Predominance of Procedural Knowledge and Between-Operation Interference as Deduced from Fraction Errors of Preservice Teachers

Asia Pacific Journal of Multidisciplinary Research
Vol. 4 No.4, 75-79
November 2016
P-ISSN 2350-7756
E-ISSN 2350-8442
www.apjmr.com

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Date Received: July 8, 2016; Date Revised: September 5, 2016

Abstract - *The study of fractions in Philippine mathematics curriculum starts as early as first grade. In spite of the regular rehearsal of this mathematical topic through secondary school level, many students reach college without showing adequate skills in fraction. This study determined the performance and analyzed the errors of preservice teachers in dealing with fractions. Findings revealed that preservice teachers' performance in solving fractions reached an unacceptable level. Prevalent errors were demonstrated when adding dissimilar fractions, adding a mixed number and a fraction, and multiplying a mixed number by a fraction, because the dominant procedural knowledge in fraction addition interferes with their knowledge of fraction multiplication, and vice versa. Moreover, preservice teachers exhibit low level of content knowledge of fractions as shown in their inability to add common fractions and their failure to translate mixed numbers into equivalent fractions.*

Keywords: *learning, mathematics, fractions, preservice teachers, procedural*

INTRODUCTION

Teaching and learning fractions has been an ongoing issue in education. Actually, it is a prevalent notion that fractions are among the most difficult concepts that young learners learn in school. Some believed that learning fractions is perhaps one of the most severe hindrances to the development of mathematical dexterity [1]. Researchers and scholars recently have uncovered various determinants of students' learning disabilities in fractions. Specifically, some scholars suggested that the hindrances preventing the learners from developing functional knowledge of fractions are attributed to the structure of fractions themselves or to the methods of teaching that teachers employ [1][2]. At present, researchers agree to the idea that one of the prevalent determinants of difficulties in teaching and learning fraction lies in the widely accepted truth that fractions are composed of many constructs [3].

Schools teach fractions in different forms, being a subset of the set of rational numbers. Symbolically, the set of rational numbers is defined by expressing a ratio of two integers, where the denominator must not be equal to zero. Examples are fractions, negative integers, whole numbers, mixed numbers, and decimals. In concrete form, a rational number is a comparison of a portion of a cake with the whole cake, a ratio of books to students, an amount of money to be divided by a number of people, etc. Under different contexts, rational numbers have different qualities. Rational numbers have different constructs, i.e. part-whole, quotients, measures, ratio, rate, and operators [1]. This is an improvement of Kieren's [3] original proposal that there are four subconstructs of fractions: measure, ratio, quotient, and operator.

In the Philippine basic education curriculum rational numbers or fractions are taught as early as Grade I [4]. The sequential mathematics curriculum develops the students' knowledge of fractions from simple visualization, modeling, and representation of "halves" and "fourths" in first grade to more advanced evaluation of complex rational expressions in high school algebra. However, despite this long exposure of students to the concepts and principles of fractions and their operations, many high school graduates reach the tertiary level without evidence of mastery.

According to several researches, preservice teachers' understanding of fraction content knowledge is weak [5] [6] [7]. There has been evidence that preservice teachers find it difficult to conceptualize fractions [8] and to explain fractions to children and why computation procedures work [9], and cannot operate fractions correctly, even if their answers are correct [10].

Since preservice teachers are expected to teach few years from now in elementary mathematics, it is quite alarming that their poor understanding of fractions can cause serious problems on their pupils' learning. Hence, this concern must be dealt with significantly and should be addressed to avoid future problems.

Before solving this issue, it is imperative that identification of the common errors in understanding rational numbers be done first. Thus, this study tried to diagnose preservice teachers' errors in dealing with rational numbers, which will serve as basis for elementary and secondary teachers and tertiary professors to implement appropriate interventions or remediation strategies.

A state higher education institution in southern Bicol, Philippines offers two teacher education programs, namely: Bachelor in Secondary Education (BSEd) and Bachelor in Elementary Education (BEEd). Enrollees in these programs passed the college entrance test and interviews. This admission process ensures that students must be highly qualified to undergo teacher education trainings for four years. However, these diverse preservice teachers (teacher education students) exhibit certain learning difficulties, especially in mathematics. This study tried to find out some of these difficulties, focusing only on the basic skills in understanding rational numbers or fractions.

The objective of this study was to determine the preservice teachers' errors in performing rational number operations, including their conceptual understanding of representing fractions. Three questions are posted: 1) What is the level of performance of preservice teachers in operating fractions? 2) Which of the errors exhibited by preservice teachers in dealing with fraction operations are more prevalent? and 3) What are the implications for teaching and learning fractions?

The result of this study will be helpful in providing baseline data about mathematics learning in the elementary, secondary, and tertiary levels for schools to implement their appropriate remedial strategies. Elementary and secondary teachers will benefit from the findings by gaining consciousness about certain errors in both conceptual and procedural knowledge of fractions demonstrated by their graduates. To prevent further occurrence of the same problem in the future, they can reflect on the effectiveness of customary instructional strategies in their respective schools and devise suitable ways in addressing the cause of the problem. Tertiary educators will consider this study as a useful source of information in understanding how their students process knowledge erroneously and as a basis for conducting further studies on the more effective strategies in teaching fractions. The products of these subsequent researches will help the preservice teachers improve their knowledge, both conceptual and pedagogical, since they will be teaching in elementary schools in the future.

METHODS

Participants in this study consisted of 38 preservice teachers enrolled in their first of four-year elementary teacher education program in a state-run college in Bicol, Philippines. The participants were selected from the three class sections using systematic random sampling. Among these 38 participants, there were about 97% of the participants younger than age 25, approximately 76% were female, and about 87% graduated from public secondary schools.

In this descriptive research, the instrument used was a diagnostic pretest with eight multiple-choice items on adding and multiplying fractions. Each item task was devised to identify certain types of error. Item 1 is intended for identifying errors in adding dissimilar and common fractions, Item 2 in adding dissimilar and uncommon fractions, Item 3 in adding a mixed number and a fraction which are similar and common, Item 4 in adding similar fractions, Item 5 in adding a mixed number and a fraction which are dissimilar and uncommon, Item 6 in multiplying common fractions, Item 7 in multiplying a whole number by a fraction, and Item 8 in multiplying a mixed number by a fraction which are common.

An item exemplified as " 1/2 + 3/4= A. 4/6 B. 2/3 C. 10/8 D. 5/4 " is deemed as a procedural item through which this study, basing on participants' responses, can determine the errors in performing addition of two dissimilar fractions (or "common fractions", because these fractions, such as ½, ¾, are commonly used quantities of weighing units, usually in kilograms, in the local marketplaces). The choices are designed such that the wrong ones have their corresponding distracting properties or error categories. For example, choice A means an error in which a student performs addition by adding the numerators and denominators; choice B means a procedural error similar to choice A but reduced to the lowest term; choice C means an error in reducing the answer to the lowest term as found in choice D (the correct answer). (It was emphasized in the test direction that answers must be reduced to the lowest terms when necessary.) Similar multiple-choice design also applies to items 1 through 8.

RESULTS AND DISCUSSIONS

Two types of analyses were conducted in this study. First, the performance level of the participating preservice teachers in adding and multiplying rational numbers in the eight component items tested was determined by getting the range, mean score, and the percentage of participants whose scores are above 4 marks or 50%. Second, the more prevalent errors exhibited by the participants were identified by determining those wrong choices in which 50% or more responses occurred. An analysis of items with the percentage of occurrences in each response is shown in Table 1.

Table 1. ***Percentages of Occurrences of Responses in the Item Choices***

Item	Choice			
	A	**B**	**C**	**D**
1	7.89	71.43	2.63	23.68*
2	21.05*	5.26	73.68	0.00
3	63.16*	34.21	0.00	2.63
4	5.26	39.47	52.63*	2.63
5	52.63	26.32*	13.16	5.26
6	5.26	36.84*	21.05	36.84
7	7.89	34.21	52.63*	5.26
8	78.95	7.89	7.89	5.26*

Note: Figure with asterisk is percentage of correct response

From the table, one observes that the percentage of errors in eight items tested is relatively greater than the percentage of correct responses, and this explains why the performance of the participants in operating rational numbers was very low. The highest possible score was 8. However, from the 38 sample participants only 8 of them (21.05%) obtained 4 marks (50%) above, with the highest score obtained as 8 and the lowest 1. The mean score was 2.61 and the mean percentage score was 32.62%.

Out of the eight items tested, there were five items in which more prevalent errors were identified, namely: (Item 1) adding common fractions, (Item 2) adding dissimilar fractions, (Item 5) adding a mixed number and a proper fraction, (Item 6) multiplying dissimilar fractions, and (Item 8) multiplying a mixed number by a proper fraction. The percentages of the more prevalent wrong choices identified in each item and the correct responses are shown in Table 2.

Table 2. ***Percentages of Errors and Correct Responses among the Five Items Identified to Have More Prevalent Wrong Choices***

Item No.	Task	More prevalent wrong choices	% of error occurrences	% of correct responses
1	1/2 + 3/4	B. 2/3	71.43	23.68
2	1/3 + 3/5	C. 1/2	73.68	21.05
5	2 1/3 + 2/7	A. 2 3/10	52.63	26.32
6	1/2 x 3/4	D. 2/3	36.84	36.84
8	1 1/4 x 1/2	A. 1 1/8	78.95	5.26

As revealed in the table, the participants' task in Items 1 and 2 was to add two dissimilar fractions. The highest percentage of these participants erroneously performed addition by adding numerators and denominators and reducing the sum to its lowest term. Here one will notice that the participants confuse the procedure in fraction multiplication with that of fraction addition. Furthermore, Item 1 (though deemed as a procedural item) involves two dissimilar fractions frequently used in everyday life, as in weighing scales in the market, which can also be considered as conceptual – that is why they are referred to here as common fractions. However, the greatest percentage of participants merely relied on procedural knowledge to find an answer. Only about 24% of them applied the conceptual knowledge of fractions to arrive at the correct answer by relating the task to meaningful life situations.

In Item 5 the task was to add a mixed number and a proper fraction, in which the highest percentage of participants made an error by only adding the fractional parts, again adding numerators and denominators. The participants failed to express the mixed number into improper fraction. The reader will note that the participants lacked the knowledge of constructs of mixed numbers and depended their manipulation heavily upon fractional constructs, ignoring the whole and operating only the fractions.

The table also shows that in Item 6 the percentages of errors and correct responses are the same. As such, the error still counts because it is an indication that the participants had difficulty in choosing the appropriate procedure in performing multiplication of two fractions. For the second time as observed, interference in procedural knowledge takes place in this error; that is, the wrong method used in the first two items on addition also applies on multiplication, getting an answer by adding the numerators and denominators instead of multiplying them.

The highest percentage of errors was exhibited in Item 8 task in which the participants multiplied a mixed number by a fraction. Similar to the error in Item 5, the participants had isolated the whole part of the multiplicand and multiplied only those fraction parts. Again, the transformation of a mixed number into an improper fraction did not take place before multiplication.

The difficulties identified above show some signs of dominant procedural knowledge among preservice teachers and their interfering concepts about fraction operations. This phenomenon can be described by *integrated theory of numerical development* [11], which views interference from whole number knowledge as only one of a number of sources of difficulty in understanding fraction concepts. For example, despite whole number errors, such as 1/3 + 2/5 = 3/8 being prevalent, mixing up with other fraction operations, such as confusion between fraction addition and fraction multiplication apparent in 1/5*2/5 = 2/5, can be even more prevalent [12].

In some perspectives, these errors are viewed not as difficulty in learning fractions but as strategies used by learners to answer fraction problems. In a similar study with 6th and 8th graders, Siegler, Thompson, & Schneider [11] classified learners as using four main fraction arithmetic strategies: First, *correct strategies* which involved use of a method that generated the right answer if implemented correctly. Second, *independent whole numbers strategies* which involved performing the arithmetic operation on the numerators and denominators separately, as if they were independent whole numbers (e.g., 1/2 + 3/4 = 4/6). Third, w*rong fractions operation strategies* which involved considering the rule in one fraction operation as applicable to the rule in another fraction operation (e.g., retaining the common denominator on a multiplication problem, as in 4/5 * 2/5 = 8/5). And last, *unknown strategy* where students showed no basis for solving a fraction problem.

CONCLUSIONS

Overall performance of the preservice teachers suggests that their knowledge of rational numbers is very low. A considerable number of preservice teachers exhibited errors in adding dissimilar fractions. They tend to manipulate symbols in a linear manner; that is, numerator to numerator and denominator to denominator. A possible reason for this phenomenon is the “between-operation interference” in which preservice teachers’ knowledge of fraction multiplication procedures impedes with their ability to carry out fraction addition or vice versa.

Nevertheless, a significant number of preservice teachers demonstrated less prevalent errors in adding similar fractions. They also reduced fractions to the lowest terms with ease. Therefore, conceptual knowledge of similar fractions and reduction to lowest terms is already established.

Preservice teachers also lacked sufficient conceptual knowledge of equivalent fractions. This is apparent in their failure to express mixed numbers into improper fractions, an essential requirement before performing either operation. Furthermore, it becomes visible that preservice teachers’ procedural knowledge predominates over their conceptual knowledge [13]. They are far more concerned about finding an answer through rigid procedure than establishing link to meaningful situations. Common fraction items were included but these were merely viewed as abstract numerical symbols rather than representations of concrete examples. Perhaps preservice teachers’ poor understanding of the different constructs of rational numbers has something to do with this difficulty.

The results of this study also imply that there is a need for students to gain mastery of the processes involved in performing operations on dissimilar fractions. These students represented by the preservice teachers had not acquired as much knowledge of fractions in the elementary and secondary schooling when they entered college.

RECOMMENDATIONS

In this study, we identified the more prevalent errors in carrying out fraction addition and multiplication. Preservice teachers performed better with similar proper fractions than with dissimilar fractions and mixed numbers. We also found out that procedural knowledge interference and dominance had a role to play in this phenomenon. From the findings it implicated a stipulation for improving the quality of teaching and learning fractions in the elementary and secondary levels. Basic education teachers should develop or adopt more effective strategies (e.g., use of manipulatives, mnemonics, etc.) to help their students improve memory retention and enrich conceptual understanding of fractions. To the tertiary level educators, it is suggested that preservice teachers be given more curative interventions and trainings in mathematics to prepare them for teaching in elementary schools four or five years later. Further research on the use of generalized fraction algorithms or generalized fraction symbols for rational numbers can be tested to address the difficulties in processing and applying fraction operations.

Furthermore, there still are some limitations of this study. First is the small number of sample participants. The researchers only tested the participants enrolled in only one elementary teacher education program in the region, thus the result may not be used to generalize on the fraction knowledge of all preservice teachers in the region. Subsequent study with larger sample size and under various learning contexts is recommended to validate the results of this study. Second, the study used a multiple-choice test in which the choices are limited only to the possible responses that participants would make as predicted by the researchers. An open-ended test might be a good design to ensure that the participants have authentic responses. Modeling and problem-solving items should have been tested to identify the preservice teachers' prior conceptual and procedural knowledge of fractions.

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ACKNOWLEDGEMENT

I thank Sherwin Balbuena for data collection and analyses, the elementary teacher education students for participating in this survey, and the Research and Development Office of the DEBESMSCAT for the support.