

Analysis of Laminated Plates Subjected to Thermal/Mechanical Loads

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Abstract: Three-dimensional thermo-elastic analysis of laminated plates subjected to uniform mechanical and thermal loads. The analysis is carried out by the finite element method using ANSYS, the most known software in the domain for it, two types of modeling are proposed: the first is modeling using a type of shell element, Shell 99 and the second is an approach based on a type of solid element, Solid 46. Deflections and stress resultants are obtained for symmetric and anti-symmetric cross-ply laminates for simply-supported boundary conditions at different temperatures. The results obtained are compared with the results of the shear deformation theory.

Keywords: Bending, Laminated Composite Plates, Finite Element Method, Thermal/Mechanical Loads.

1. Introduction

Advanced composites are used extensively in aerospace and other structural applications because of their low density, high strength and high stiffness. Their superior strength and stiffness properties are often compromised by the environment to which they are exposed. Moisture and temperature may be distributed through the volume of the structure and may induce residual stresses and extensional strains. These residual stresses and extensional strains may also affect the gross performance of the structure. In particular, the bending characteristics, buckling loads and vibration frequencies can be modified by the presence of moisture, temperature or both. Therefore, to utilize the full potential of advanced composites, it will be necessary to analyze the effects of moisture and temperature in composite structural components.

The vibration characteristics of thick isotropic rectangular plates under an arbitrary state of initial stress were investigated by earlier researchers [1–3]. Yang and Shieh [4] considered the free vibration of anti-symmetric crossply laminates in presence of a non-uniform initial stress, where the effects of transverse shear and rotary inertia were also included. Whitney and Ashton [5] used the classical laminate plate theory to study the effect of hygrothermal environment on the stability, vibration and bending behaviour of laminated composite plates. Pipes et al. [6] presented the

distribution of inplane stresses through the thickness of symmetric laminates subjected to moisture absorption and desorption. Sai Ram and Sinha [7,8] studied the hygrothermal effects on the bending and free vibration behaviour of laminated composite plates.

Mukherjee and Sinha [9] developed the micro-mechanics model to obtain the thermo-mechanical properties for 3D multidirectional composites and used those properties for analyzing the thermostructural problems. The transient analysis of isotropic, orthotropic and layered composite plates using the Newmark's direct integration scheme and considering shear flexible finite element was carried out by Reddy [10, 11].

Meimaris and Day [12] assessed the performance of 20 noded elements for the static and dynamic response analysis of laminated composite plates. To the best of author's knowledge, no results were reported on the transient response of multidirectional composite plates under the action of hygrothermal environment.

In the present work attention is focused primarily on investigating the effects of temperature on the bending characteristics of laminated composite plates using finite element. Using ANSYS, the response of laminated plates exposed to thermal environment. The results obtained are compared with the results of the shear deformation theory.

2. Example problems and numerical results

To test the validity of the finite element analysis technique and to establish its range of applicability, several numerical examples are investigated. Comparisons were made with analytical results obtained by the theory of Reddy. The boundary conditions used for the following examples are specified in terms of the generalized nodal displacements, as shown in Table 1. All the problems studied in this paper are symmetric cross-ply or anti-symmetric cross-ply laminates.

Table 1: Boundary conditions

Type of restraint	Conditions along $x = \text{constant}$	Conditions along $y = \text{constant}$
Simply supported	AB : $u_x=0, u_z=0$ CD : $u_x=0, u_z=0$	BC : $u_z=0, u_y=0$ DA : $u_y=0, u_z=0$

2.1. Bending of multiply square laminated plates

A simply-supported square laminated plate with two, three and four layers of equal thickness is considered here, as shown in Fig. 1. The subjected transverse loading is uniform defined as: $q(x,y) = q_0$.

Table 2. Nondimensionalized deflections in cross-ply squared laminate under uniform loading, for $a/h=10$.

Laminates	Element	Reference [11]	Ansys	Ratio
0/90	Shell 99	0.19469	0.20164	0.96
	Solide 46	0.19469	0.19487	0.99
0/90/90	Shell 99	0.10220	0.10371	0.98
	Solide 46	0.10220	0.10246	0.99
0/90/90/0	Shell 99	0.10251	0.10390	0.98
	Solide 46	0.10251	0.10337	0.99

Table 3. Nondimensionalized deflections in cross-ply squared laminate under uniform loading, for $a/h=100$.

Laminates	Element	Reference [11]	Ansys	Ratio
0/90	Shell 99	0.16980	0.17179	0.98
	Solide 46	0.16980	0.16586	1.02
0/90/90	Shell 99	0.066970	0.06782	0.98
	Solide 46	0.066970	0.066360	1.00
0/90/90/0	Shell 99	0.068331	0.069192	0.98
	Solide 46	0.068331	0.067594	1.00

Where L signifies the fiber direction, T the transverse direction. To validate the accuracy of the present method, the results are compared with the shear deformation plate theory of Reddy. The predicted results of displacements are listed in Tables 2 and 3 for different span-thickness ratios ($S = a/h = 10, 100$). Generally the predictions obtained by the present elements (Shell99 and Solid46) are in good agreement with the analytical solution.

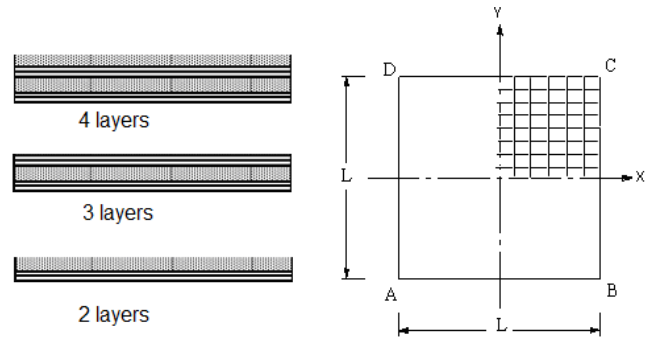


Figure 1: Stratification of a composite plate to 2, 3 and 4 layers.

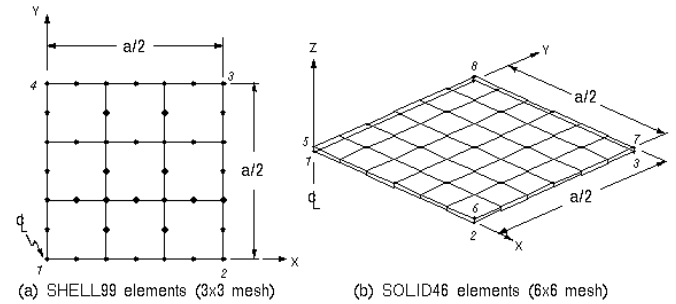


Figure 2: Representative finite element model (Shell 99 and Solid 46).

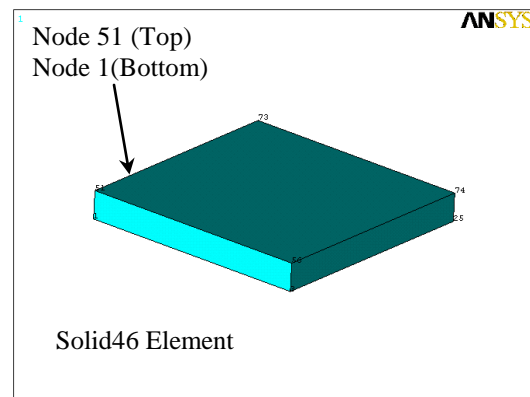


Figure 3: Nodes at the center of the plate modeled with Solid 46

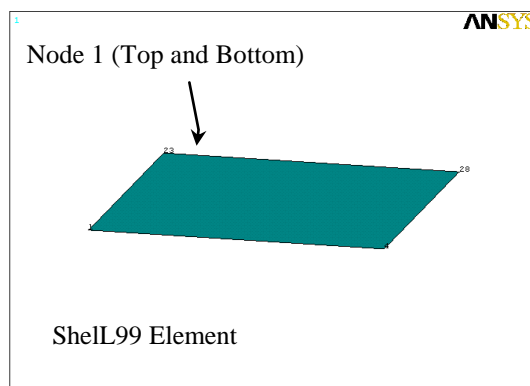


Figure 4: Nodes at the center of the plate modeled with Shell 99.

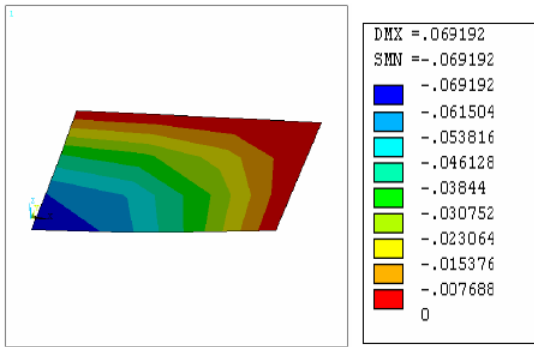


Figure 5: Variation of deflection for a cross-ply laminate, through the thickness of the plate using Shell 99 Element.

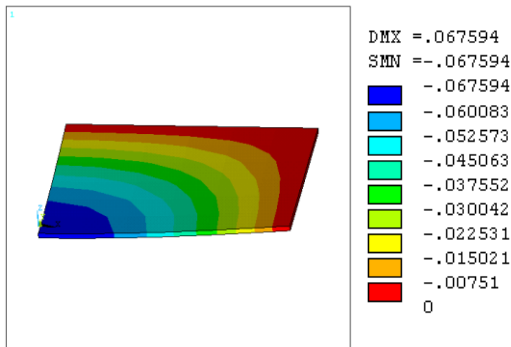


Figure 6: Variation of deflection for a cross-ply laminate, through the thickness of the plate using Solid 46 Element.

The node 1 is the node to the center of the plate when the plate is modeled with by the Shell 99 element (figure 4). Node 1 and 51 are nodes to the center of the plate when the plate is modeled with Solid 46 elements (figure 3). In addition, Fig. 5-6 shows the comparison of the deflections computed by Shell element 99 and Solid element 46, respectively, with stacking sequences (0/90/90/0). We note that results obtained with Shell 99, point of view displacement, are nearly similar those obtained with Solid 46.

2.2. Effect of temperature

Firstly, the longitudinal stresses to the center of the plate are calculated for cross-ply laminates plates, with stacking sequences (0/90/90/0) under mechanical and uniform temperature rise, and are plotted in Fig. 7. Fig. 7 shows the longitudinal stresses vs the thickness to span ratio a/h for $T=25^{\circ}\text{C}$. It is seen that the longitudinal stresses increases monotonically as the relative thickness a/h increases. The values of the longitudinal stresses calculated by using the Solid 46 are lower than those calculated by using the Shell99, especially for thin plates.

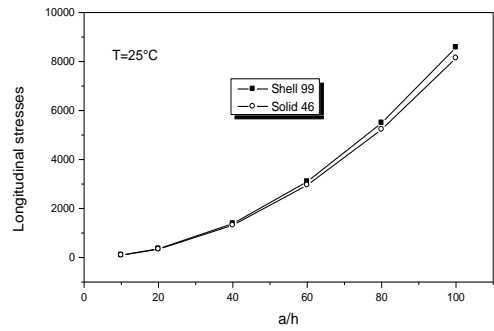


Figure 7: Longitudinal stresses to the center of the plate under thermal–mechanical load vs relative thickness of the plate ($T=25^{\circ}\text{C}$).

The variations of the transverse stresses to the center of the plate under thermo-mechanical loading are plotted in Figs. 8-10. In Figs. 8 -10, the transverse stresses to the center of the plate increases, when the geometric parameter a/h is increased. On the other hand, the transverse stresses to the center of the plate decreases, when the temperature is increased.

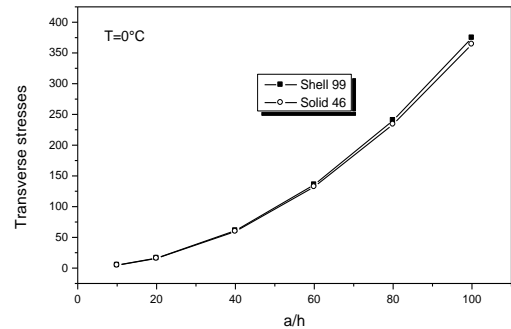


Figure 8: Transverse stresses to the center of the plate under thermal–mechanical load vs relative thickness of the plate ($T=0^{\circ}\text{C}$).

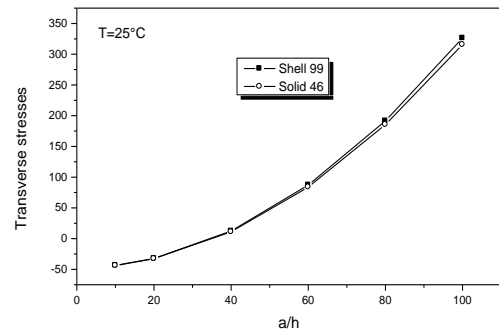


Figure 9: Transverse stresses to the center of the plate under thermal–mechanical load vs relative thickness of the plate ($T=25^{\circ}\text{C}$).

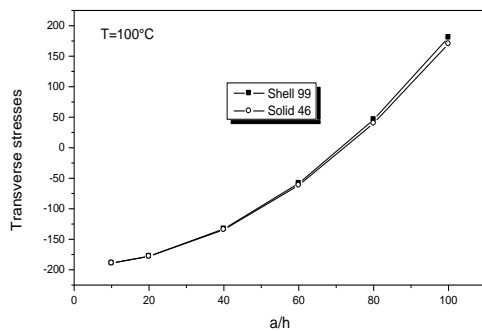


Figure 10: Transverse stresses to the center of the plate under thermal–mechanical load vs relative thickness of the plate ($T=100^{\circ}\text{C}$).

3. Conclusions

Effects of temperature on the bending analysis of cross-ply square plates have been performed using finite element method. For verification of the present analysis, several numerical examples are solved and compared with the third-order shear deformation plate theory and is found to be in good agreement. Based on the results reported here of cross-ply laminate plates, the following conclusions may be drawn.

- 1- The finite element method results are in good agreement with those obtained with the theory of Reddy [11].
- 2- Deflections are nearly identical for the two types of elements: Shell 99 and Solid 46.
- 3- The stresses for Solid 46 are generally lower than the corresponding values for Shell99, especially for thin plates.
- 4- The distribution of stresses depends on the type of stacking. In addition, longitudinal stresses are higher than that the transverse stresses.
- 5- The longitudinal stresses and transverse stresses are increased when the plate aspect ratio or the thickness to span ratio increases. However, when the temperature increases, the longitudinal constraints increase. While the transverse constraints decrease.

Finally, this method of modelling for quick solutions opens new avenues for the conception of structural composites making it possible to determine the necessary stresses instantaneously.

References

- [1] Herrmann G, Armenakas AE. Vibrations and stability of plates under initial stress. *Trans Am Soc Civil Eng* 1962;127:458–91.
- [2] Brunelle EJ, Robertson SR. Initially stressed mindlin plates. *AIAA J* 1974;12:1036–45.
- [3] Brunelle EJ, Robertson SR. Vibrations of an initially stressed thick plate. *J Sound Vibrat* 1976;45:406–16.
- [4] Yang IH, Shieh JA. Vibrations of initially stressed thick, rectangular, orthotropic plates. *J Sound Vibrat* 1987;119(3):545–58.
- [5] Whitney JM, Ashton JE. Effect of environment on the elastic response of layered composite plates. *AIAA J* 1971;9:1708–13.

- [6] Pipes RB, Vinson JR, Chou TW. On the hygrothermal response of laminated composite systems. *J Compos Mater* 1976;10:129–48.
- [7] Sai Ram KS, Sinha PK. Hygrothermal effects on the bending characteristics of laminated composite plates. *Comput Struct* 1991;40(4):1009–15.
- [8] Sai Ram KS, Sinha PK. Hygrothermal effects on the free vibration of laminated composite plates. *J Sound Vibrat* 1992;158(1):133–48.
- [9] Mukherjee N, Sinha PK. Three-dimensional Thermo structural analysis of multidirectional fibrous composite plates. *Compos Struct* 1994;28(3):333–46.
- [10] Reddy JN. Dynamic (transient) analysis of layered anisotropic composite material plates. *Int J Numer Methods Eng* 1983; 19:237–55.
- [11] Reddy J.N., *Mechanics of laminated composite plate: theory and analysis*, New York: CRC Press; 1997
- [12] Meimaris C, Day JD. Dynamic response of laminated anisotropic plates. *Comput Struct* 1995;55(2):269–78.
- [13] Bouazza M , Hammadi F and Khadir M. Bending Analysis of Symmetrically Laminated Plates. *Leonardo Journal of Sciences*. Issue 16, January-June 2010 p. 105-116

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