt)$

$\geq M(Sz, Tz_1,t) * ½(M(Sz,Az,t) + M(Tz_1, Bz_1, t)) * M(Sz, Bz_1,t)$

$\geq M(z, z_1,t) * ½ (M(z, z_1, t) + M(z_1, z_1,t)) * M(z_1, z_1,t).$

$M(z, z_1,kt) \geq M(z,z_1,t)$

And,

$N(z,z_1,kt) = N(Az,Bz_1,kt)$

$\leq N(Sz, Tz_1,t) \diamond ½(N(Sz,Az,t) + N(Tz_1, Bz_1, t)) \diamond N(Sz, Bz_1,t)$

$\leq N(z, z_1,t) \diamond ½ (N(z, z_1, t) + N(z_1, z_1,t)) \diamond N(z_1, z_1,t).$

Hence, from Lemma 2.1, we get $z = z_1$. This completes the proof

Corollary 3.2

Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric space and let A,B,S and T be a self.. mappings of X satisfying (1) –(3) of theorem 3.1 and there exist $k \in .(0,1)$ such that

$M(Ax, Bx,kt) \geq M(Sx,Ty,t) * M(Ax, Sx,t) * M(By,Ty,t) * M(By, Sx, 2t) * M(Ax, Ty,t)$

$N(Ax, Bx,kt) \leq N(Sx,Ty,t) \diamond N(Ax, Sx,t). \diamond N(By,Ty,t). \diamond N(By, Sx, 2t). \diamond N(Ax, Ty,t)$

For.every $x,y \in X$ and $t > 0$.Then A,B,S and T.have a unique common fixed point in X.

Corollary 3.3

Let (X,M,N,*,◊) be a complete intuitionistic fuzzy metric space and let A,B,S and T be a self. mappings of X satisfying (1) –(3) of theorem 3.1 and there exist k ∈.(0,1) such that

$M (Ax, Bx,kt) \geq M (Sx,Ty,t) * M(Sx, Ax,t) * M(Ax,Ty,t)$

$N(Ax, Bx,kt) \leq N(Sx,Ty,t) \diamond N(Sx, Ax,t).\diamond N(Ax, Ty,t)$

For.every x,y ∈ X and t > 0.Then A,B,S and T.have a unique common fixed point in X.

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